Abstract

Currently, computed tomography (CT) is one of the most important imaging methods in medical technology. Although CT examinations only make up a small proportion of X-ray examinations, they do make a great contribution to the civilization radiation exposure of the population. In particular, the use of modern CT techniques has made it possible to lower the mean radiation dose per examination over the past few years. This also includes the use of statistical iterative reconstruction methods (SIRM). While SIRM enable the modeling of physical imaging properties, the user can also decide freely and independently about the choice of numerous free parameters. However, every decision regarding parameterization has an influence on the final image quality (IQ). Accordingly, the main goal of this work was to examine the influence of some SIRM parameters in a more detailed manner.

In this work, initially all essential and necessary mathematical relationships of the data acquisition in addition to the basics of SIRM in CT were explained. A description of the basics of the IQ assessment followed, including a quantitative evaluation using basic metrics and task-based metrics.

When defining a SIRM, the definition of the modeling of the forward projection model plays a central role. In a preliminary study, various linear interpolation methods and basis function techniques were examined. Based on these results, an extensive study based on linear interpolation techniques was carried out. The choice of the interpolation method fundamentally influences IQ. Depending on both the pixel size of the reconstructed volume and on the choice of the linear interpolation method, the reconstruction error can be reduced by more than 65%. However, the task-based IQ assessment showed that the differences observed were not statistically significant. Furthermore, the influence of statistical weighting matrices (SWM) together with two different regularization methods (RM) was examined. In the first approach, the SWM carried the information about differently shaped bowtie filters and, in the second approach, the SWM carried the information about the handling of redundant data. Regularization was carried out by specifying a maximum number of iterations and by using a stopping criterion based on the absolute change in pixel values in the reconstruction volume. Both factors, the definition of the SWM and the RM, have a considerable, sometimes unexpected, impact on the final IQ in terms of reconstruction error and noise effects, i.e., for a non-regularized SWM solution the mean standard deviation was reduced by 45% whereas that value increased by 11% for the regularized SWM solution. In summary, the investigation of some SIRM parameters shows that the definition of iterative reconstruction parameters is not always trivial and must always be understood comprehensively in order to obtain an optimal IQ.

Many currently available SIRM have relatively long reconstruction times due to their underlying design. For this reason, a SIRM called FINESSE was developed that combines the properties of the FISTA algorithm with the block-based descent method. FINESSE showed promising results in both a simulation study and in a study with real data.

Kurzfassung

Heutzutage zählt die Computertomographie (CT) zu einem der wichtigsten bildgebenden Verfahren in der Medinzintechnik. Obwohl CT Untersuchungen insgesamt nur einen geringen Anteil der Röntgenuntersuchungen ausmachen, wird durch sie dennoch ein großer Beitrag zur zivilisatorischen Strahlenexposition der Bevölkerung geleistet. Insbesondere durch den Einsatz moderner CT-Techniken konnte die mittlere Strahlendosis pro Untersuchung über die letzten Jahre gesenkt werden. Hierzu zählt auch der Einsatz statistischer iterativer Rekonstruktionsverfahren (SIR). Einerseits ermöglichen SIR die Modellierung physikalischer bildgebender Eigenschaften, andererseits kann der Anwender selbstständig und unabhängig über die Wahl zahlreicher freier Parameter entscheiden. Jedoch hat jede Entscheidung der Parametrierung einen Einfluss auf die finale Bildqualität (BQ). Das Hauptziel dieser Arbeit war die genaue Untersuchung mehrerer Einflussfaktoren von SIR.

In dieser Arbeit wurden zunächst alle erforderlichen und notwendigen mathematischen Zusammenhänge der Datenakquise sowie die Grundlagen von SIR in der CT erläutert. Nachfolgend wird detaililert auf die Grundlagen der BQ-Beurteilung eingegangen, die eine quantitative Auswertung mittels grundlegender Metriken sowie aufgabenbasierter Metriken beinhaltet.

