

Contents

Acknowledgement	iii
Introduction	1
1 Preliminaries	9
1.1 Notations and Conventions	9
1.2 Group Theory	11
1.3 Function Space Theory	12
1.3.1 Banach, Hilbert and Fréchet Spaces	13
1.3.2 Function Spaces on Groups	14
1.4 Representation Theory	19
1.5 Coverings	20
1.6 Sequence Spaces	22
1.7 Continuous Frames	24
1.8 Kernel Spaces	27
1.9 Discretization of Function Spaces	30
2 Classic Coorbit Theory	33
2.1 Coorbit Spaces	33
2.2 Discretization	39
2.2.1 Atomic Decompositions	39
2.2.2 Banach Frames	40
2.3 Examples	41
2.3.1 Homogeneous Besov Spaces	41
2.3.2 Homogeneous Shearlet Coorbit Spaces	45
3 Coorbit Theory with Non-Integrable Kernel	51
3.1 Coorbit Spaces	52
3.1.1 Kernel in a Fréchet Space	52
3.1.2 Non-Integrable Kernel	55
3.2 Discretization	59
3.2.1 An Assumption on the Kernel	62
3.2.2 Atomic Decompositions	65
3.2.3 An Example: Paley-Wiener Spaces	76
3.3 Obstructions to Discretization	81
3.3.1 Necessary Assumptions on the Kernel	82
3.3.2 Revisiting Paley-Wiener Spaces I	86
3.4 Improved Discretization Results Under Additional Assumptions	89
3.4.1 Banach Frames	91

3.4.2	Atomic Decompositions	96
3.4.3	Revisiting Paley-Wiener Spaces II	102
4	Generalized Coorbit Theory	107
4.1	Coorbit Spaces	107
4.2	Discretization	112
4.2.1	Atomic Decompositions	113
4.2.2	Banach Frames	114
4.3	An Example: Inhomogeneous Besov Spaces	115
5	Generalized Coorbit Theory with Non-Integrable Kernel	121
5.1	Coorbit Spaces	122
5.2	Discretization	129
5.2.1	An Assumption on the Kernel	131
5.2.2	Atomic Decomposition	134
5.3	Obstruction to Discretization	143
5.4	Improved Discretization Results Under Additional Assumptions	145
5.4.1	Banach Frames	147
5.4.2	Atomic Decompositions	151
5.5	An Example: Inhomogeneous Shearlet Coorbit Spaces	155
5.5.1	Inhomogeneous Shearlet Frame	156
5.5.2	Conditions on the Reproducing Kernel	160
5.5.3	Inhomogeneous Shearlet Coorbit Spaces	177
5.5.4	Conditions on the Oscillation	177
5.5.5	Discretization	192
6	Conclusion and Outlook	195
Zusammenfassung		201
Bibliography		207