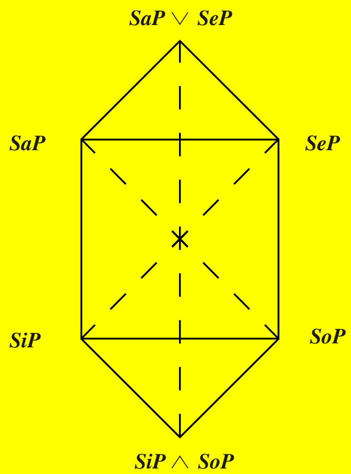
Logische Philosophie

Pavel Materna

Conceptual Systems



λογος

Die Open-Access-Stellung der Datei erfolgte mit finanzieller Unterstützung des Fachinformationsdiensts Philosophie (https://philportal.de/)

FACHINFORMATIONSDIENST
PHILOSOPHIE

DOI: https://doi.org/10.30819/0636

Pavel Materna

Conceptual Systems

Logische Philosophie
Herausgeber:
H. Wessel, U. Scheffler, Y. Shramko, M. Urchs

λογος

Herausgeber der Reihe Logische Philosophie

Horst Wessel

Unter den Linden 61
D-14621 Schönwalde
Deutschland

Uwe Scheffler

Institut für Philosophie
Humboldt-Universität zu Berlin
Unter den Linden 6
D-10099 Berlin
Deutschland

SchefflerU@philosophie.hu-berlin.de

Yaroslav Shramko

Lehrstuhl für Philosophie
Staatliche Pädagogische Universität
UA-50086 Kryvyj Rih
Ukraine

shramko@rocketmail.com

Max Urchs

Fachbereich Philosophie
Universität Konstanz
D-78457 Konstanz
Deutschland

max.urchs@uni-konstanz.de

Bibliografische Information Der Deutschen Bibliothek

Die Deutsche Bibliothek verzeichnet diese Publikation in der Deutschen Nationalbibliografie; detaillierte bibliografische Daten sind im Internet über http://dnb.ddb.de abrufbar.

ISSN 1435-3415
ISBN 3-8325-0636-5

Logos Verlag Berlin
Gubener Str. 47, 10243 Berlin, Tel.: +49 030 42 85 10 90
INTERNET: http://www.logos-verlag.de

Conceptual Systems

Pavel Materna

www.phil.muni.cz/~materna

materna@lorien.site.cas.cz

Table of contents

PREFACE

There are two major trends in the history of the logical analysis of (natural) language ('LANL'). The first one can be traced to Bolzano, Frege, Carnap (in his semantic period), and Wittgenstein (in his tractarian period); the main representatives of the second are the later Wittgenstein, Quine, his numerous followers, Brandom, etc. The former tendency is older, which may give the impression (which seems to have actually happened) that it is *outdated*. The latter is younger and gives the impression that it is a fundamental correction of the errors made by the former. Among those errors are: *realism, atomism* ('myth of museum'), and, of course, the 'two dogmas', which have been so resolutely criticised by Quine.

As a result of this criticism we have inherited *anti-realism*: a widespread misuse of Occam's razor, i.e., cutting out even such abstract entities which make it possible to explain linguistic phenomena from the vantage point of LANL; replacing any use of 'objective' by the coy term 'intersubjective'; selling *pragmatic definitions* as *semantic analyses.*

We have also inherited *holism*: the view (first clearly formulated by Quine in his *Two dogmas*) that asking for *meanings* of particular expressions is futile since only knowledge systems as a whole can be semantically evaluated. (No positive results of such a holistic evaluation have been offered—unless the numerous scattered aphorisms in the later Wittgenstein's works can be taken to be such results.)

Since it is claimed that 'atomistic', 'informational' semantics has been shown to be mistaken, *inferential semantics* has come into being: in full harmony with anti-realism, for if we refuse the 'vertical' linkages between expressions and extra-linguistic world what remains are 'horizontal' linkages connecting expressions *inter se*: the 'so-called' meanings are the ways we use expressions according to some rules of our 'language games'. (But "...it doesn't seem that languages are a lot like games after all: queens and pawns don't mean anything, whereas 'dog' means *dog*.", see Fodor, p.36.)

One of the consequences of the 'new' (or rather 'post-') tendency is that various kinds of *relativism* have become most popular. (More about this can be found in Tichý's book, Preface and Epilogue.) Allegedly it follows that the distinction between analytic and empirical expressions is doubtful (the boundary between them is made fuzzy) and radical contextualism should prove that isolated expressions do not possess an identifiable meaning.

There is a natural (albeit unwilling) ally of the post-trends: *formalism*. Even *philosophical logic* can be hospitable to it: starting our analyses with formal axioms may support the illusion that some good formal properties of the respective system (like completeness or decidability) are the main goal of the analysis. Incomplete systems are something like exiles from the logical paradise and—as we are told by some logicians—logic should not be concerned with such systems (as if logic came into existence till with the idea of formal axiomatics). See Bealer's 1982 book for a criticism of this approach to logic.

We must admit, however, that the post-analytic tendency, as characterised above, has influenced mainly the philosophy of language; the LANL proper proved more immune (Montague, for example.) What is important for understanding the present book is that the author has decided not to systematically criticise theories based on post-analytic assumptions: instead, he intends to show that a *positive* theory based on rejecting these assumptions is possible. Not only that, the author admits that the change of the 'classical semantic paradigms' has been made possible by some shortfalls in their application. Something was missing, so the classics remained defenceless when the Quinean attack started: Quine surely put his finger on the weaknesses of Carnap *et alii*.

The 'missing link', the absence of which led to the easy success of the post-wave, was detected by Pavel Tichý (see the References), who has not hesitated to "go back to where Frege and Russell left off and go on from there"; he was able to do it because he enriched the classics with fundamentally new principles of logical analysis. These new principles could have been formulated (in his *transparent intensional logic,* TIL) since Tichý was not shy about his realism, and therefore did not see any reason not to introduce abstract entities (in harmony with the counterpart of Occam's razor, i.e., 'Menger's comb', which says that *entities should not be omitted unless necessary*). Thus one of the leading principles of TIL (quoted also in the main text of the present book) is that *language is a code*, and therefore meanings are not created but detected by language:

> *The notion of a code presupposes that prior to, and independently of, the code itself there is a range of items to be encoded in it. Hence…meanings cannot be conceived of as products of the language itself. They must be seen as logical rather than linguistic structures, amenable to investigation quite apart from their verbal embodiments in any particular language. To investigate logical constructions in this way is the task of logic. The linguist's brief is to investigate how logical constructions are encoded in various vernaculars.*

Needless to say, this principle is radically anti-Quinean and hence outside the mainstream of contemporary semantics. Yet Tichý did not waste his time in 'proving' this principle: instead he worked out a positive theory that has been able to solve fundamental problems of LANL; besides he has shown that the 'rival' theories were not able to solve them. TIL, as a general approach to solving problems of LANL, differs from other approaches also by not being an *ad hoc* theory that should solve a particular problem *via* building a special axiomatic system. Its tools are sufficiently strong and can be, of course, further developed.

