Abstract

This thesis presents methods that use model reduction to reduce the complexity of a dynamical model to a practicable level. In addition to procedures for model reduction, we introduce algorithms for bounding the error between the detailed and reduced model as well as predictive control using reduced models for prediction. All approaches provide guarantees either for the reduced model or while using the reduced model. At the same time, we mind the computational tractability of the procedures. The applicability of the proposed methods is demonstrated by means of a nonisothermal tubular chemical reactor.

The first contribution is a model reduction procedure to approximate the input-output map of continuous-time nonlinear ordinary differential equations. The reduced model is parameterized with the observability normal form. Using a sample of simulated input-output trajectories, the parameters are computed by convex optimization. A low complexity of the functional expression is promoted by sparsity enhancing ℓ_1-minimization. In addition, we extend the method to preserve the location and local exponential stability of multiple steady states.

Furthermore, we improve an existing a-posteriori bound of the model reduction error for linear models. The generalized error bound is given by an asymptotically stable scalar ordinary differential equation, which results, in general, in a considerably tighter bound with a comparable computational demand.

Finally, we propose a novel model predictive control scheme using reduced models for linear time-invariant systems. This model predictive control scheme uses the developed bound of the model reduction error to guarantee asymptotic stability as well as satisfaction of hard input and state constraints despite the error between the reduced model used for the prediction and the high-dimensional plant. Moreover, we show that the proposed model predictive control scheme minimizes the infinite horizon cost functional for the plant for a common choice of design parameters. In this case, the proposed scheme ensures an upper bound for the cost functional value of the closed loop with the detailed plant model despite using a reduced model for the prediction. For discrete-time plants we show that the optimization problem of the model predictive control scheme can be reformulated as a second-order cone program.

Deutsche Kurzfassung

In dieser Arbeit werden verschiedene Methoden entwickelt, die durch den Einsatz von Modellreduktion die Komplexität eines dynamischen Modells beherrschbar machen. Neben einem Verfahren für die Modellreduktion stellen wir sowohl einen Algorithmus für die Beschränkung des Fehlers zwischen dem detaillierten und dem reduzierten Modell als auch eine Methode für die modellprädiktive Regelung basierend auf einem reduzierten Modell vor. Für alle Ansätze leiten wir Garantien entweder für das reduzierte Modell oder für die Verwendung des reduzierten Modells her. Gleichzeitig achten wir darauf, dass die entwickelten Algorithmen mit derzeitigen Computern gelöst werden können. Die Anwendbarkeit der Methoden wird anhand eines nicht-isothermen chemischen Rohrreaktors demonstriert.

Als erste Methode schlagen wir ein Verfahren vor, das das Ein-/Ausgangsverhalten einer zeitkontinuierlichen nichtlinearen gewöhnlichen Differentialgleichung approximiert. Das reduzierte Modell wird mit der Beobachternormalform parametrisiert. Basierend auf einer Stichprobe von simulierten Ein-/Ausgangstrajektorien sind die Parameter durch die Lösung eines konvexen Optimierungsproblems bestimmt. Eine geringe Komplexität der Funktion des reduzierten Modells wird durch eine ℓ_1-Minimierung erreicht. Darüber hinaus erweitern wir die Methode, so dass die Position und lokale exponentielle Stabilität der stationären Zustände erhalten bleiben.

Außerdem verbessern wir eine bestehende a-posteriori Schranke für den Modellreduktionsfehler für lineare Modelle. Die verallgemeinerte Fehlerschranke wird durch eine asymptotisch stabile skalare gewöhnliche Differentialgleichung beschrieben und führt, im Allgemeinen, zu einer deutlich genaueren Schranke mit einem vergleichbaren Rechenaufwand.

Weiterhin stellen wir ein neuartiges modellprädiktives Regelungsverfahren basierend auf reduzierten Prädiktionsmodellen für lineare zeitinvariante Systeme vor. Das entwickelte Verfahren verwendet die verallgemeinerte Schranke für den Modellreduktionsfehler, um asymptotische Stabilität sowie die Einhaltung von Eingangs- und Zustandsbeschränkungen trotz der Abweichung zwischen dem reduzierten Prädiktionsmodell und dem detaillierten Streckenmodell zu garantieren. Zudem beweisen wir für eine übliche Wahl von Entwurfsparametern, dass das modellprädiktive Regelungsverfahren, trotz des reduzierten Prädiktionsmodells, das Kostenfunktional für die Strecke minimiert und eine obere Schranke für den Wert des Kostenfunktionals des geschlossenen Kreises mit der Strecke gewährleistet. Für zeitdiskrete Strecken zeigen wir, dass das Optimierungsproblem des modellprädiktiven Regelungsverfahrens in ein Second-Order Cone Programm umgeformt werden kann.

Chapter 1

Introduction

1.1 Motivation

In the industrial environment, model predictive control (MPC) is getting more and more popular. Two reasons are the increasing computational power and more demanding performance requirements for controlled systems. The latter results in more difficult control task, which render the well-known proportional-integral-derivative control inappropriate.

MPC uses a model of the plant for the prediction. Based on a specified cost criterion, the control inputs are computed such that this cost criterion is minimal for the predicted behavior. To ensure robustness with respect to a model plant mismatch and measurement noise a feedback is introduced by measuring the current state and recomputing the optimal control inputs repeatedly. MPC is well studied in the academic environment leading to important theoretical results including rigorous stability proofs. Furthermore, MPC is successfully applied in numerous industries ranging from, e.g., process industry to electrical power conversion and automotive industry. This success is based on the advantages of MPC. MPC can be applied to many plants including nonlinear models and multiple inputs. Moreover, hard input as well as state constraints can be easily incorporated. Furthermore, a time domain performance criterion is approximately minimized.

For MPC, typically the solution of an optimization problem in real time is required. Hence, a computationally tractable prediction is essential for the application of MPC. Moreover, the requirements for controlled systems are continuously increasing, which results in more complex models. For example, the required fast adaption of production targets in the chemical industry to the market demand necessitates more frequent load changes [Agar et al., 2017], which calls for models that represent a whole operation regime and not only the area around one particular steady state. Or the desired increase in energy efficiency of chemical processes to reduce the CO_2 footprint results in more couplings between the reactors, e.g., by using the waste heat of one reactor for another reaction [Agar et al., 2017]. Another example is the increasing share of renewable energies in the power grid, which leads to an increase in complexity of the controlled system. This increase in model complexity makes the solution of the optimization problem in real time computationally challenging.