Bei der Festlegung eines SIR spielt die Definition der diskreten Modelierung der Forwärtsprojektion eine zentrale Rolle. In einer vorläufigen Studie wurden zunächst verschiedene lineare sowie auf Basisfunktionen basierende Interpolationstechniken (IT) untersucht. Daraufhin wurde eine umfangreiche Studie basierend auf rein linearen IT durchgeführt. Die Wahl der IT hat grundlegenden Einfluss auf die finale BQ. Eine Reduktion um mehr als 65% des Rekonstruktionsfehlers ist je nach Wahl der linearen IT möglich. Die aufgabenbasierte BQ-Beurteilung zeigte jedoch, dass die ermittelten Unterschiede statistisch nicht signifikant sind. Des Weiteren wurde der Einfluss statistischer Gewichtsmatrizen (SGM) im Zusammenhang mit zwei verschiedenen Regularisierungsmethoden (RM) untersucht. Die SGM trug im ersten Ansatz die Information über verschieden geformte Bowtie Filter und im zweiten Ansatz die Information über die Handhabung redundanter Messdaten. Regularisiert wurde einerseits durch die Vorgabe einer bestimmten Anzahl von Iterationen, anderseits mittels eines Stopkriteriums basierend auf der absoluten Veränderung der Pixelwerte im Rekonstruktionsvolumen. Sowohl die SGM als auch die RM haben erheblichen und teils unerwarteten Einfluss auf die finale BQ. Je nach Wahl von SGM und RM hat sich gezeigt, dass die mittlere Standardabweichung um bis zu 45% reduziert bzw. um bis zu 11% erhöht wird. Zusammenfassend zeigt die Untersuchung der Einflussfaktoren von SIR, dass die Festlegung von iterativen Rekonstruktionsparametern nicht immer trivial ist und stets intensiv verstanden sein muss um eine optimale BQ zu erhalten.

Viele aktuell verfügbare SIR haben relativ lange Rekonstruktionszeiten aufgrund ihres zugrundeliegenden Designs. Aus diesem Hindergrund wurde ein SIR, genannt FINESSE, entwickelt, das die Eigenschaften des FISTA Algorithmus mit der blockbasierten Abstiegsmethode kombiniert. FINESSE lieferte sowohl in einer Simulationsstudie als auch auf Anwendung realer Daten vielversprechende Ergebnisse.

CHAPTER 1

Introduction

Wilhelm Röntgen discovered X-rays on November 8, 1895. At that time, the research in that field was enormous, and is still. Less than a month later Röntgen's publication X-rays were used in medical imaging [Spie 95]. Since then, the number of X-ray examinations has increased steadily, i.e., five billion medical imaging examinations had been conducted worldwide by the year 2010 [Roob 10].

X-ray examinations are used in various fields in medical imaging, i.e., dental medicine, radiotherapy, and mammography. Computed tomography (CT) is just one field of the application of X-rays, but CT has become one of the most important tools for radiologists. However, even though X-ray examinations help radiologists in their diagnoses, each examination is accompanied with a certain radiation exposure to the patient that is said to be unhealthy. Therefore, a significant amount of research is still ongoing in the direction of reducing the patient's radiation dose.

In this chapter, Section 1.1 and 1.2 report the facts and concerns about CT examinations that motivated the topic of the present dissertation. A summary of the achieved scientific contributions to the progress of research follows. The chapter ends with an overview of the individual dissertation chapters in Section 1.4

1.1 Computed Tomography Examinations

The number of CT examinations has increased worldwide dramatically over the last two decades [Smit 09]. In the United States of America, approximately 72 million scans were performed in 2007 [De G 09]. In Germany in 2009, about 4.88 million people received at least one CT scan [hila 11]. The *Bundesamt für Strahlenschutz* reported an increase in CT examinations of about 40% between 2007 and 2016 [Bund 20]. The statistics of the Organization for Economic Co-operation and Development (OECD) provide the number of computed tomography scans per country in 2020 or latest available per 1 000 000 inhabitants for various countries [OECD 21]. According to their statistics, the number of CT scans is between 6 in Colombia and

Mexico and 111 in Japan (see Figure 1.1), whereas Germany sits somewhere in between with 35 CT scans.

The most likely reason for the popularity is that CT has many advantages over traditional 2D medical radiography. For example, interfering superimposition of structures outside the area of interest can be eliminated. In general, a three-dimensional volume is displayed using cross-sectional 2D images that are used for both diagnostic and treatment purposes. Depending on the diagnostic task, images can be viewed in axial, coronal, or sagittal planes and with different resolution levels. These advantages make CT an essential tool that allows reliable diagnoses for lesions, bone fractures, bleedings, bruises, swelling, and/or inflammation.