The author has been inspired by three sources: the main source is TIL; further, some profound thoughts in Bolzano's *Wissenschaftslehre*, and, finally, Church's generalisation of the notion of concept. The idea resulting from putting these sources together is simple: *Concepts are abstract procedures that lead (in the best case) to some object.*

The notion that makes it possible to realise this general idea is that of *construction*, as defined in TIL. Armed with this notion the author published in 1998 a monograph (*Concepts and Objects*). Many discussions with his friends and students have convinced him that his explication of *concept* was fruitful and that it could be exploited even in some more practical areas (such as conceptual modelling). On the other hand, the importance of one chapter of *Concepts and Objects*, viz. 'Conceptual systems', proved to be much greater than could have been shown within the space available. Therefore, the author decided to show that a theory of *conceptual systems* might influence the solution of some broader questions, even some that would be of interest for the theory of science. The theory of conceptual systems itself belongs, however, to LANL and its non-empirical character should be clear (which does not exclude the building of 'bridges' with, say, the cognitive sciences.)

Among the results of our analysis the following observation is interesting: the tendency to relativism characteristic of the 'post-wave' needs not be dangerous if the necessity of *relativisation* in justified cases is explained; the explanation can be nearly universally formulated in terms of conceptual systems. Thus the dangerous 'conceptual relativism' can be replaced by admitting 'conceptual plurality' definable in terms of conceptual systems; the boundary between analytic and synthetic statements does not disappear (as Quine has claimed) but is relative to conceptual systems; incommensurability (see Kuhn) does exist but is not as damaging to our intuition of continuity of knowledge as we sometimes think when reading our post-authors, etc.

The author himself was sometimes rather surprised when recognising that some consequences of his approach were totally unexpected (so, e.g., the claim that there are synthetic concepts *a priori*; to get this result it was necessary, of course, to modify—in accordance with what we now know and what Kant could not have known—the idea of analyticity).

One point can never be sufficiently emphasised: The theory of concepts worked out in *Concepts and Objects* and further developed in the present book is based on the TIL notion of *construction*. It is just this notion that is most difficult to swallow for most mainstreamers. The term itself may be not the happiest: TIL constructions are not the 'constructions' of intuitionists, not to speak about connotations from other areas. But I still think that the respective definitions unambiguously determine what they are *and what they are not*. The most widespread misunderstanding consists in interpreting constructions as a kind of expressions of an artificial language. Allow me to use the present Preface to explain once more why this interpretation is wrong.

Constructions are extra-linguistic abstract 'procedures' that consist of unambiguously determined 'steps'. We have to define them, of course, and to do so we need some means of unambiguously fixing them. These means *are* a kind of an artificial language but constructions are not expressions of this language but what these expressions *denote*. Thus whereas—according to our approach—constructions are the meanings of the given expressions of natural language and are (using Fregean terminology) *expressed* by these expressions, our artificial expressions (of the 'language of constructions') *denote* the constructions in question.

(The Quinean prejudices strongly influence the misunderstandings concerning constructions. In harmony with these prejudices some philosophers say: We cannot 'jump out' of language; semantics can only study 'horizontal' relations between expressions. Well, try at least conceive of constructions in the same way as when we use the expression 'elephant' to talk about extra-linguistic elephants. We will see in the main text that, e.g., while the *names* of constructions in our artificial language may contain brackets and letters, the constructions themselves can contain neither brackets nor letters.)

Some analogies may elucidate what is meant by the key notion of constructions. Imagine a computer program. This program is an *expression*. The *denotatum* of this expression is the function that has to be calculated. Yet between the program and the function there is an *algorithm*: the latter corresponds to construction; it could be said that it is the *meaning of* the program. (Needless to say the algorithm is not the same as any of its concrete executions; the former is *abstract,* i.e., neither spatially nor temporally localisable, the latter is always a concrete event, hence *concrete*.) What is characteristic of this algorithm is that it consists of particular 'steps' (instructions), which are at the same time fundamentally different from the *set* of these steps. The set cannot be executed, while the algorithm can be executed to realise a function. (Who could forget Bolzano's distinction between the content (*Inhalt*) of a concept and the concept itself! NB Bolzano knew this distinction in the year 1837, see p.244 of [Bolzano 1837], Vol.I.) See also [Moschovakis 1990], and for a criticism of Cresswell's tuples [Tichý 1996a, pp.74-80], [Jespersen 2003].

Once we accept constructions as *explicans* for 'abstract procedures' we break through the circularity that, according to Quine, makes impossible a definition of the boundary between analytic and synthetic statements. Indeed, the *meaning* of an expression E (of any language) can be identified with that *concept* which is the best analysis of E, where

i) an analysis of E is a concept that is the result of synthesising (in a definable way) the sub-concepts underlying the particular sub-expressions of E
ii) one of the possible analyses is the best one
iii) which analysis is the best one is unambiguously determined by a given conceptual system
iv) concepts are (closed) constructions.

This is, of course, a very coarse characterisation of how the “obscure entity” (Quine, e.g., his [1953]) *meaning* can be defined independently of defining analyticity or synonymy. For details, see *Intermezzo: Parmenides Principle* in the present book. As soon as the definition is accepted, synonymy is easily defined independently of analyticity (see 1.4.3.4); the rest is obvious.

If some followers of Quine really believe that we sink into the metaphysical mud if we accept the notion of construction / concept, we can only regret that in semantics the metaphysical sin is what for mathematics the normal working method is. But of course, this is no argument for the Quineans: since his early works Quine behaviourizes and pragmatizes semantics, which in his hands becomes an empirical discipline; as such it is dramatically distinct from LANL, and Occam’s razor rather than Menger’s comb can be emphasised (or at least such an emphasis can be more tolerated).

Another point deserves attention. The term *concept* is frequently used in semantics, psychology, the theory of science etc. Strangely enough, mostly we are informed about various facts in the respective area but we are not able to appreciate the information, for we are not informed what is meant by the term ‘concept’. Even analytic philosophers and semanticists use such terms as *intension, concept, reference* extremely carelessly, even, I would say, sloppily. It sometimes seems to me that whenever an author writes on semantics he presupposes that everybody understands the meaning of *intension* or of *concept*, although it is a banal truism that these terms are used in very different senses in different articles and books. (In this respect I have to appreciate the tradition of Vienna Circle, where every new term has been defined and then used in the same sense.)

An example of such a context: concerning *conceptual schemes* we get the following information from The Oxford Dictionary of Philosophy:

> The general system of concepts with which we organize our thoughts and perceptions. The outstanding elements of our everyday conceptual scheme include spatial and temporal relations between events and enduring objects, causal relations, other persons,...

The meaning of the term *concept* is here simply taken as being well-known and uniformly used. Using in this careless way the term *concept* we cannot decide, e.g., whether we can have more concepts for the same object (even Bar-Hillel in his [1950] has not understood Bolzano in this respect), whether the set of primes is a concept, whether some concepts are not *universalia*, what does it mean when we say that a concept is empty (or whether there are more kinds of conceptual emptiness), whether we can rationally speak about the ‘development’ of concepts, etc. etc.

The necessity of various explications of the term *concept* became obvious—see *Introduction*. My explication has been worked out in 1998; the present book recapitulates this explication and then tries to exploit it in analysing conceptual systems and their connections with languages.