One remedy described in numerous articles is the use of reduced models for the prediction. Model reduction algorithms for nonlinear systems are much less developed than for linear systems. Furthermore, the complexity of the functional expression of the ordinary differential equation (ODE) is limited by the model order for linear time invariant (LTI) models but not for nonlinear models (compare with [Rewieński, 2003, Section 2.3]). Hence, we cover methods that reduce the order and complexity of nonlinear models.

Due to the mismatch between the reduced model and the plant, constraint satisfaction and stability of the closed loop are not guaranteed any more. To recover the important guarantees of MPC, it is essential to take the error of the model reduction into account. An intermediate step towards an MPC approach with robustness against the model reduction error is a bound for the error between the original and the reduced model while simulating the reduced model. There are only few results for MPC using reduced models that provide guarantees for the closed loop with the plant even for LTI systems. Hence, a bound for the model reduction error and an MPC approach using reduced models are developed for LTI systems.

Altogether, in this thesis we focus on three research directions. First, model reduction procedures that result in models with a reduced order and reduced complexity. Second, bounds for the model reduction error that can be utilized during simulation. Third, MPC using reduced models with a guaranteed behavior of the closed loop with the original model. For the last research direction, the fields model reduction, error bounds, and MPC have to be combined as visualized in Figure 1.1.

A connection between all methods developed in this thesis are *reduced models* that are either computed by model reduction methods or used in MPC schemes. In addition to the connection in form of reduced models, the methods proposed in this thesis are motivated by two shared features, which are, inter alia, important for the applicability in industry: *computational efficiency* and rigorous *guarantees*. For the model reduction methods, the aspect of computational complexity appears twice: the procedure needs to be computationally tractable and the resulting models possess a low complexity. Guarantees for the model reduction are, e.g., preservation of stability. By simply using reduced models it is unclear how the approximation error deteriorates the application at hand. Hence, it is crucial to provide rigorous bounds for the error between the original and the reduced model. For these bounds, it is important to establish a good compromise between a low computational complexity and a small conservatism. For MPC using reduced models the common guarantees of nominal MPC such as constraint satisfaction, bounds for the cost functional value, and asymptotic stability should be recovered. The computational efficiency shows up for MPC by using reduced models, error bounds with a low computational burden, and the way these ingredients are combined in the optimization problem.

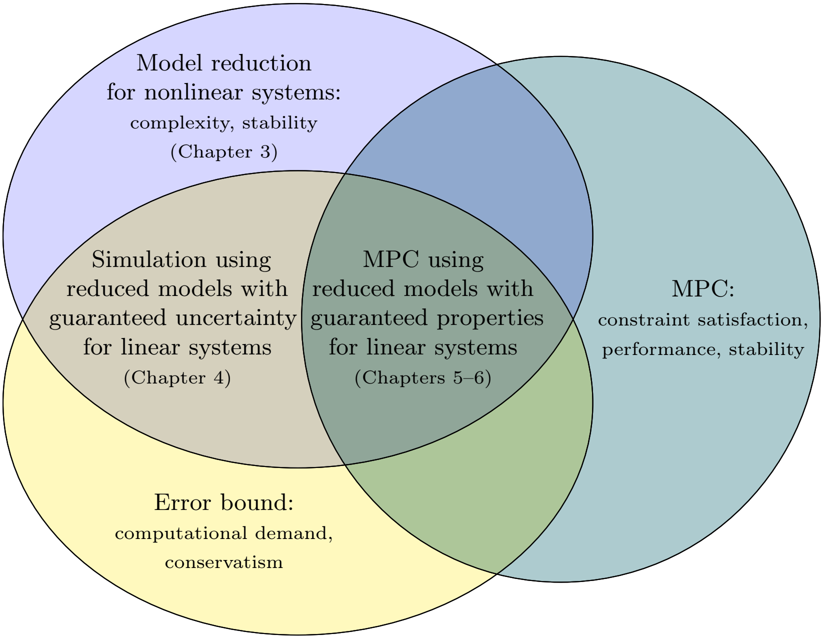

Figure 1.1: Research directions and aspects from each research field covered in this thesis.

1.2 Overview of the Research Area

In the last section, the three research areas of this thesis have been motivated:

i) model reduction

ii) error bounds for simulation using reduced models

iii) MPC using reduced models

In this section, we give a brief overview of related work in this three research areas and challenges that are addressed in this thesis. Hereby, we provide the basis for the contributions of this thesis, which are stated in Section 1.3.

An introduction into the three research areas is given in Chapter 2. If few prior knowledge of one research area exist, it is beneficial to read the corresponding section of Chapter 2 before this section.

Small parts of Section 1.2.2 and 1.2.3 have already been presented in [Löhning et al., 2014].

1.2.1 Model Reduction

In this section, we give a brief overview of the field of model reduction. For a more thorough introduction we refer for linear model reduction to [Antoulas, 2005a,b; Baur et al., 2014; Benner et al., 2015, 2017; Besselink et al., 2013; Obinata and Anderson, 2001] and for nonlinear model reduction to [Baur et al., 2014; Marquardt, 2002; Đukić and Sarić, 2012].

For linear systems, well-developed methods for model reduction are described in the literature.

The well-known method of balanced truncation introduced in [Moore, 1981; Mullis and Roberts, 1976] uses a state space realization in which each state is as well controllable as it is observable. Then the states that are simultaneously difficult to observe and difficult to control are truncated. A survey of model reduction by balanced truncation can be found in [Gugercin and Antoulas, 2004]. Under mild assumptions, balanced truncation preserves stability [Pernebo and Silverman, 1982] and an error bound [Enns, 1984; Glover, 1984] exists. Furthermore, it has been generalized to unstable [Kenney and Hewer, 1987; Therapos, 1989; Zhou et al., 1999] systems, passivity preservation [Desai and Pal, 1984], and frequency-weighted balanced truncation [Enns, 1984]. Closely related to balanced truncation is the Hankel norm approximation developed by Glover [1984].

Another established method is moment matching, which finds a reduced model that matches the first derivatives of the transfer function at a certain number of points. In the 1990s, numerically efficient algorithms have been developed for moment matching [Feldmann and Freund, 1995; Grimme, 1997; Odabasioglu et al., 1998]. Algorithms are available for preserving passivity [Odabasioglu et al., 1998]. Further details can be found, e.g., in [Benner et al., 2017, Chapter 7] and [Antoulas, 2005b; Freund, 2003].

A frequently used model reduction method is modal truncation [Antoulas, 2005a; Varga, 1995]. Modal truncation utilizes the eigenvalue decomposition of the state matrix and preserves the n_R dominant poles in the reduced model. Several measures for the dominance of poles exist, e.g., distance from the imaginary axis or the Hankel singular values of the subsystems corresponding to each block of the state matrix [Varga, 1995]. Another method is the proper orthogonal decomposition (POD) [Holmes et al., 1996; Lumley, 1967; Rathinam and Petzold, 2003; Sirovich, 1987], which uses state trajectories to compute a subspace on which the detailed model model is projected. For more details of the POD we refer to Section 2.1.2. Both, the modal truncation as well as the POD are applicable to high-dimensional dynamical system.