Nevertheless, each CT examination comes with an additional radiation exposure to the patient. Although only 9% of all X-ray examinations in Germany are done with CT, their overall contribution in total collective effective dose in 2016 was more than 65% [Bund 20]. The total radiation dose depends highly on multiple factors: volume scanned, number, and type of scan sequences, desired resolution, and image quality. The typical effective dose to the body for a chest X-ray scan is about 0.02 mSv, for a Head CT scan $1 - 2$ mSv, for an abdomen CT scan approximately 8 mSv, and for a cardiac CT angiogram $9 - 12$ mSv [Furl 10, Food 17]. Compared to the world average dose rate from naturally occurring sources of 2.4 mSv per year [Cutt 09], a single abdominal CT examination may easily add a radiation dose equal to three years of average background radiation.

1.2 Towards Low-Dose CT

Due to the fact that additional radiation dose may lead to damaged body cells, including DNA molecules [Bren 07], doctors always have to decide if a CT examination comes with a real benefit for the patient. Many publications state that there is an increased risk of cancer caused by the additional radiation dose in CT examinations. Some studies predict that, in the future, between three and five percent of all cancers worldwide will result from medical imaging [Food 17]. Another Australian study of 10.9 million people reported than one in every 1800 CT scans was followed by an excess cancer which in turn led to an increase of lifetime risk of developing cancer from 40.00% to 40.05% after a CT scan [Sasi 11, Math 13]. On the other hand, McCollough et al. [McCo 15] state in their publication that studies that indicate an increased risk of cancer are plagued with serious methodological limitations and several highly improbable results. They conclude that there is no evidence that low doses, i.e., radiation doses is below 2 mSv, can cause any long-term harm [Padd 14, Expe 14]. Baysson et al. also came to similar conclusions [Bays 12].

These controversial discussions have opened a wide field of research and development worldwide. Not only are charitable and medical organizations involved in special campaigns, but manufacturers of medical imaging devices also show real interest in attempting to lower patients' radiation dose with multiple approaches while keeping high image quality. Among many other examples, the development of detectors with little or no electronic noise, adapted bowtie filters, and the application of statistical iterative reconstruction may provide significant improvements toward obtaining that goal.