Among some interesting results of the present study I can adduce:

1. (To be found already in *Concepts and Objects*):
   Distinctions of various kinds of emptiness of concepts (1.4.1.1)
   An analysis of the distinction between *using* and *mentioning* concepts (1.4.2.3)
   To be identified by a concept *C* = to be constructed by *C*.
   Some (non-empirical) concepts do not identify any object.
   Empirical concepts identify intensions (never their values in the actual world-time).
2. (New):
   Simple expressions do not necessarily express simple concepts. (2.1)
   Given a conceptual system ***C*** *the best* semantic analysis of an expression w.r.t. ***C*** can always be found. (Intermezzo *Parmenides Principle.*)
   At least one empirical concept in an empirical conceptual system ***C*** must be primitive in ***C***. (2.4)
   The area of an empirical conceptual system can be creatively extended either '*inessentially*' or '*essentially*'. (Def.26)
   If the set of problems (= of non-simple concepts!) that can be posed in a conceptual system ***C*** is a proper subset of the set of problems that can be posed in a conceptual system ***C'*** then ***C'*** is a creative extension of ***C***. (Follows from 3.2.2 and 3.2.3)
   Analyticity is relative to conceptual systems. (3.6)
   There are synthetic concepts *a priori.* (3.9)

The next (final) remark is rather important. TIL, which is the 'philosophical' and 'technical' base of the present theory of concepts and conceptual systems, is a *logic*; contemporary logicians expect that every logical system is given by a system of *axioms* and *rules*. Is TIL given by such an axiomatic system? We have to admit that what we call transparent intensional *logic* is not and cannot be given by an axiomatic system. TIL is a theory whose logical character is rather clear; what could be called *semantic proofs* (which corresponds to *meta-proofs* in the standard logic) is unambiguously determined in TIL (see, e.g., good examples in [Tichý 1986]) and the following characteristics of logic from [Tichý 1978] should be recognised as being compatible with the standard theory:

> Logic is the study of logical objects (individuals, truth-values, possible worlds, propositions, classes, properties, relations, and the like) and of ways such objects can be constructed from other such objects. The logician makes it his business to explain, for example, how Bill, the individual, and walkerhood, the property, combine to yield or construct the proposition that Bill walks, and walkerhood combines with some other objects to yield or construct the proposition that everything walks. The point of investigating logical constructions of objects is

> two-fold. *In the first place, the nature of such constructions often guarantees noteworthy properties or relationships between the objects generated by those constructions*. For instance, the two constructions mentioned above assure that the proposition generated by the former is weaker than (i.e. is implied by) the proposition constructed by the latter. *In the second place, logical constructions can be assigned to linguistic expressions as their analyses.* For example, the former construction will serve as the logical analysis of the sentence "Bill walks" and the latter as the logical analysis of "Everything walks". Provided that those analyses are correct, the aforementioned relationship between the constructions legitimises an argument from "Everything walks" to "Bill walks". (Emphasis mine. —P.M.)

On the other hand, it should be clear that TIL is an open-ended *theory* (on this point see [Bealer 1982], 219-221) that cannot be exhaustively determined by any particular axiomatic system (NB completeness of such a system cannot be achieved, of course). Such a particular (Gentzen-like) system can be found in [Tichý 1982] for the 1st order (unramified) theory of types as formulated in TIL. Particular axiomatisations for the ramified version (see Definition 5) can be realised, of course, but any such particular axiomatic system can serve only for particular goals; no such system can be conceived of as *defining TIL*.

Therefore, neither in *Concepts and Objects* nor in the present book can any axiomatic system be found. Since the present theory of concepts and conceptual systems defines concepts as procedures ('constructions') not reducible to set-theoretical entities, any axiomatisation will be highly imperfect: classical models are set-theoretically oriented. This does not mean that no attempts should be made, of course.

The present book can be naturally divided into four parts.

The first part (Chapter 1) begins with general problems of semantics; it defines and defends *transparent intensional logic* (TIL) and recapitulates—with some modifications—the explication of the term 'concept' in *Concepts and Objects.*

The second part (Chapter 2) defines—in the same way as in *Concepts and Objects* —conceptual systems and presents the first fundamental application thereof in the *Intermezzo* ("Parmenides Principle").

The third part (Chapter 3) connects conceptual systems with languages and tries to define notions needed for the application of the theory of conceptual systems to a diachronic view which takes into account the development of language. We show that the results thereof can be interesting for the philosophy of science (in particular, the 'incommensurability' problem), for epistemology (analytic vs. synthetic) and suchlike.

Appendix 1 recapitulates the main symbols used in the book, Appendix 2 summarises some specific features of TIL, Appendix 3 does the same for the theory of concepts, and Appendix 4 shows an application of the approach presented to the solution of Putnam's 'Carnapian vs. Polish Language' problem.

Acknowledgments

First of all, I would like to express my deep gratitude to Marie Duží, who not only took care of technical problems with implementation of my text but—more importantly—read the text and contributed to its positive traits by thinking over and often correcting my views.

I am very much obliged to Timothy Childers, who corrected my linguistic faults and formulated numerous recommendations concerning not only linguistic moments. I am even anxious that wherever I did not obey his recommendations I missed an opportunity of making my ideas more readable and even more precise.

I owe many inspiring ideas to the Slovak philosopher and logician Pavel Cmorej, one of the friends of Pavel Tichý. I did not cite his numerous works (also concerning concepts) since they are written in Slovak or Czech, which makes them unfortunately inaccessible to philosophers and logicians outside the former Czechoslovakia.

I had time for essential modifications of my manuscript from 2003 due to the support of the Grant Agency of Czech Republic doing this work under the head of my general project No 401/04/2073.

Institute of Philosophy (Academy of Sciences of Czech Republic) ensured good conditions for my undisturbed work.

Finally I appreciate the moral and material support provided by my wife Helena.

0. Introduction

At least three monographs more or less concerning concepts appeared in the same year, 1998 (Bartsch, Fodor, Materna). It is as if philosophers, psychologists and logicians wanted to simultaneously confirm the claim that

> *a host of issues in logic, philosophy of language, philosophy of mind, and metaphysics come together in the theory of concepts and related issues of intensionality and ontology.* (G.Bealer in a personal letter.)

This 'interdisciplinary' character of the entity named *concept* is nearly commonly agreed on. Also Rey in his [Rey 1998] writes:

> *The topic of concepts lies at the intersection of semantics and philosophy of mind.*

Reading similar formulations we are tempted to say: What a chameleon a concept must be! Yet such an impression is misleading. It arises, of course, if somebody says:

> My topic is what concepts *are.* (Fodor in his [1998, 1]. Emphasis mine.)

Actually, what any author of a theory of concepts does is articulate some (optimal) *explication* of the term 'concept', explication being meant in Carnap's sense. Now any Carnapian explication should take into account the most frequent contexts containing the given term. And we will surely agree that there are many distinct kinds of context which contain the term 'concept', as well as that some of these contexts are mutually incompatible in the sense that what should correspond to the term 'concept' in one kind of context cannot satisfy the requirements given by another kind of context. (A typical example: the way the psychologists often use the term is incompatible with its use by, say, Frege.)