Optimization based approaches to model reduction connected to the POD are proposed in [Bui-Thanh et al., 2007; Kunisch and Volkwein, 2008]. Bui-Thanh et al. take the ODE of the reduced model into account in the computation of the subspace used for projection. Kunisch and Volkwein consider the case that the reduced model is used for optimal control and consider the dependence of the subspace used for projection on the control input.

The above methods for linear model reduction rely on a projection of the detailed model as discussed below in Section 2.1.1. If the internal dynamics are unknown and only input/output (I/O) data is available, e.g., a time-domain Loewner approach [Peherstorfer et al., 2017] or transfer function fitting [Sootla, 2013; Sou et al., 2008] can be used.

In contrast to model reduction for linear systems, model reduction for nonlinear systems is much less developed [Benner et al., 2015]. Many model reduction methods for nonlinear systems are extensions of methods for linear systems [Baur et al., 2014], e.g., balanced truncation [Fujimoto and Scherpen, 2010; Scherpen, 1993] and moment matching [Astolfi, 2010; Ionescu and Astolfi, 2016]. Unfortunately, both methods require the solution of partial differential equations (PDEs). Hence, they are computationally involved. A remedy for balanced truncation is the empirical balanced truncation [Kawano and Scherpen, 2017; Lall et al., 2002; Pallaske, 1987]. Empirical balanced truncation as well as the POD require trajectories generated by simulation and rely on a linear projection. But, according to Gu [2011], "nonlinear projection is natural and appropriate for reducing nonlinear systems, and can achieve more compact and accurate reduced models than linear projection". The extensions of balanced truncation [Fujimoto and Scherpen, 2010; Scherpen, 1993] and moment matching [Astolfi, 2010; Ionescu and Astolfi, 2016] to nonlinear systems can be interpreted as a nonlinear projection. But these methods are computationally challenging.

There exist several methods relying on a linear mapping between the states of the detailed and reduced model. The nonlinear Galerkin projection [Matthies and Meyer, 2003] assumes a (linear) subspace for the dominant states. The nonlinear Galerkin projection results in a differential algebraic equation (DAE). In contrast to the empirical balanced truncation and the POD performing a transformation followed by truncation, the nonlinear Galerkin projection is strongly connected to residualization, which also results in a DAE as discussed briefly in Section 2.1.1. The trajectory piecewise linear approximation [Rewieński and White, 2006] linearizes the nonlinear system around several points in the state space. Then, the nonlinear system is approximated with a weighted sum of the linear systems. The order reduction is done by a projection of this sum of the linear systems onto one (linear) subspace. Gu [2009] suggests a method that deduces an equivalent quadratic-linear DAE first, which is then reduced using a linear projection.

The above cited methods either utilize a linear mapping between states of the detailed and the reduced model or a nonlinear transformation, which is difficult to compute. Alternative approaches rely on data sampled from trajectories. Lohmann [1994] presented a model reduction procedure assuming that the dominant states are known and the nonlinearity is kept in the reduced model. The linear couplings between the state variables and the nonlinear functions are computed by minimizing the equation error. Another optimization based approach proposed in [Bond et al., 2010] allows to, first, utilize a reduced order training set of the states computed by a linear projection and, second, enforce incremental stability. The reduced basis methodology provides a framework for iteratively adding new trajectories to the training set (known as greedy sampling) and algorithms yielding a basis used for projection [Haasdonk and Ohlberger, 2008]. In [Wood et al., 2004], derivatives of I/O data up to the second-order are used to determine an implicit nonlinear ODE. Vargas and Allgöwer [2004] presented a procedure that is applicable to systems admitting a discrete-time Volterra representation and suggested an iterative approach for the construction of the reduced model. In this thesis, we build upon the work in [Lohmann, 1994; Vargas and Allgöwer, 2004; Wood et al., 2004].

Above, the aspects of a small approximation error and computational efficiency of the procedure are discussed. Another important issue of a model reduction procedure is preserving properties of the model such as stability or passivity [Antoulas, 2005b; Astolfi, 2010] and thereby providing guarantees for the model reduction procedure. For nonlinear systems, preserving local asymptotic stability of the equilibrium was shown for balanced truncation [Scherpen, 1993]. For moment matching conditions exist that ensure a locally asymptotically stable equilibrium for the reduced model [Astolfi, 2010; Ionescu and Astolfi, 2016]. Local asymptotic stability of the equilibrium is preserved for the POD if the detailed model is appropriately transformed in advance [Prajna, 2003]. For the trajectory piecewise linear approximation small-signal finite-gain $\mathcal{L}_p$ stability can be guaranteed [Bond and Daniel, 2009]. Model reduction preserving incremental stability is covered in [Besselink, 2012; Bond et al., 2010]. For incremental stable systems, state trajectories corresponding to a given input signal converge to each other [Angeli, 2002]. Hence, systems with multiple equilibria considered in Section 3.3 cannot be handled by incremental stability.

Another important issue is the computational complexity of the reduced model. For model reduction of linear systems the order of the model is often used as measure for model complexity. For general nonlinear systems, applying a linear projection reduces the order of the model but the complexity of the nonlinear expression remains similar. Hence, for nonlinear model reduction the complexity of the functional expressions is also important [Astolfi, 2010; Ionescu and Astolfi, 2016]. Simplification of the nonlinear expression before a linear projection can be achieved by a Taylor series expansion, which allows to compute the coefficients of the polynomials offline [Phillips, 2003]. Using the trajectory piecewise linear approximation [Rewieński and White, 2006] also results in a simplification of the nonlinear expression since it results in a weighted sum of linear systems. Furthermore, to reduce the complexity of the nonlinearity the empirical interpolation [Barrault et al., 2004; Chaturantabut and Sorensen, 2010; Drohmann et al., 2012b; Haasdonk et al., 2008; Peherstorfer et al., 2014] has been proposed.

1.2.2 Bounds for the Error of Model Reduction

Using a reduced model for simulation introduces uncertainty due to the mismatch between the detailed and reduced model. For the application of the reduced model it is important to quantify the uncertainty, especially in safety critical situations.