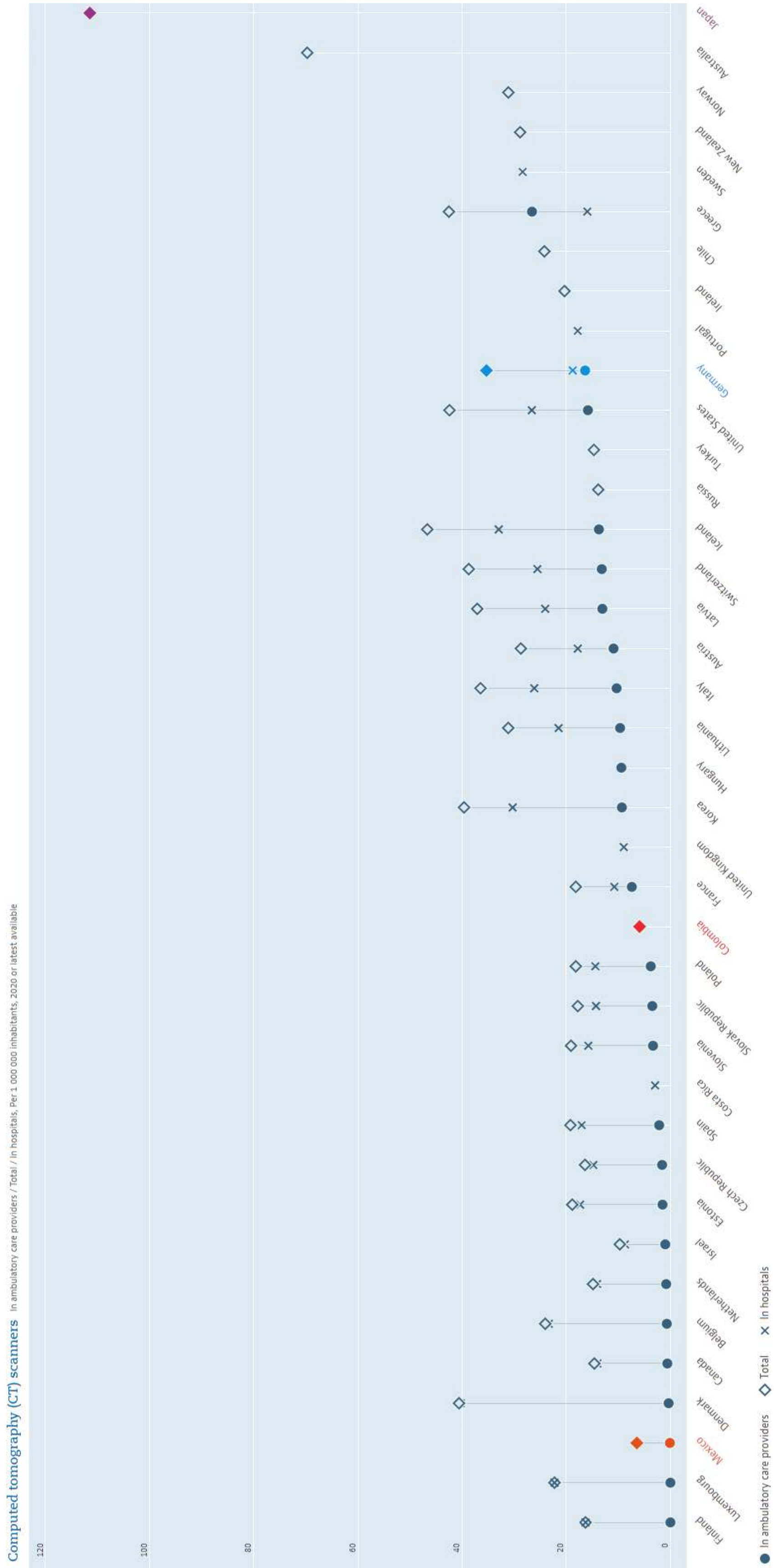

Figure 1.1: Computed tomography scans per country in 2020 or latest available per 1 000 000 inhabitants. This figure was published in [OECD 21].

Statistical iterative methods have long been commonplace, especially in nuclear medicine, since fewer data are available here and the image quality could be massively improved. In CT, analytical methods have long been the gold standard. But that has changed. The area of statistical iterative reconstruction has received a lot of attention in recent years. Unlike with analytical reconstruction methods, statistical iterative reconstruction methods have the advantage of allowing the modeling of any physical aspect of the CT data. In that way, they could be more exact than analytical methods. That advantage is coupled with the multiple degrees of freedom that iterative reconstruction methods have. Therefore, a thorough analysis of all possible influencing factors should be a natural part of each study. Up to now, to the best of the author's knowledge, there is a major lack of literature regarding fundamental investigations on the influence of single iterative reconstruction parameters on final image quality.

Another challenging factor in statistical iterative reconstruction is the reconstruction speed. Currently, almost all statistical iterative reconstruction methods are slow, making it difficult to use them in clinical routine. This is mainly due to the huge amounts of data that needs to be available during the whole reconstruction process. This can be easily understood with the following example based on the data sheet of the Siemens Healthineers CT scanner SOMATOM Drive. A typical chest or thorax examination includes up to 10 rotations. Depending on the scan settings, up to 2304 readings are transferred for the entire detector that is of size 736×32 pixels. Hence, the matrix size of the data is already of size $23040 \times 736 \times 32$ which corresponds to several Gigabytes depending which accuracy is preferred (float or double). In contrast to the analytical methods a so-called system matrix is required for all calculations. That matrix has the dimensions reading per rotation multiplied by the size of the detector times size of the reconstructed target volume. Most of the system matrix elements are zero, nevertheless the information about at least all non-zero elements need to be available during the iterative reconstruction process. Depending on the storage accuracy and design of the system matrix, several gigabytes of data are added. Overall, therefore, many gigabytes of memory may already be blocked. Having sufficient memory available is one problem, the other is that the system matrix must be available for every reconstruction. The calculation of the system matrix elements can either be done on the flow or the required information is read from the disk. Both options cost additional time. In addition, the reconstruction process is also slowed down by the fact that widely distributed elements of the data set have to be accessed again and again during the reconstruction.

Both, having a good understanding of the influencing factors in iterative reconstruction and handling the big data is essential to bring statistical iterative reconstruction in clinical routine. In the current dissertation, we try to fill at least some of these gaps.