So there are obviously many notions associated with the term 'concept', which means that there is no one such entity which could be said to be *the* concept. The present study is an attempt at such an explication that takes into account mainly logico-semantic contexts. The explication, based on 1.2.2 and closed in 1.4.2, is essentially the one articulated in [Materna 1998]. The purpose of the present study is, however, different; I would like to show that our Platonic approach can contribute to the rational analysis of some problems connected with what I call *conceptual systems* and that are at the same time relevant from the diachronic view. I mean some problems whose formulation uses the word *development* and which are interesting from the epistemological viewpoint. The 'focal point' of the study lies therefore in Chapter 3. The explanations, or, if you like, 'models', offered in that section are, of course, not empirical explanations: the history of languages is not the subject of logico-semantic analyses. But if I am permitted to adduce an analogy, dynamics of natural events can be successfully studied by exploiting the non-empirical static tools given by mathematics.

Before beginning our own explication we should, however, mention some other options. I have chosen two independent bivalent criteria of classifying these options, so that I get four kinds of explication (which is indeed a very coarse-grained classification, but sufficient for our purposes):

Criterion A: I. Concepts as ***mental entities***.

II. Concepts as ***non-mental entities***.

Criterion B: I. Concepts as ***set-theoretical entities***.

II. Concepts as ***structured entities*** (procedures).

AI BI	AI BII	AII BI	AII BII
Traditional logic (?)	Intuitionists	Frege, Gödel, Church	Bolzano (?), Cresswell (?), Bealer (?), Materna

Now we will comment this classification.

1. Concepts

1.1. Mental entities

The main difficulty to be resolved by any mentalistic theory of concepts consists indeed in the fact that if concepts are ‘constituents of thought’ (see, e.g., [Rey 1998]), then they should be ‘shareable’ (*ibid*): otherwise one could not explain the obvious fact that people, in general, understand each other. (Or one would have to give up the fundamental intuition that communication proceeds *via* concepts.)

This difficulty has been recognised long ago. Ziehen [1920, 459] tries to resolve it as follows:

> Jede Vorstellung, die als unveränderlich mit Bezug auf denselben Gegenstand gedacht wird, wird dadurch aus dem Bereich des *Psychologischen* herausgerückt und zu einer *logischen* Vorstellung — Normalvorstellung, Begriff — umgedacht.

This kind of solution is, of course, not viable. *Vorstellung* in the ‘normal’ (not Bolzanian, not Fregean) sense is a psychological phenomenon. There is no guarantee that it is thought about one and the same object; it is a concrete (i.e., temporally and spatially localisable) phenomenon; concepts, unlike thoughts, should be abstract and their status should be independent of the way they are thought (even *umgedacht*) about.

There are some other difficulties, perhaps still more devastating, if we take concepts to be images. First, images are normally distinct in distinct brains (this is the general problem of shareability). Second, no image can correspond to an abstract object whereas everybody knows concepts of such objects (concepts of numbers, functions, properties etc. etc.). Third, we surely agree that an image “lasts”, i.e., that it exists just then and there when and by whom it is possessed, whereas we would hardly agree that a concept “lasts”, that a concept “exists” only in so far as it is possessed by somebody.

Notwithstanding these and similar problems, the mentalistic explication is not completely absurd. It has inspired some theories interesting from a psychologico-philosophical viewpoint; for example, one can be interested in ‘genetic theory of concepts’ and build up an essentially empirical theory based on the notion *stabilisation of possible expansions of sets under a perspective* (see [Bartsch 1998, 2]). We will take mentalistic explications to be acceptable as concerning such entities which may be interesting in some contexts (not in the logical ones).

1.1.1 Mental set-theoretical entities

The notion of *set-theoretical entity* has been explained in [Materna 1998]. To say that some entity is set-theoretical is to say that it is not complex, not an abstract procedure. In particular, a set cannot be executed. More details can be found in Tichý's excellent [1995] wherefrom it follows that to be a complex is more than to have constituents. (Bolzano seems to have been aware of this distinction, see 1.2.2.)

Set-theoretical explications make up a great majority of the explications of the term *concept*. Within the framework of mentalistic conceptions it is probably so-called *traditional logic* which could serve as a representative of set-theoretical explications. I do not think that the heterogeneous group of works classifiable with traditional logic must be always characterised as 'mentalistic'. As soon, however, as a textbook of logic 'defines' concept as a reflection (of essential features of the object) we can see that concept has to be mental (otherwise, how could it 'reflect' anything?). As for the set-theoretical character of the traditional explications, it is clearly visible from the doctrine of the reverse proportion between the so-called intension and extension of a concept. The extension of a concept is traditionally construed as a class/set of objects 'falling under' the concept. The intension of a concept is then the set of *Merkmale* (features) of the concept, which can be set-theoretically defined as follows: Let the extension of a concept C be the intersection of the extensions of concepts $C_1,\ldots,C_n$. (These are the *Merkmale* of C.) Then the intension of C is the set $\{C_1,\ldots,C_n\}$. The doctrine of the reverse proportion becomes in this way a trivial consequence of this definition. Notice that a) this conception presupposes that all concepts are *universalia,* i.e., general concepts, b) the intension is defined in terms of extension, and c) the *concept itself* (not only *its* extension and *its* intension) is, properly speaking, not defined, so that what remains is to take refuge in a 'psychological' characteristics ('reflection…').

Being extensionalist is not the same as being set-theoretical. Kauppi in her [1967] has formulated a theory of concepts that is perhaps the most modern formalisation of the traditional theory, and she tries to emphasise the 'intensional character' of this theory in Leibniz's sense. The relation *intensional containment*, which should guarantee this intensional character, is, however, deliberately undefined, it is a primitive notion in Kauppi's calculus. Yet even if its explication were given in the spirit of Carnapian intensions (as Kauppi seems to intend in [1967, 26]), Kauppi's notion of concept would remain to be set-theoretical. The point is that *intensions* are mostly modelled as *functions,* as *mappings* with possible worlds as arguments; mappings are, of course, set-theoretical entities, unlike procedures. (See [Materna 1998, esp. 81-82].)

Kauppi herself was no mentalist. It is mainly 'traditional logic' which can be associated with mentalistic set-theoretical systems; this does not mean that the set-theoretical approach would have been consciously applied. The mentalistic moment consisted, as we already suggested, in the pseudo-definitional claim according to which concepts were said to be 'reflections' of .... The set-theoretical moment is represented by the doctrine of extension and intension of a concept (see above). Indeed, the intension of a concept is a set of *Merkmale*: in at least this respect a concept cannot be said to be structured—a traditional logician cannot tell the difference between a concept and its intension (therefore, if we say that a *concept is a set of 'Merkmale'* instead of saying it of the *intension of the concept*, no traditional logician would probably protest). See 1.2.2.

One could object that the 'traditional concepts' are, of course, structured, since they contain components, viz. the mentioned *Merkmale*. In this sense we could, however, say that every set is structured, since its components are its members. Tichý's and my answer is: sets are properly speaking, *dichotomies*: they divide the respective area into members and anti-members which *together* determine the set. This situation does not give us the right to speak about *components* of a set. We will see that (abstract) *procedures* can be contrasted with set-theoretical entities. (For more details see [Tichý 1995].)