In the model reduction community several bounds for the model reduction error are known that are satisfied for all inputs. For linear systems, an a-priori error bound exists for balanced truncation with zero initial condition [Enns, 1984; Glover, 1984] and inhomogeneous initial condition [Baur et al., 2014; Heinkenschloss et al., 2011]. Error bounds for moment matching are proposed in [Panzer et al., 2013; Wolf et al., 2011]. In contrast to the a-priori error bounds, the a-posteriori error bounds are applicable after the reduced model has been computed. An a-posteriori error bound for stable (parameterized) linear ODEs is presented in [Haasdonk and Ohlberger, 2011]. In [Haasdonk and Ohlberger, 2011], the error is bounded by a scalar ODE, which depends only on the input and state of reduced model. Hence, this error bound takes the input trajectory into account. But the error bound monotonically increases with time. This is overcome by the generalized error bound introduced in [Ruiner et al., 2012]. For this error bound, the computational demand in the offline phase can be reduced significantly [Grunert et al., 2020]. Unfortunately, the error bound of [Grunert et al., 2020; Ruiner et al., 2012] requires the computation of a convolution integral for every time point since it cannot be written as an ODE. This significantly increases the computational demand. A-posteriori bounds for the error of the transfer function have been presented in [Antoulas et al., 2018; Feng et al., 2017]. These a-posteriori error bounds can be used to refine the reduced model [Antoulas et al., 2018].

A-posteriori error bounds exist also for nonlinear systems. For the trajectory piecewise linear approximation of stable systems an error bound is introduced in [Rewieński and White, 2003]. An error bound for the Galerkin projection with a POD basis is presented in [Volkwein, 2011]. An error bound for any projection based model reduction and approximation of the nonlinearity with the discrete-empirical interpolation method is introduced in [Wirtz et al., 2014]. Furthermore, an error bound for incremental balanced truncation exist [Besselink, 2012].

When reduced models are used for MPC as considered in the subsequent section, several error bounds have been utilized. Narciso and Pistikopoulos [2008] proposed to use the a-priori error bound of balanced truncation. In [Dubljevic et al., 2006], the model reduction method is limited to modal truncation, where the states of the reduced model and the neglected dynamics are only coupled by the input. Then, input-to-state boundedness of the error system is exploited to establish a constant error bound. In [Kögel and Findeisen, 2015; Lorenzetti et al., 2019; Sopasakis et al., 2013], the error bounds are based on robust positive invariant sets while no assumptions on the model reduction method are stated. Kögel and Findeisen [2015] compute, in a first step, box constraints for the error of the performance output and the error in the dynamics of the estimated states of the reduced model. These box constraints bound these errors for all possible trajectories in a chosen time interval. In a second step, these box constraints are utilized to compute a robust positive invariant set for the estimated state of the reduced model. Finally, the error bounds on the input and performance output are established using the box constraints and the robust positive invariant set. In [Lorenzetti et al., 2019] the error bounds are directly computed based on the given dynamics. This results in less conservative error bounds. In the recent publication [Lorenzetti and Pavone, 2019], an error bound is proposed that bounds the contribution from the most recent time interval by polytopic sets and all prior contributions by the a-posteriori error bound presented in our work [Löhning et al., 2014]. To compute the polytopic set, all inputs and states in a bounded set are taken into account. To allow for the a-posteriori error bound, a projection-based model reduction method is assumed in [Lorenzetti and Pavone, 2019]. Altogether, in [Dubljevic et al., 2006; Kögel and Findeisen, 2015; Lorenzetti and Pavone, 2019; Lorenzetti et al., 2019; Narciso and Pistikopoulos, 2008; Sopasakis et al., 2013], only error bounds that are fulfilled for all inputs or all inputs and states in a bounded set have been utilized for MPC using reduced models. Hence, the known input and state of the reduced model is not taken into account in the prediction of the error bound, which, in general, results in a considerable conservatism.

1.2.3 MPC Using Reduced Models

In this section, we discuss the literature in the field of MPC related to this thesis. At the beginning, we discuss the literature ensuring stability of the closed-loop system. Afterwards, we concentrate on MPC using reduced models. For a general overview of MPC we refer to the books [Camacho and Bordons, 2007; Grüne and Pannek, 2017; Kouvaritakis and Cannon, 2016; Rawlings and Mayne, 2009] and survey articles [Findeisen et al., 2003; Joe Qin and Badgwell, 2003; Magni and Scattolini, 2004; Mayne, 2014; Mayne et al., 2000].

In MPC, the infinite horizon optimal control problem is approximated by a finite horizon optimal control problem. Hence, stability of the closed-loop system is not guaranteed a-priori. Many versions of MPC have been proposed in the literature in order to ensure stability. A zero terminal state constraint was used in [Chen and Shaw, 1982; Keerthi and Gilbert, 1988; Mayne and Michalska, 1990]. The zero terminal state constraint was extended to a terminal constraint set [Chisci et al., 1996; Michalska and Mayne, 1993; Scokaert et al., 1999]. This MPC version is known as dual mode since inside the terminal constraint set a local control law is used instead of the model predictive controller. A terminal cost (without terminal state constraint) was proposed for linear, constrained, and stable systems in [Rawlings and Muske, 1993]. The combination of a terminal constraint set and terminal cost emerged to a common framework to guarantee stability for MPC [Chen, 1997; Chen and Allgöwer, 1998; Chmielewski and Manousiouthakis, 1996; De Nicolao et al., 1998; Fontes, 2001; Scokaert and Rawlings, 1998]. The terminal constraint set is often defined by a local stabilizing controller. In contrast to dual mode MPC, this local controller is never applied. But, the local stabilizing controller is used to prove recursive feasibility, i.e., from feasibility of the finite horizon optimal control problem at the initial time instant follows feasibility for all subsequent sampling instants.

Together with results from tube-based robust MPC [Bemporad and Morari, 1999; Mayne et al., 2005], the above references are the basis for the results about

MPC using reduced models of this thesis. Further results about stability of MPC are, e.g., contractive MPC [de Oliveira Kothare and Morari, 2000; Polak and Yang, 1993a,b; Yang and Polak, 1993] and unconstrained MPC [Grimm et al., 2005; Grüne, 2009; Grüne and Pannek, 2017; Grüne and Rantzer, 2008; Jadbabaie and Hauser, 2005; Reble, 2013; Reble and Allgöwer, 2012]. Unconstrained MPC relies on certain controllability assumptions to compute a sufficiently large prediction horizon in order to guarantee stability without a terminal constraint.