1.3 Scientific Contributions

All evaluations were based on mostly MATLAB code, whereas all statistical iterative reconstructions were based on C code. For all iterative reconstruction algorithm implementations, no third-party software was used.

An overview of the original contributions of this dissertation along with the corresponding scientific publications is provided below. Please note that, due to my name change in 2014, there will be references to *Schmitt* (former name) and *Hahn*.

1. **Impact of Discrete Image Representation Techniques.** An essential basis in each statistical iterative reconstruction algorithm is the design of the forward projection model. Several forward projection models have been published over the last two decades. Which of the forward projection models the user should apply is still often a question of personal preference rather than based on quantitative evaluation using basic and/or task-based metrics. Chapter 6 tries to bridge that knowledge gap with an extensive investigation. For the quantitative evaluation, meaningful metrics had to be defined as part of Chapter 5. The results of that study were presented at four international conferences [Schm 12b, Schm 13, Schm 14a, Hahn 15a] and in one publication [Hahn 16].

2. **Challenges posed by Statistical Weights and Data Redundancies.** There are many ways to formulate a statistical reconstruction method for X-ray CT. In particular, the maximum likelihood solution without and with constraints on the image both appear highly popular. In the first approach, the user formulates an iterative algorithm that converges towards the maximum likelihood solution and defines the reconstruction as the application of a finite number of iteration steps. Using this approach, the iteration number is essentially seen as a regularization means. In the second approach, the regularization is not left to the iteration number; it is enforced directly by the constraint, and the user iterates as long as needed to reach the minimum of the objective function. The question which of these approaches is most appropriate in CT cannot be answered easily and needs to be analyzed thoroughly. In Chapter 7, we studied the effectiveness of regularization including essential aspects of CT imaging. The results of that investigation were presented at two international conferences [Schm 13, Hahn 15b].

3. **A Fast Iterative Non-linear Exact Sub-space SEarch based Algorithm (FINESSE).** Most existing statistical iterative reconstruction algorithms can be classified according to the number of voxels that are updated within one iteration. Some algorithms update only one voxel at a time, yielding an algorithm that converges quickly but is poorly parallelizable and thus time consuming. Other methods update all voxels simultaneously and are thereby much more amenable to parallelization, but these algorithms require many more iterations. Last, there are the third group of algorithms that aim at updating several but not all voxels within each iteration. These algorithms suffer from the drawback of requiring the solution of a complex sub-problem, which is often achieved using an approximate, monotonic update that slows down convergence. Currently, none of these algorithms is seen as being satisfactory for routine clinical usage. With the development of a novel algorithm, called *Fast Iterative Non-linear Exact Sub-space SEarch based Algorithm* (FINESSE), we want to fill that gap. First results have been shown on one international conference [Schm 14b]. The algorithm has also been patented [Schm 17].

1.4 Dissertation Outline

In the present dissertation, all background descriptions as well as comprehensive chapter-spanning contents were bundled in separate individual chapters. The main reasons for this were to facilitate readability and mainly to concentrate on the substantial content(s) of the subsequent chapters although the number of chapters was increased with that structure. Thus, Chapter 1 through Chapter 4 describe essential basics that will be referenced in subsequent chapters. Then, Chapter 5 to 9 will primarily focus on my own scientific contributions. A short description of the organization of the dissertation chapter-by-chapter is provided below.

Chapter 1 - Introduction

The introductory chapter starts with a description of the benefits of CT followed by the motivation of the dissertation. Finally, an overview of the original contributions as well as the organization of this dissertation is given.

Chapter 2 - Principles of Computed Tomography

The second chapter gives a brief introduction in the basic components of a computed tomography scanner. That is followed by a short summary of the development from the first clinically available CT scanner in 1972 up to modern commercially available CT scanners. Along with the evolution of the third generation of CT scanners, important gold standard techniques, such as scan modes, beam filtration, quarter detector offset, and flying focal spot were established. As most of those techniques serve as a basis of the conducted research and throughout the dissertation a rough description of them is given. That chapter concludes with the definition of the Hounsfield units.