1.1.2 Mental procedures

I am not sure which of the theories of concept or concept theories (for this distinction see [Palomäki 1994]) could be construed as taking concepts to be mental *procedures*. Allow me, therefore, to imagine *intuitionists* as theorists of concept. If they agreed to conceive of concepts as *constructions*, which I think would be a very natural step, they would be good representatives of the possible doctrine "*Concepts are mental procedures".* Let the expression "construction" in the following text be substituted for by the expression "concept":

> If one had to define constructions in general, one would surely say that a type of construction is specified by some *atoms* and some *combination rules* of the form 'Given constructions $x_1,\ldots,x_k$ one may form the construction $C(x_1,\ldots,x_k)$, subject to certain conditions on $x_1,\ldots,x_k$'. [Fletcher 1998, 51]

We can see that the procedural character of such a theory of concepts is obvious. On the other hand, the intuitionistic conception makes constructions dependent on our ability to 'find' them. Sundholm adduces a classical example in his [2000, 7]:

> We consider, with Kronecker, a classical function $f \in \mathrm{N} \rightarrow \mathrm{N}$ that is defined by a non-decidable separation of cases:
>
> $f(k) =_{def}$ 1 if the Riemann Hypothesis is true
> $f(k) =_{def}$ 0 if the Riemann Hypothesis is false.
>
> According to Kronecker, and I agree, f is *not* well defined, that is, the rule does not give a function from N to N. Because consider f(14), say. There is at present

> no way of evaluating this to primitive form as an Arabic numeral, since we cannot (yet) decide the Riemann hypothesis.

This quotation, so characteristic of intuitionists, proves that constructions—and, in our thought experiment, concepts—cannot be independent of human creativity; thus concepts would be for intuitionists mental constructions.

Remark: 'Mental' means here 'mental' in the usual psychological sense. If 'being mental' were related to 'infinite mind', 'Absolute Intellect' or so, our considerations would be senseless. See [Köhler 2000].–

1.2 Non-mental entities

1.2.1 Non-mental set-theoretical entities

The most classical representative of non-mentalistic theories of concepts is obviously Frege. His [1892] defines concepts as characteristic functions of classes of objects; as functions they are 'unsaturated' so that the expressions denoting concepts cannot occupy another position in a sentence than that of predicate. If such an expression ('Begriffswort') stands in the subject position, then the respective concept becomes an object in virtue of replacing the original function by the 'Wertverlauf' of the latter (which is somewhat mysterious).

Some important points can be adduced as objections to Frege's theory (see, e.g., [Tichý 1988] or [Materna 1998] from the viewpoint of transparent intensional logic, and many critical articles biased in another way). Some of them, relevant for our purposes, are:

1) Only general (universal) concepts are taken into account. THE HIGHEST MOUNTAIN is not a concept for Frege *contra* our intuitions.
2) The same example can serve to show that Frege is the victim of what Tichý called *Frege's Thesis*, for example in [Tichý 1996a, 23]:

   > Special contexts aside, a descriptive phrase does not refer to the determiner linguistically associated with it but to the object (if there is one) which the determiner singles out.

   In the case of the empirical concepts this means, e.g., that for Frege the expression *the highest mountain* denotes the object that happens to be the highest mountain, i.e., Mount Everest, so it is not a *Begriffswort*. Or take a *Begriffswort*, say, the phrase *a black cat*. The respective concept is—according to Frege's Thesis (which is still tacitly accepted by most contemporary semanticists) —a function which associates a concrete object with **T(rue)** or **F(alse)** dependently on whether this object is or is not a black cat. This means, however, that the expression *a black cat* denotes distinct concepts at distinct time points, and that what concept is denoted by it depends on contingent facts, on the given state of the world; this consequence strongly clashes with our intuitions connected with the word *concept*.

3) Consider a mathematical concept, say, A PRIME NUMBER. To Frege the concept is the characteristic function of the class of prime numbers. One could ask, however, what difference there is between the class of prime numbers and a concept of this class. Properly speaking this class would be identical with its concept. (Well, one could agree, saying that mathematical concepts are just classes; my intuition resists accepting this view: there are many concepts of one and the same class.)

4) Knowing about Frege's idea of *sense*, which should be *a mode of presentation* of the object denoted and which would be *expressed* by the expression, one would expect that this idea, together with a common intuition associated with the word *concept*, would lead Frege to let the concept be *expressed* by the respective expression and serve as the *sense* of the latter. Yet the *Begriffswörter* do not express concepts: they *denote* them. Practically, they simply denote classes (as we already stated, the only distinction between a class and the respective concept would be the distinction between a class and its characteristic function). Briefly, concept is, for Frege, not a *way to the object*: rather it is an object (*sui generis*) itself.

On the other hand, Frege's conception is an anti-psychologist one. Concepts are not mental entities; rather they are inhabitants of the Platonic realm (although some formulations are not strictly Platonic). For the most part, Frege's theory belongs to the box AII BI.

Frege's problems are shared by all set-theoretical conceptions. At least points 1) and 4) were, however, unacceptable to the great Fregean Alonzo Church. In his [1956] Church deliberately ignored Frege's construal of concepts and—avoiding thus the objections under points 1) and 4)—placed concepts where Frege would place his *Sinn*. Briefly, Church has identified concepts with *senses* (today we would talk promiscuously about *meanings*). Then, of course, a mentalistic view is untenable. Cf. [Peacocke 1992, 237]:

> If 'meaning' is used correlatively with 'sense', meanings are the concepts expressed, not the mental representations of them.

Church's universalism in this respect is remarkable: concepts are for him expressed not only by universal expressions (as it would follow from Frege's theory), and not only by universal expressions and names/descriptions (as Bolzano had it) but by all (meaningful) expressions: as a consequence sentences also express concepts. This last point sounds provocative enough: traditionally nouns are supposed to denote something whereas sentences do not *denote* (and they cannot, therefore, possess a sense in the Fregean interpretation): sentences *claim* something, so that they can be—unlike the other kinds of expression—true or false. And it would be incompatible with the normal use of the word *concept* to say that concepts—since connected with sentences—lead to a truth-value: usually we say that concepts cannot be true or false. But this objection can be successfully dealt with as follows: in the case of empirical sentences concepts are ways of identifying what the sentence says, viz. the respective proposition. Whereas this *proposition* can be, indeed, true or false, the identifying procedure ("way") is not. So for example the proposition given by the sentence

There are living beings outside of our Solar system

is of course true or false, but the concept expressed by it only *identifies* this proposition, far from determining its truth-value. (See [Materna 1998, 65].) A somewhat modified consideration can be applied in the case of mathematical sentences.

Now the fact that Church has modified Frege's conception of concepts (since if every kind of expression expresses a concept, then the latter cannot be a characteristic function of a class) cannot change the set-theoretical character of Church's conception. For if we try to find out what exactly Church has meant by concepts, we can see that a most cogent interpretation could be delivered by P(ossible-)W(orld) S(emanticists) who would offer *intensions* as functions from possible worlds as *explicans.* One of pernicious consequences thereof would be that it would be impossible to define mathematical concepts.

Remark: To see this consider two examples. First, which intension would correspond to, say, the expression *a prime number*? Obviously it would be a *constant function* that would associate every possible world (and time) with one and the same class of numbers. Such trivial, constant intensions are hardly something what would distinguish the *class* from the *concept*. Second, still worse, if true mathematical claims were to express concepts as intensions, then one and the same concept would correspond to every such claim: the function **TRUE**, which associates every possible world with the truth-value **T**. –

Gödel in his [1990, 128] tried to define concepts set-theoretically as opposed to the procedurally construed *notions* of intuitionists: his concepts are

> properties and relations of things existing independently of our definitions and constructions.