When MPC is applied to high-dimensional systems, solving the high-dimensional optimization problem is a large computational burden. Therefore, reduced models are frequently used for the prediction in MPC, e.g., [Agudelo et al., 2007a; Balasubramhanya and Doyle III, 2000; Dubljevic et al., 2006; Froisy, 2006; Hovland and Gravdahl, 2008; Hovland et al., 2008a; Huisman and Weiland, 2003; Jarmolowitz et al., 2009; Marquez et al., 2013; Nagy et al., 2000; Narciso and Pistikopoulos, 2008; Ou and Schuster, 2009; Shang et al., 2007; Touretzky and Baldea, 2014; Xie and Theodoropoulos, 2010]. Furthermore, linear reduced models can be exploited in MPC to facilitate the solution of the online optimization problem [Huisman and Weiland, 2003; Marquez et al., 2013]. For explicit MPC either reduced models or a projection of the state may be used to reduce the number of regions [Hovland et al., 2008a; Johansen, 2003]. Alternatively to using reduced models for MPC, the computational efficiency can also be increased by reformulating the optimization problem. Examples are move-blocking [Cagienard et al., 2007], generalized input parameterizations [van Donkelaar et al., 1999], or the approximation of the optimization problem [Kouvaritakis et al., 2002].

By using a reduced model within the model predictive controller, a mismatch between the plant and the prediction model is introduced. As a result, satisfaction of constraints or asymptotic stability of the closed loop may be lost. Since these are important properties of the closed-loop system, robustness against the model reduction error has to be ensured by the MPC scheme.

Narciso and Pistikopoulos [2008] proposed to utilize the a-priori error bound of balanced truncation to tighten the output constraints. However, recursive feasibility and asymptotic stability are not established in [Narciso and Pistikopoulos, 2008]. When the high-dimensional system is given by a PDE, under the assumption of recursive feasibility, asymptotic stability of the closed-loop system and hard state constraint satisfaction can be guaranteed [Dubljevic et al., 2006]. However, in [Dubljevic et al., 2006], the model reduction method is limited to modal truncation.

Alternatively, one may think of methods from robust MPC [Bemporad and Morari, 1999; Mayne et al., 2005]. As noted in [Hovland et al., 2008a], "the applicability of these methods to establish robustness in the context of MPC with reduced-order models remains a challenging open research question". In the meantime, results of robust output feedback MPC [Løvaas et al., 2007, 2008a,b] have been specialized to MPC using reduced models. Hovland et al. [2008b] guarantee robust stability despite the model reduction error by choosing the cost functions such that a Lyapunov function for the closed-loop system decreases with time. The work of Hovland et al. does not rely on an explicit bound on the model reduction error with the result that it applies only to stable systems and furthermore soft state constraints. In [Sopasakis et al., 2013], it is suggested to use methods from tube-based robust MPC [Mayne et al., 2005, 2006] to establish constraint satisfaction and asymptotic stability of a (possibly large) set around the origin. Tube-based robust MPC is exploited in [Kögel and Findeisen, 2015; Lorenzetti and Pavone, 2019; Lorenzetti et al., 2019] to prove constraint satisfaction and asymptotic stability of a (possibly large) set around the origin despite the model reduction error and a bounded additive disturbances on the system dynamics and measurement. Furthermore, robust MPC is utilized in [Bäthge et al., 2016] to show recursive feasibility when using a coarse or reduced model for the long-term prediction. The error from using the coarse model is approximated by an additive uncertainty and an uncertain initial condition. Hence, constraint satisfaction and recursive feasibility is not proven when a reduced model is used for the long-term prediction.

Altogether, asymptotic stability and satisfaction of hard state constraints for MPC using a reduced model is treated, to the best of our knowledge, only in [Dubljevic et al., 2006; Kögel and Findeisen, 2015; Lorenzetti and Pavone, 2019; Lorenzetti et al., 2019; Sopasakis et al., 2013]. However, in all these references, satisfaction of hard state or output constraints is guaranteed by tightening the constraints according to a bound on the error between the detailed and the reduced model. To predict this bound, all inputs or all inputs and states in a bounded set are taken into account. Thereby, significant conservatism is introduced, since this possibly large error bounding sets are used to tighten the constraints.

The existing methods for MPC using reduced models with guarantees are compared in Table 1.1 with respect to the applicable model reduction method, conservatism of the error bound utilized to satisfy hard state constraints, asymptotic stability of the origin, computational efficiency of the optimal control problem, and the possibility to use output feedback. The method proposed in this thesis is also shown in Table 1.1 to ease the comparison with existing methods.

1.2.4 Summary

In the last sections, we have given an overview about the literature concerning model reduction, bounds for the reduction error, and MPC using reduced models.

While for model reduction of linear systems several widely accepted methods that result in models with reduced order and reduced computational complexity exist, methods for nonlinear systems are less developed. Challenges in nonlinear model reduction are, first, that a nonlinear mapping between the states of the detailed and the reduced model can be required to achieve a reduced model of low order despite a small approximation error. Second, for nonlinear systems besides a reduced order also a simplification of the functional expression is often necessary to achieve computationally efficient reduced models. Third, system properties such

Table 1.1: Properties of methods for MPC using reduced models with robustness against the model reduction error. A ✓ (−, ✗) denotes that the property is satisfied largely (with restrictions, not at all or with considerable restrictions).

	Model reduction	Error bound and constraint satisfaction	Asymptotic stability of origin	Computational efficiency	Output feedback
Modal truncation, error bound for worst-case input, recursive feasibility assumed [Dubljevic et al., 2006]	✗	−	−	✓	✗
Balanced truncation, a-priori error bound of balanced truncation [Narciso and Pistikopoulos, 2008]	−	−	✗	✓	✗
Any model reduction method, no explicit error bound, asymptotic stability of the origin by constraints on the cost function [Hovland et al., 2008b]	✓	✗	✓	✓	✓
Any model reduction method, error bound for worst-case inputs, asymptotic stability of a set around the origin [Sopasakis et al., 2013]	✓	−	−	✓	✗
Any projection-based model reduction method, error bounding system taking the actual input and state into account, asymptotic stability of the origin [Löhning et al., 2014]	✓	✓	✓	✓	✗
Any model reduction method, error from model reduction approximated by additive bounded uncertainty and uncertain initial condition [Bäthge et al., 2016]	✓	✗	✗	✓	✗
Any (projection-based) model reduction method, error bound for all inputs and states in a bounded set, asymptotic stability of a set around the origin [Kögel and Findeisen, 2015; Lorenzetti and Pavone, 2019; Lorenzetti et al., 2019]	✓	−	−	✓	✓

as stability need to be preserved in many cases. Fourth, it is desired to obtain a computationally efficient model reduction procedure.

For the application of reduced models we have looked at MPC using reduced models. For this field, the main challenge is to provide guarantees for the closed loop with the plant such as asymptotic stability of the origin and satisfaction of constraints with a reasonable conservatism while allowing for a computationally efficient solution of the online optimization problem. Even for linear systems only few and limited results exist. To tackle the challenge, bounds for the model reduction error are required that are both tight and computationally efficient. Moreover, existing applicable a-posteriori error bounds are only marginally stable, which prevents to establish asymptotic stability of the origin for the closed loop with the plant. Hence, another challenge is to derive error bounds that converge to zero asymptotically for vanishing input.