Chapter 3 - Mathematical Description of the Data Acquisition Process

Chapter three covers the mathematical description of the measurement process that is used throughout the dissertation. Several coordinate systems, such as the world coordinate system, the detector coordinate system, and the image coordinate system, are introduced. Since the definition of the source coordinate system changes with the projection sampling geometry, this coordinate system is introduced separately for each geometry. The underlying coordinate systems are used to give a mathematical description for the calculation of a line integral, which is either called the Radon transform for fan-beam or cone-beam geometries.

Chapter 4 - Iterative Reconstruction Techniques

The basics of iterative reconstruction techniques, the description of the definition of the objective function, and the definition of a non-constrained and constrained image reconstruction problem are described in Chapter Four. Moreover, two commonly known iterative reconstruction methods, the Landweber method and the ICD method, are explained. Finally, the strengths and weaknesses of iterative reconstruction algorithms in general are discussed.

Chapter 5 - Image Quality Assessment

Image quality assessment is a central concept for evaluating the key performance parameters of a CT scanner, for example, resolution or noise. It is very important that the final reconstructed image is related to how well it conveys all anatomical or functional information. The signs of a disease or an injury need to be clearly visible so the interpreting radiologist can make an accurate diagnosis. Different quantitative metrics exist to evaluate certain IQ properties, such as basic metrics and task-based metrics, which are both described in detail in that chapter. Furthermore, details about the phantom used to evaluate image quality are given. This chapter ends with a discussion explaining the pros and cons for the basic and task-based IQ assessment for the choices being made throughout the dissertation.

Chapter 6 - Impact of Discrete Image Representation Techniques on Image Quality

In literature, many definitions of discrete image representation techniques that may affect the image quality of the final reconstructed images exist. Conceptually, the choice of the forward projection model is a major step in the design of an IR algorithm, particularly because the decision being made at this level affects both bias and noise properties of the reconstruction. In addition, the selection of additional parameters that might appear in the cost function or the regularization also influences image quality. However, optimizing a variety of parameters is complicated and accompanied with the need to perform several reconstructions to account for different noise realizations and variations in geometry, which is essential for meaningful observations. Therefore, the focus in Chapter 6 is exclusively on the impact of discrete image representation techniques.

This chapter starts with a mathematical formulation of various image representation techniques that have been extensively investigated. In order to allow a fair comparison of all methods, a description of the experimental comparison conditions is given. Based on the results of a preliminary study, three linear forward projection models, namely the Joseph's method, the distance-driven method, and the bilinear methods were selected to continue with a more detailed analysis including both basic and task-based metrics. The chapter concludes with a discussion and conclusion.

Chapter 7 - Challenges posed by Statistical Weights and Data Redundancies

Iterative reconstruction methods allow modeling of the properties of the line integral measurement using a statistical weighting matrix. In the current chapter, the statistical weighting matrix was formed by: i) taking the effect of different bowtie filters; and, ii) considering data redundancies. The solution of the iterative reconstruction method was then found using two different regularized reconstruction methods, namely the

Landweber method and the ICD method. The Landweber method was regularized by stopping after a certain number of iterates, whereas the ICD method was stopped when the change in pixel value of the reconstructed volume became smaller than a predefined number. Subsequently, the impact of the regularization method together with the respective statistical weighting matrix was investigated.

After a description of the experimental setup, the concept of the ICD parameter selection is given. The results are given in two subsequent sections where each of them describes the definition of the statistical weighting matrix, the parameter selection, and the results individually. Finally, the chapter ends with an overall summary discussion.

Chapter 8 - FINESSE: a Fast Iterative Non-linear Exact Sub-space SEarch based Algorithm

In Chapter 8, a novel reconstruction algorithm, called FINESSE, is presented. That algorithm tries to fill a gap between iterative reconstruction algorithms that update all voxels within one iteration and between algorithms that update all voxels simultaneously. Whereas the first group of algorithms is poorly parallelizable and thus time consuming, the second group of algorithms is much more amenable to parallelization, but also generally requires many more iterations.

This chapter starts with a description of the algorithm design. Then, details about the experimental setup and the results of the simulation study are given. The subsequent section contains the results obtained with real data. A discussion and conclusion completes that chapter.

Chapter 9 - Summary and Outlook

The final chapter provides a summary of the conducted research and the scientific progress achieved by the work presented in this dissertation. The chapter concludes with a perspective on open research challenges.