To explain the difference between this definition and the construal of intuitionists' *notions* he says (*ibidem*):

> Any two different definitions of the form $\alpha(x) = \varphi(x)$ can be assumed to define two different notions α in the constructivistic sense.

Interestingly enough, Bealer in [1982] uses the fact that there are distinct definitions of 'the same object' as a criterion of there being distinct concepts. Bealer is no intuitionist and — similarly as Gödel — he strongly opposes mentalism. Thus he more or less belongs to AII BII.

1.2.2 Non-mental abstract procedures

To be a realist who does not construe concepts as set-theoretical entities is a rare property. It seems that Dummett's 'anti-realism' (see [Putnam 1983]) stems just from his inability to imagine something like that. Gödel's set-theoretical conception of concepts may have the same roots: Gödel obviously thought that a procedural construal of concepts is necessarily connected with mentalistic philosophy. Perhaps the first suggestion of another view can be found in Bolzano in his [1837]. This suggestion in § 56 is highly remarkable. Bolzano defines here *intension* of a concept (*Inhalt*) —explicitly of a *Vorstellung an sich*, but concepts (*Begriffe*) are the most important kind of Vorstellung *an sich*—and as a realist he derives complexity of a concept from complexity of the respective expression. The intension (better perhaps: content) of a concept is for Bolzano

> die Summe der Theile, aus denen eine gegebene Vorstellung an sich bestehet. (244)

Now we could say that a similar definition could be found in any textbook of traditional logic. Yet even neglecting the anti-mentalistic spirit of Bolzano's logic we have to state that a principal difference of Bolzano and tradition concerning theory of concepts consists in the fact that Bolzano somehow derives the elements of *Inhalt* from the linguistic structure of the expression whereas the traditional logicians were able to analyse the structure of an expression only as far as the elements of the structure were connected *via* conjunction. The famous doctrine about the reverse proportion of intension and extension of a concept is based just on this assumption. Bolzano obviously was aware of the fact that such a 'conjunctive' analysis is very poor and admitted such analyses that led to components ("*Theile"*) that were connected in another way than *via* conjunction. This can be seen as soon as we read his §120 of [1837], where he criticises (unjustly, in an obvious sense) the doctrine mentioned above and takes into account such *Merkmale* that the traditional logic could not recognise as *Merkmale*. (Cf. his famous example of the concepts A MAN WHO UNDERSTANDS ALL EUROPEAN LANGUAGES vs. A MAN WHO UNDERSTANDS ALL LIVE EUROPEAN LANGUAGES.) This point is very important, since — as we now know very well — a logical analysis of an expression cannot be indeed reduced to finding the 'conjunctively connected' components. But not only that. Bolzano—unlike the tradition—is able to precisely distinguish the intension of a concept from the concept itself: He says (244):

> Da unter diesem Inhalte nur die *Summe* der Bestandtheile, aus denen die Vorstellung bestehet, nicht aber die *Art, wie* diese Theile untereinander verbunden sind, verstanden wird: so wird durch diese blosse Angabe ihres Inhaltes eine Vorstellung noch nicht ganz bestimmt…

Thus we can say that a concept consists in *combining the elements of its 'intension'.* (So that distinct concepts can share their intension, as Bolzano exemplifies in the same §. This holds, of course, only if the elements of the intension of the given concept are *simple* at least in the Bolzanian sense.)

Bolzano's suggestion remained to be a suggestion; Bolzano himself could not have elaborated his idea so that it would be acceptable according to the contemporary standards. Approximately after one hundred years Church — himself belonging to AII BI — invented an ideal tool for this *combining* in [1940].

Properly speaking not many explicit articulations of a procedural theory of concepts can be found in the history of logic after Bolzano. If, however, Church's identification of concepts (of a *denotatum*) with senses (meanings) (of the expression) is accepted one can see that such a theory can be derived from the attempts at defining *structured meanings*. The important part of the story begins probably with Carnap's [1947], where Carnap became aware of the fact that the L-equivalence criterion of synonymy does not work as soon as propositional attitudes are to be analysed. Carnap's notion of *intensional isomorphism* was intended to derive *semantic structure* from the *syntactic structure* of the given expression. The result is, however, *too close* to the grammatical structure, and Carnap's notion can be criticised not only from Church's viewpoint (see Church's surely inspiring objections in his [1951]) but — more fundamentally — from the viewpoint of a realistic semantics: see [Tichý 1988, 8-9].

David Lewis in [1972] identified meanings with interpreted phrase marker trees; so he gets a structure in a sense but the usual theory of trees makes it possible to reduce them by definition to set-theoretical entities.

The same objection can be applied to the theory that has been systematically explained in [Cresswell 1985]. Cresswell, who has coined the phrases *hyperintensionality* and *structured meaning*, tried to save the particular components of this 'structured meaning' by creating tuples embedding, as the case may be, other tuples. To adduce a most simple example, the tuple

$$\langle -, 9, 5\rangle$$

should be the meaning of the expression

$$9 - 5.$$

The point against this solution can be formulated as follows: Cresswell is well aware of the fact that the meaning of the expression above cannot be "the result", i.e., the number 4. (A very good and simple argument is that meaning enables us to understand the given expression; to explain what the expression above means we would certainly not cite the number 4.) Thus he defines such semantics where the particular components of the expression are associated with their semantic (better: 'ontological') counterparts and such a counterpart of the whole expression is defined as the tuple of those particular counterparts (taking into account various levels of embedding). Yet tuples are tuples, nothing more. Even when we neglect the fact of their reducibility to set-theoretical objects (mappings from the set of natural numbers), the notion of a tuple does not contain the specific *roles* of the particular components of the tuple, so that we cannot speak about (abstract) *procedures*. This point will be much clearer in 1.3.3.

Remark: Much more details in this respect can be found in [Tichý 1996a], [Jespersen 2003], where some further problems with Cresswell's solution are analysed. –

The set-theoretical character of Cresswell's 'meanings' is explicitly stated already in his [1975]: introducing (p.30) $I(\alpha)$ as the intension of α, $M(\alpha)$ as the *meaning* of α and V as the value assignment for his language Λ he defines $I(\alpha)$

for α simple: $I(\alpha) = M(\alpha) = V(\alpha)$,

for $\alpha = \langle\delta, \alpha_1, \ldots, \alpha_n\rangle$: $I(\alpha) = I(\delta)(I(\alpha_1),\ldots, I(\alpha_n))$,

$$M(\alpha) = \langle M(\delta), M(\alpha_1),\ldots, M(\alpha_n)\rangle,$$

and says:

> The point is that the intension of a complex expression is obtained by allowing the intension of its functor to operate on the intensions of the arguments of the functor. *The meaning however is simply the* n+1*-tuple consisting of the meaning of the functor together with the meanings of its arguments.* (Emphasis mine.)

And on p.32 we read:

> [T]ruth-conditional semantics is sufficient to determine meaning.

So we get again **the set-theoretical paradigm**: *what counts is always the result of applying a procedure rather than the procedure itself.*

Nevertheless, Cresswell could be classified with AII BII, since he explicitly stated the thesis that meaning (and, therefore, concept) has to be *structured,* and made an attempt — although a problematic one — to satisfy this requirement.