1.3 Contributions of the Thesis

In this thesis, we address the challenges described in the previous section. Thereby, we contribute to the fields of model reduction and MPC. More precisely, we present novel methods for the research topics model reduction of nonlinear systems, bounds for the model reduction error, and MPC using reduced models.

With respect to model reduction, we propose in Chapter 3

- a model reduction method for nonlinear continuous-time dynamical systems, which allows to obtain models of low order and low computational complexity.

The method is an I/O trajectory-based approach that — in contrast to many other existing model reduction methods — relies on parameter optimization and not on a projection. This allows us to use a nonlinear mapping from the state variables of the detailed model to the state variables of the reduced model determined by the observability map. A low complexity functional expression of the reduced model is achieved by sparsity enhancing ℓ_1-minimization. Moreover,

- we extend the proposed method to preserve the location and local exponential stability of multiple steady states.

For this purpose, we derive a necessary and sufficient condition for the simultaneous stability of a set of steady states. We relax the resulting optimization problem to a sequential convex optimization problem, which admits an efficient optimization. The reduced model with low complexity obtained using the proposed method can be used for MPC of complex nonlinear systems. MPC using reduced models with guaranteed asymptotic stability and constraint satisfaction is demanding even for LTI systems as indicated by the few and limited results. To overcome the limitations depicted in Table 1.1, we focus on the class of LTI systems for the error bound and the MPC schemes.

In Chapter 4, we improve an existing a-posteriori bound for the model reduction error. This results in

- an asymptotically stable system that bounds the error between the detailed and the reduced model.

Due to the asymptotic stability instead of marginal stability, the improved error bound is considerably tighter than the original one while at the same time possesses a comparable computational demand. For the application in model based control, we achieve the asymptotic stability even for unstable systems by prestabilization. Furthermore, we compare the proposed error bound with existing ones using a model of a tubular reactor.

In Chapter 5, we utilize the improved error bound to derive

- an MPC scheme using a reduced model that guarantees asymptotic stability and satisfaction of hard input and state constraints

when the model predictive controller is applied to a high-dimensional and possibly unstable plant. Besides asymptotic stability and constraint satisfaction, obtaining a good performance with respect to the cost functional is an important advantage of MPC. Hence, we show that for a common choice of design parameters, first, the infinite horizon cost functional is preserved while replacing the plant with the reduced model and, second,

- the proposed MPC scheme implicitly minimizes the quasi-infinite horizon cost functional of an MPC scheme using the plant model despite using a reduced model for the prediction.

In this case, the proposed MPC scheme also guarantees an upper bound for the cost functional value of the closed loop with the plant. To achieve the mentioned guarantees, the online optimization problem includes the nonlinear error bounding system. This raises the question of the computational complexity of the online optimization problem. Therefore, in Chapter 6 we show for discrete-time plants that

- the online optimization problem can be reformulated as a second-order cone program,

which can be solved efficiently. Existing MPC schemes using reduced models with comparable guarantees have been applied to a practically motivated example, to the best of our knowledge, only in [Kögel and Findeisen, 2015]. We demonstrate the applicability of the developed MPC scheme to control the model of a tubular reactor. In contrast to [Kögel and Findeisen, 2015], we show in the simulation study that

- the proposed MPC scheme achieves a good trade-off between computational efficiency and conservatism

while at the same time providing important guarantees for the closed-loop behavior.

Summarizing, we provide novel methods concerning model reduction and the utilization of reduced models with the common goal of reduced computational complexity. Since the model reduction error can compromise the application at hand, we provide methods with rigorous guarantees in this thesis. Our main contributions are a novel method for model reduction of nonlinear systems, improved a-posteriori bounds for the model reduction error as well as novel algorithms for the utilization of reduced models in MPC.

1.4 Design Workflow of the Proposed Model Predictive Control Scheme

One major contribution of this thesis is the proposed MPC scheme using a reduced model and error bound. This MPC scheme combines several elements introduced in different chapters. As a consequence, the workflow to design the proposed model predictive controller using a reduced model depicted in Figure 1.2 serves the reader also as a guide while reading the thesis. Hence, we present the workflow already here.

The first step to design the proposed model predictive controller using a reduced model is preprocessing of the plant model in order to end up with an asymptotically stable preprocessed model. Then, the reduced model is computed by projection of the preprocessed model. In this thesis, the POD is used for this step but any projection based methods are applicable. In the third step, the error bound is derived and evaluated. If the error bound is too conservative, the preprocessing or the model reduction has to be adapted. If the error bound is tight enough, the proposed model predictive controller can be designed. To achieve a satisfactory closed-loop performance, further adaptions of the design parameters of all steps can be required.

1.5 Outline of the Thesis

The background for this thesis is provided in Chapter 2. This includes the framework of model reduction by projection, model reduction by POD, existing a-posteriori bounds for the model reduction error, a common framework to guarantee stability for nominal MPC, and the control problem of a tubular reactor.

In Chapter 3, we present a method for model reduction of nonlinear systems based on input-output data. After the problem statement, the procedure is described and utilized to reduce the model of a mitogen-activated protein kinase (MAPK) cascade. In the third section of Chapter 3, the procedure is extended to preserve the location and local exponential stability of multiple steady states.

The model reduction introduces an error between the detailed model and the reduced model. Hence, in Chapter 4 we show for LTI systems how this error can be bounded while simulating the reduced model. Although the error bound can be used in a broad context, we aim at the application for MPC. After stating the problem setup we introduce a preprocessing of the plant including a prestabilization and a state transformation. Then, a bound for the norm of the matrix exponential