CHAPTER 2

Principles of Computed Tomography

Computed tomography is an important tool in medical imaging for generating a non-overlapping and non-invasive image of a single slice through the scanned object. Besides a single, two-dimensional image slice, it is also possible to generate a three-dimensional sequence of images. This sequence consists of adjacent two-dimensional image slices. Such three-dimensional images facilitate the examination of the physical configuration and expansion of anatomical structures and of abnormal tissue alterations in the volume of the scanned object. Since the introduction of computed tomography and its first clinical application in 1972, research and development has propelled technological progress in a variety of ways.

This chapter starts with a description of the setup of a CT scanner, followed by a description of Lambert-Beer's Law and an overview of the development of CT scanners. Next, Section 2.4 describes the sampling geometries. Subsequently, some practical aspects are described such as scan modes, X-ray beam filtration, quarter detector offset, and flying focal spot. The chapter concludes with the definition of Hounsfield units.

2.1 Setup of a Computer Tomograph

A typical setup of a modern CT scanner is shown in Figure 2.1. The gantry of a CT scanner hides the power supply, most of the electronics necessary for controls and data pre-processing, as well as the data acquisitions system. The data acquisition system consists of an X-ray source and a detector. Both are mounted opposite to each

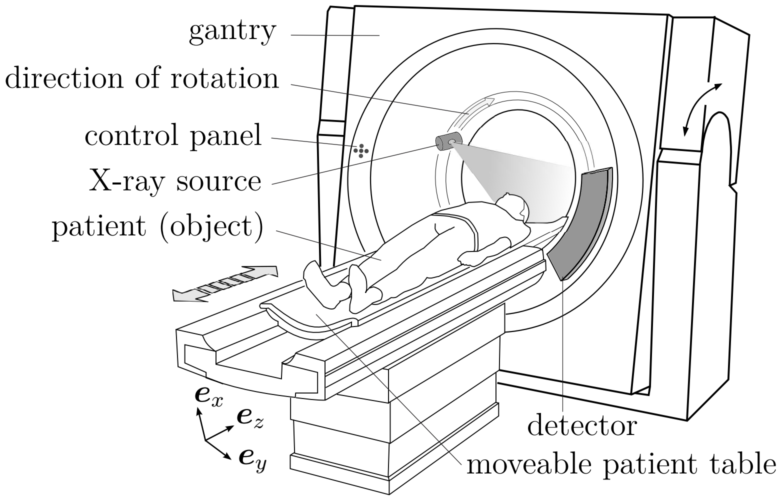

Figure 2.1: Typical setup of a modern CT scanner (3rd generation).

other on a rotating frame. The slip ring technique enables the transfer of electrical power and data so that cable entanglement through continuous rotation is avoided. This also allows taking measurements from any desired start angle.

In most CT scanners, the gantry can be tilted up to ±30° with respect to the axis of rotation. This enables the taking of a CT measurement through structures of interest, i.e., the base of the skull or the lumbar spine. The gantry opening itself has a width of about 70 cm with a field of measurement of about 50 cm to allow also an examination of patients with a wide profile.

The patient lies on a patient table. The patient table is designed to be movable with a high level of positioning precision and in motion speed along the z-axis, i.e., orthogonal to the gantry rotation plane. During an examination, the X-ray source and the detector rotates with a constant speed with up to 4 rotations per second around the patient. The possibility of translating the patient table during the scan enables different scanning trajectories. A detailed description of possible scanning geometries is described in Chapter 2.4.

2.2 Lambert–Beer's Law

X-ray photons are absorbed or scattered when they pass through a material. This attenuation effect is due to interactions between the irradiated material and the photons, i.e., through the photoelectric effect, the Compton Effect, and the coherent scattering effect [Buzu 04, Hsie 09].