A special attention has been paid to 'structured propositions'. The idea of structured propositions has been articulated first by Russell in his [1903]. Briefly, the idea consists of two points: first, propositions contain constituents, second,

> [e]very proposition has a unity which renders it distinct from the sum of its constituents. ([1903, 52])

Notice that this second point is closely related to what Bolzano said about the distinction between content of a concept and the concept itself (see [1837, 244], quoted above). The weak points of Russell's notion are well-known (see, e.g., King in [1997], Tichý in [1988, 68-70]).

Referring to and criticising Kaplan's, Salmon's, Soames' and Zalta's attempts at introducing the notion of structured propositions King ([1997]) asks a key question (p.6):

What Binds Together the Constituents of Structured Propositions?

This is indeed a (or even *the*) key question. If *concept* should be explicated as a structured entity, then an adequate answer to a generalisation of this question—an answer that would confirm the non-set-theoretical character of concepts—would entail an adequate answer to the question above. The generalised question is:

What Binds Together the Constituents of a Concept*?*

Since one of the kinds of concept is concept of a proposition, our answer to the generalised question implies the answer to the question

What Binds Together the Constituents of a Concept of a Proposition?

Bealer's theory of P(roperties)R(elations)P(roposition) is also a theory of concepts, as we can see reading his [1982]. While his 'qualities', 'connections' and 'conditions' (let them be called 'intensions of the 1^{st} kind') correspond essentially to set-theoretical intensions known from PWS (which does not mean that Bealer would define them as PWS do), his 'concepts' and 'thoughts' (intensions of the 2^{nd} kind) are counterparts of concepts as procedures, which again does not mean that they would be defined in this manner. The comparison I just formulated is supported by the fact that the intensions of the 1^{st} kind are — according to Bealer — sufficient for analysing modalities, whereas intensions of the 2^{nd} kind are what is needed for treating intentional matters (attitudes). We will see that this distinction can be explained just *via* taking intensions of the 1^{st} kind as set-theoretical objects (functions *qua* mappings) and intensions of the 2^{nd} kind as concepts *qua* procedures. (See also [Materna 1998, 75-77].)

As far as I know it is just my [1998] which has explicitly defined concepts as abstract procedures. In what follows the philosophical background and the respective technical tools will be introduced.

1.3 Basic philosophy of 1.2.2: TIL *(informal exposition)*

The main problem with construing concepts as abstract procedures non-reducible to set-theoretical entities consists in finding (more or less 'standard') logical tools for handling such procedures. Surprisingly (for many philosophers and logicians) such tools have been found in the unjustly neglected work of Pavel Tichý (see particularly [Tichý 1988], [Tichý 2004]).

The purpose of the present study is not to look for the reasons of the highly unfortunate fact that transparent intensional logic (TIL), as Tichý coined his system, has been nearly completely ignored. One of these reasons (perhaps a secondary one) is however important, at least if we want to understand the claims and definitions essential from the viewpoint of the genuine purpose of our study. Using a Kuhnian terminology, we could say that in a sense TIL means a *change of paradigm*; the change is however only partial, it can be characterised (not defined) by a succinct quotation from [Tichý 1988, viii]:

> The theories of Frege and Russell are far from 'noble ruins', interesting only from an historical point of view. They are, rather, the most advanced theories of objectual logic we have. Those who believe that there is more to logic than the study of finite strings of letters, have to go back to where Frege and Russell left off and go on from there.

Thus the change of paradigm I talk about is not directed against some 'paradigm of logic'. Rather it opposes the more or less formalistic trends, sometimes connected with the 'linguistic turn' but characteristic mainly of contemporary mathematical logic, which became simply a branch of mathematics—unlike the original symbolic logic from the 'Russellian times': as Tichý says *ibidem* about Frege and Russell:

> [t]hey themselves were not symbolic logicians; a symbolism to them was not the *subject matter* of their theorizing but a mere shorthand facilitating discussion of extra-linguistic entities.

This struggle for restoring the (original) objectual character of logic, often articulated in a rather provocative style, could have brought about a misleading impression, as if TIL fought against exact methods which make up the distinction between traditional and modern logic. Not in the least. The philosophy of TIL differs from some nearly standardly accepted assumptions made by logicians but using exact methods usually called 'formal' is a self-evident part of what TIL does. On the other hand, this philosophy does have an essential influence on the way the logical problems are stated and solved. The core of this 'philosophy of TIL' will be explained in the following sections. Now only a brief quotation from [Tichý 1978, 275]:

> Logic is the study of logical objects (individuals, truth-values, possible worlds, propositions, classes, properties, relations, and the like) and of ways such objects can be constructed from other such objects.

Now we have to admit that for many contemporary (post-)analytic philosophers like the followers of Quine, not to mention Rorty et al., the position characterised in this way looks entirely outdated; like if it were a sort of philosophical superstition to believe that there were some such abstract extra-linguistic entities like properties, relations, let alone possible worlds. Logic should look for as 'successful' 'models' of our using language as possible: to believe that *there are* ('literally') such entities as properties etc. means to violate Ockham's razor. Yes, we have to use such expressions as *property* etc., but we must not forget that such *universalia* are "conceived in sin" (as we are told by Quine).

Unfortunately, these philosophers are unable to answer some rather simple questions:

a) What do you mean by 'model' in this case? Is it an empirical description of our linguistic behaviour? Then it is irrelevant for our *logical* purposes. Is it some *axiomatisation*? Well, this is meaningless without an 'intended interpretation', and such an interpretation exploits at least sets/classes, in any case some extra-linguistic entities. Should they be only irrelevant means of our modelling?

b) When do we say that our model is successful? Is successfulness something what does not need an explanation? ("Yes, it is", is what a neopragmatist says, but a logician should steer clear of any form of pragmatism.)

c) According to the attitudes mentioned above I am allowed to *speak* about properties etc., I only must not assume that they 'literally exist'. Now what is the meaning of this 'literally'? Does anybody really believe that a realist (for this is essentially a modern form of the dispute between nominalists and realists) thinks of abstract entities that they have some spatial and temporal existence? It is just realists who know that abstract entities are not spatially and/or temporally localisable. (Bolzano would say "sie haben keine Wirklichkeit"). So why this quarrelling as concerns the 'genuine existence'?

d) As for the famous Ockham's razor, the best answer is given in [Tichý 1995, 175]:

> The vision informing 20th Century philosophy has been aptly described as one of a desert landscape. Philosophers behave as if in expectation of an ontological tax collector to whom they will owe the less the fewer entities they declare. The metaphysical purge is perpetrated under a banner emblazoned with Occam's Razor. But Occam never counselled ontological genocide at all cost. He only cautioned against multiplying entities *beyond necessity*. His Razor is thus in full harmony with the complementary principle, known as Menger's Comb, which cautions against trying to do with less what requires more. The two methodological precepts are just two sides of the same coin.

So let me say "No!" to this post-fashion and suppose that logic is about abstract entities, that these entities should not be said 'to exist' but rather 'to be objective', independent of particular minds, and that concepts are a kind of abstract entity.