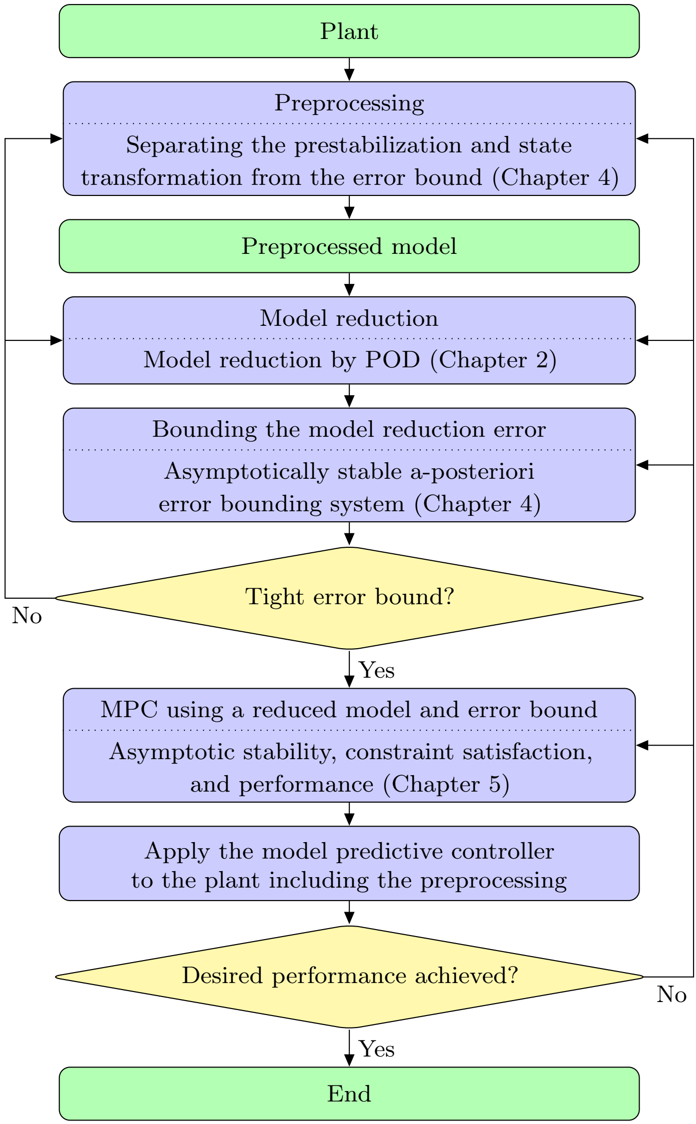

Figure 1.2: Workflow to design the model predictive controller using a reduced model and error bound.

is introduced and the connection between the preprocessing and the parameters of this bound is discussed. The a-posteriori error bound is presented in the fourth section together with a discussion of several ways to achieve an asymptotically stable error bound and the relation to existing error bounds. Finally, the proposed error bound is compared with existing ones using the model of the tubular reactor.

In Chapter 5, the error bound is utilized for MPC using reduced models. At the beginning we state the problem setup and apply the preprocessing and model reduction. Then, we ensure constraint satisfaction for the preprocessed model in a computationally efficient way. Furthermore, we show that for a certain choice of design parameters the model reduction error can be eliminated in the cost functional. The main theoretical result of Chapter 5 is the proof of recursive feasibility and asymptotic stability of the plant model in closed loop with the proposed MPC scheme. Finally, we apply the proposed MPC scheme to the tubular reactor and compare the performance with an MPC scheme using the plant model as well as an MPC scheme using only the reduced model without the error bound. Using the plant model for MPC is possible in this simulation study but would in general be prohibitive in an industrial application due to the real-time requirements.

The application of MPC requires optimization in real time. Thus, the computational demand of the proposed MPC scheme is considered in Chapter 6 in order to enhance the applicability. To have a finite-dimensional static optimization problem, we consider discrete-time systems. The main goal is to deduce a computationally efficient formulation for the optimization problem. Thus, we show that the optimization problem can be reformulated as a convex optimization problem. Afterwards, the convex optimization problem is reformulated such that it is independent of the dimension of the plant model. Furthermore, the optimization problem is stated as a second-order cone program (SOCP), which allows to utilize dedicated and more efficient solvers. Finally, the computational demand of the three MPC schemes considered in Chapter 5 is assessed by means of the tubular reactor.

This thesis concludes with a summary and discussion followed by an outlook of possible future research directions.

Some results of this thesis have already been published in a very similar form. Parts of Chapter 2 have already been presented in [Löhning et al., 2014]. Chapter 3 is very similar to [Löhning et al., 2011a,b]. Chapter 4 is partially based on [Hasenauer et al., 2012; Löhning et al., 2011c, 2014]. Preliminary results of Chapter 5 have already been published in [Löhning et al., 2014].

Chapter 2

Background

In the previous chapter, we have given an overview of the research areas related to this thesis and we have outlined the contributions of this thesis. In this chapter, we describe existing results underlying the contributions in more detail to provide the knowledge required subsequently. In Section 2.1, we present a common framework for model reduction and show one particular method for model reduction known as POD. For this model reduction framework, we discuss in Section 2.2 existing a-posteriori error bounds. An introduction to MPC and a common framework to guarantee convergence to the origin for nominal MPC is given in Section 2.3. Furthermore, we introduce an MPC approach using a reduced model that neglects the model reduction error. Finally, in Section 2.4, we introduce the model of a nonisothermal tubular reactor, which is used in Chapters 4–6 to evaluate the presented methods.

Parts of Sections 2.3 and 2.4 have already been presented in [Löhning et al., 2014].

2.1 Model Reduction

The field of model reduction deals with algorithms that simplify dynamical models. This system to be reduced will be called detailed model in this thesis and abbreviated using the subscript D. We consider detailed models described by n first-order ODEs and a set of n_{Y} algebraic equations defining the output $y_{\mathrm{D}}(t; x_0, u) \in \mathbb{R}^{n_{\mathrm{Y}}}$ at time $t \in \mathbb{R}_{0+}$ for the initial condition $x_0 \in \mathbb{R}^n$ and input $u(\cdot) \in \mathcal{L}_2^{n_{\mathrm{U}}}$ of dimension n_{U} as in [Antoulas, 2005b]. Hence, the detailed models are of the form

$$\Sigma_{\mathrm{D}} : \begin{cases} \dot{x}(t) = f\big(x(t), u(t)\big)\,, \qquad x(0) = x_0\,, \\ y_{\mathrm{D}}(t) = h\big(x(t), u(t)\big)\,, \end{cases} \tag{2.1}$$

in which $x(t) \in \mathbb{R}^n$ is the state of the system at time $t \in \mathbb{R}_{0+}$ and n the order of the model. To ensure existence and uniqueness of solutions the vector field $f : \mathbb{R}^n \times \mathbb{R}^{n_{\mathrm{U}}} \to \mathbb{R}^n$ is assumed to be globally Lipschitz continuous.

Given a detailed model Σ_{D}, the objective is to find a model of reduced complexity Σ_{R} that provides a good approximation of the I/O behavior of Σ_{D}. In this thesis, the complexity of a system contains the number of states as well as the computational complexity of the right-hand side of the dynamics. The approximation quality can be measured, for example, by the norm of the output error, i.e., the difference in the output for the same input and corresponding initial condition. While for LTI models a good approximation for all inputs is often considered, this choice may lead to unnecessarily complex reduced models for general nonlinear systems. Hence, we consider the output error only for a finite time interval $[0, T_{\mathrm{end}}]$ as well as important initial conditions and input trajectories. The important initial conditions and inputs can be specified by a subset of the state space and input space or importance weights as used in Chapter 3.