Let λ_x be the wavelength of photons of a monochromatic X-ray beam along the line $\mathcal{L}$, let $I_0(\lambda_x)$ and $I(\lambda_x)$ be the incident and transmitted X-ray intensity, and let $\mu(\boldsymbol{x}, \lambda_x)$ be the attenuation coefficient distribution of the irradiated material, where $\boldsymbol{x}$ denotes the position vector in Cartesian coordinates, i.e., $\boldsymbol{x} = [x, y]^T$ in two dimensions or $\boldsymbol{x} = [x, y, z]^T$ in three dimensions, respectively. The attenuation of photons that are traveling along the line $\mathcal{L}$ can be described by the Lambert–Beer's law that is given by

$$I(\lambda_x) = I_0(\lambda_x)\, e^{-\int_{\mathcal{L}} \mu(\boldsymbol{x},\lambda_x)\, d\boldsymbol{x}} \quad . \tag{2.1}$$

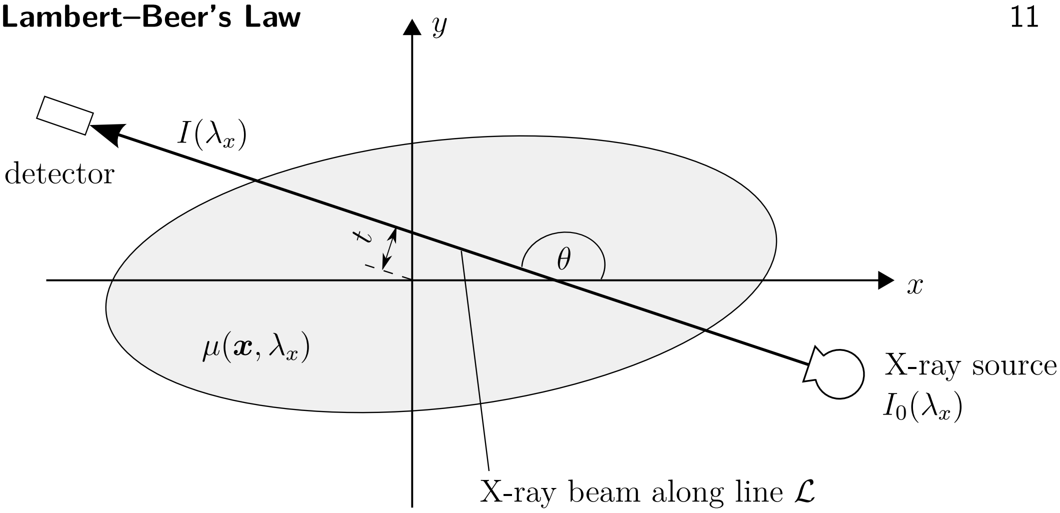

Figure 2.2: Mathematical model of Lambert–Beer's law for monochromatic emission. The effective intensity, I_0, from the x-ray source is attenuated by the material along $\mathcal{L}$. Finally, the remaining intensity, I, is measured by the detector. The parameters t and θ are used to describe the direction of the X-ray beam along the line $\mathcal{L}$ (see also Chapter 3).

An illustration of Lambert–Beer's law is shown in Figure 2.2. Note that Equation 2.1 represents a monochromatic measurement process whereas the output energy spectrum of the X-ray tube of a real measurement process is quite broad. To address this aspect, the measurements are pre-processed before any reconstruction is applied. The pre-processing step includes among others a correction for polychromatic X-ray radiation. After this correction step, the data can be assumed to be equivalent to a measured data set from a monochromatic X-ray radiation, meaning that all photons emitted by the source are of the same energy, called effective energy E_{eff}. Under this assumption Equation 2.1 simplifies to:

$$I = I_0 \, e^{-\int_{\mathcal{L}} \mu(\boldsymbol{x}) \, d\boldsymbol{x}} \ , \tag{2.2}$$

where the attenuation coefficient μ is now adopted to be independent of the X-ray photon energy, i.e., μ varies only with x.

By applying the logarithm in Equation 2.2, a projection measurement, g, can be calculated. The measurement of g represents a single line integral over the attenuation coefficient along the X-ray path $\mathcal{L}$ and is given by

$$g = -\ln\left(\frac{I_0}{I}\right) = \int_{\mathcal{L}} \mu(\boldsymbol{x}) \, d\boldsymbol{x} \ . \tag{2.3}$$

In CT, the attenuation coefficient distribution for an ensemble of lines is calculated based on solving Equation 2.3. Since the attenuation coefficient is heavily material dependent it is possible to distinguish materials from each other. Materials with a high μ value attenuate the X-ray beam more than materials with a low μ value, for example, the attenuation coefficient of bones is higher than that of soft tissue [Kale 06, Buzu 08, Kak 01].

Note that under the assumption of the ideal monochromatic Lambert–Beer's law different reconstruction artifacts may appear in the reconstructed images. For exam-