2.1.1 Model Reduction by Projection

In this section, we discuss a very common framework for reducing detailed models of the form (2.1), which we denote model reduction by projection. This framework is used by many well-known methods, among other things, balanced truncation, POD, modal truncation, and Krylov methods. Model reduction by POD will be explained in Section 2.1.2. For the other model reduction procedures we refer to [Antoulas, 2005b]. In addition to the popularity, model reduction by projection is important in this thesis since the error bounds discussed in Section 2.2 and Chapter 4 allow for any model reduction method relying on projection. Since the error bounds are derived for linear systems, only linear transformations are considered in this section.

For model reduction by projection, we consider the full-column rank matrices $V \in \mathbb{R}^{n \times n_{\mathrm{R}}}$ and $W \in \mathbb{R}^{n \times n_{\mathrm{R}}}$ satisfying $W^{\mathsf{T}} V = I_{n_{\mathrm{R}}}$. We derive the reduced model of order n_{R} similar to [Antoulas, 2005b, Section 1.1.1]. Consider the nonsingular matrix $T = \begin{bmatrix} V & \tilde{V} \end{bmatrix} \in \mathbb{R}^{n \times n}$ with $T^{-1} = \begin{bmatrix} W & \tilde{W} \end{bmatrix}^{\mathsf{T}}$ and the state transformation $x(t) \coloneqq Tz(t)$. Then, we partition

$$z(t) =: \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} = \begin{bmatrix} W^{\mathsf{T}} x(t) \\ \tilde{W}^{\mathsf{T}} x(t) \end{bmatrix}$$

into the dominant states $z_1(t) \in \mathbb{R}^{n_{\mathrm{R}}}$ and the nondominant states $z_2(t) \in \mathbb{R}^{n - n_{\mathrm{R}}}$. By inserting $x(t) = Tz(t)$ into (2.1), we get

$$\begin{aligned} \dot{z}_1(t) &= W^{\mathsf{T}} f\big(V z_1(t) + \tilde{V} z_2(t), u(t)\big) , \qquad & z_1(0) &= W^{\mathsf{T}} x_0 , \\ \dot{z}_2(t) &= \tilde{W}^{\mathsf{T}} f\big(V z_1(t) + \tilde{V} z_2(t), u(t)\big) , \qquad & z_2(0) &= \tilde{W}^{\mathsf{T}} x_0 . \end{aligned}$$

The model reduction occurs by neglecting the dynamics of the nondominant states $z_2(t)$. Truncating the nondominant states results in the reduced model

$$\Sigma_{\mathrm{R}} : \begin{cases} \dot{x}_{\mathrm{R}}(t) = W^{\mathsf{T}} f\big(V x_{\mathrm{R}}(t), u(t)\big) , \qquad x_{\mathrm{R}}(0) = z_1(0) = W^{\mathsf{T}} x_0 , \\ y_{\mathrm{R}}(t) = h\big(V x_{\mathrm{R}}(t), u(t)\big) . \end{cases} \tag{2.2}$$

An estimate for the state of the detailed model $x(t) = V z_1(t) + \tilde{V} z_2(t)$ is given by $V x_{\mathrm{R}}(t)$. Since the summand $\tilde{V} z_2(t)$ is neglected in the reduced model, the state of the reduced model $x_{\mathrm{R}}(t)$ and $z_1(t) = W^{\mathsf{T}} x(t)$ are, in general, not equal for $t > 0$.

In the last paragraph, we have seen that model reduction by projection is a truncation of states in an appropriate basis. If only the matrices V and W are given and not the whole transformation matrix T, as in model reduction by POD described below, an alternative derivation of the same reduced model Σ_R can be used. This alternative uses the projection VW^T onto the subspace spanned by the columns of V parallel to the kernel of W^T. In this approach, the state of the detailed model $x(t)$ is replaced in (2.1) by the estimated state $Vx_\mathrm{R}(t)$. Then, the resulting dynamics $V\dot{x}_\mathrm{R}(t) = f\big(Vx_\mathrm{R}(t), u(t)\big)$ are projected along the kernel of W^T. This results in the same reduced model Σ_R since $W^\mathsf{T}V = I_{n_\mathrm{R}}$.

Model reduction by projection is visualized in Figure 2.1. The initial condition of the detailed model is projected along the kernel of W^T onto the subspace spanned by the column of V. The reduced model evolves in the subspace spanned by the column of V. The black lines depict the connection between the state of the detailed and reduced model at the time instants $0, 0.5, 1, \ldots$. Clearly, the majority of these lines are not parallel to the kernel of W^T. This demonstrates that the state of the reduced model is, in general, not equal to the projected state of the detailed model for $t > 0$.

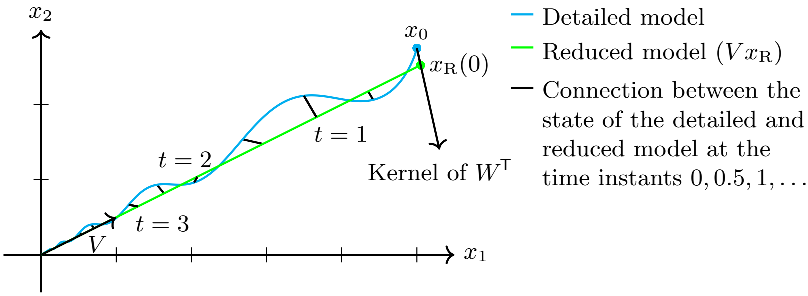

Figure 2.1: Visualization of model reduction by projection for $V = [1\ 0.5]^\mathsf{T}$ and $W = [0.9\ 0.2]^\mathsf{T}$.

Beyond truncation, the nondominant states can also be determined by an algebraic equation, as discussed in a similar context in [Marquardt, 2002]. Thereby, a connection to methods mentioned in Section 1.2 is obtained. One example is residualization, which assumes $\dot{z}_2(t) = 0 = W^\mathsf{T} f\big(Vz_1(t) + Vz_2(t), u(t)\big)$. Another example is the nonlinear Galerkin projection [Matthies and Meyer, 2003], which uses an explicit relation $z_2 = \eta(z_1)$ in which η is a vector field of appropriate dimension. These methods are not considered as model reduction by projection in this thesis as in [Antoulas, 2005b].

To evaluate the right-hand side of the dynamics of the reduced model (2.2), we have to compute, first, the n-dimensional estimated state $Vx_\mathrm{R}(t)$, then the vector field $f(\cdot)$, and finally the multiplication with W^T. Hence, in general, without any simplification of the functional expression $W^\mathsf{T} f(V\cdot)$, the computational complexity