Remark: Köhler's conception in [2000] is not as incompatible with these claims as it could seem at first sight. The form of Platonism suggested above could be hard to swallow for somebody: let him/her read Köhler's formulation. –

Now we will explain basic definitions and principles of TIL. Our aim in the first part of the present study is to explicate *concept* in terms of TIL; some terminological (perhaps even not only) deviations from Tichý's [1988] arose during our working out the explication in question. Many essential points have been already explained in our [1998].

1.3.1 What is Frege's *sense*?

We can read hundreds of articles and monographs commenting Frege's [1892a]. The core of the famous 'Frege's problem' is the question

> *How to explain the fact that a true sentence of the form a = b can be informative unlike the sentence of the form a = a.*

(Indeed, it *is* a problem: assuming — as the condition does — that the sentence is true we have to accept that ***a*** *denotes* ('*bezeichnet'*) the same object as ***b***.)

Frege's attempt at resolving this puzzle is well-known: the informative character of the sentences of the form ***a*** = ***b*** is explained by the claim that the expressions ***a***, ***b***, although they denote the same *Bedeutung* (*denotation/reference*; later we will see that these options of translation can be fundamentally distinguished) but they do it in another way: they differ by *expressing* distinct *senses*.

The idea is clear: the sense is *the way the object is given* (Frege says: *die Art des Gegebenseins*, the standard translation is *the mode of presentation*). The idea itself is however no definition: rather it determines a *task*: the task of *defining this 'way to the object'*. Frege never fulfilled this task; so host of attempts to find an adequate definition can be found in the post-Fregean literature.

This is not a historical study. We want to reproduce basic assumptions of TIL, so let us summarise only the points important from this point of view. The two options of defining sense that are relevant for our contrasting AII BI, AII BII are:

A. Senses are intensions (as defined in PWS).

B. Senses are constructions (as defined in TIL).

To analyse these two options is of key importance for achieving the aim of the first part of the present study: we must not forget that accepting (as we do) Church's idea of identifying senses with concepts we can equivalently formulate our options as follows:

A'. Concepts are intensions.

B'. Concepts are constructions.

Yet our answer is — among other things — dependent on the analysis of the *denoting relation*. So we come to the following point.

1.3.2 Denotation vs. reference. Intensions and extensions

The word *Bedeutung* chosen by Frege to name the object *denoted* (*bezeichnet*) by an expression is a very bad choice. The most verbatim translation would probably be *meaning*, but meaning is commonly conceived of as that entity which makes it possible to *understand* a given expression, so *meaning* would best correspond to what Frege had in mind when talking about *Sinn*. Thus, Church translation uses *denotation*, ([Church 1956]) whereas Geach and Black ([Geach, Black 1952]) introduce *reference*. We will see that denotation is actually not the same as reference, so let us use meanwhile the neutral expression *object*; it will be used in the sense *object denoted by the given expression*.

Let us return to the famous example of Frege's problem, viz. to the sentence

morning star = evening star.

When we go through the host of articles and books that try to analyse this example we can see that nearly every author accepts Frege's careless assumption that the object commonly denoted by both nouns is the celestial body Venus. One of the most important claims articulated by TIL rejects this assumption. Many arguments for this rejection can be found in Tichý's book and articles—for very concise argumentation see his [1978a]. Here we try to formulate the core of this argumentation:

Frege's problem would not exist (in connection with the example above) if *morning star* and *evening star* were proper names (i.e. something like Millian labels) like *Venus*. The equality above would be simply a linguistic statement about synonymy. The informative character of the sentence is, however, confirmed by the fact that the sentence is an *empirical* sentence which has been verified by astronomers. But then, of course, both nouns cannot denote Venus, the body; rather they denote two distinct *conditions* an individual has to fulfil to become the morning star (the evening star).

True, these conditions are commonly interpreted as being just Fregean *senses*, and are explicated as intensions as defined in PWS. Then, under this interpretation, we get the following 'solution' of Frege's problem: both nouns *denote* one and the same object (Venus) but *via* distinct senses (conditions—intensions). A strong objection to this solution can be formulated. The first place in [Frege 1892a] where the need of *Sinn* is argued for is not the popular example with the morning star; the first place introduces the example with medians of a triangle. The sentence that states that the point of intersection of medians *a* and *b* is the same as the point of intersection of medians *a* and *c* is only formally analogous to the sentence about morning star and evening star. The distinction is given by the commonly neglected or underestimated fact that the former, unlike the latter, is *not an empirical sentence.* Being the point of intersection of the respective medians is not dependent on the state of the world, and the sentence stating the identity of both points of intersection is a mathematical claim true in all possible worlds. Thus the sense of the sentence cannot be an intension (or it would have to be a 'trivial' intension with constant value in all the worlds, which would make the idea of sense needless).

Another point is that the expression *morning star* was surely not created in English (as well as *Morgenstern* in German) to enable us to name Venus in another way. (Another example: the expression *the highest mountain* is a well formed English expression; it surely did not come into being only after it became known that the condition it encodes is fulfilled by Mount Everest.)

These and similar arguments support the general claim: *Empirical expressions denote intensions (in the PWS sense), never the actual objects that are their values in the actual world.*

One of the consequences of this claim is a principle which we will call *Parmenides' Principle* (a term coined by Tichý in a manuscript) which can be found already in [Frege 1884]; see [Duží, Materna 2003]:

(*Parmenides' Principle*)

An expression is not about an object X unless it contains a name of X.

Consider the sentence

The highest mountain is in Asia.

This sentence is among others about the property to be in Asia and about the highest mountain. *It is, however, not about Mount Everest.* This can be seen from the simple logical fact that our sentence *does not imply* the sentence

Mount Everest is in Asia.

True, many people are ready to infer this sentence from the premiss above, but this is only because of their implicit knowledge of which object is the highest mountain. The inference is therefore logically correct only if another premiss is added:

Mount Everest is the highest mountain.

This sentence (as well as the now correct conclusion) is about Mount Everest (and about the highest mountain as well).

Setting aside (for the time being) the question of what would correspond to Frege's *sense* we will show how TIL classifies the area of objects that can be *denoted* by an expression of a natural language. This classification, based on *a type-theoretical hierarchy,* harmonises well with our intuitions.

Remark 1: Problems with the type-theoretical approach are well-known. Attempts to solve logico-semantic problems within type-less (type-free) conceptions are referred to for example in [Orilia 1999]. All the same, the way natural languages create complex expressions from more simple ones is best understood when the underlying ontology is explained in terms of *functions* of various orders and their applications to arguments. Montague (see his [1974]) knew this; and categorial grammars, based on Ajdukiewicz' similar intuition, corroborate this conviction. Besides, while the semantics of the typed λ-calculus is simple and natural, this cannot be said of the semantics of the type-free λ-calculus (Scott's domains are an excellent but unfortunately unintuitive interpretation.) –

Remark 2: The next pages could discourage those readers (most probably philosophers) who are not accustomed to using symbolic expressions. Yet we should not forget that the philosophy of our conception is strongly opposed to the formalistic abuse of symbolism. Symbols are for us only 'shorthand' which helps us to *understand* more exactly what is being said. To give an example, imagine that we would express our knowledge of arithmetic of natural numbers in terms of verbal expressions like three times four is the same number as five plus seven, and, e.g., learn the multiplication table in this way. When introducing symbolic expressions we will always give intelligible definitions without any non-trivial tacit assumptions. An effective cooperation between logic and analytic philosophy is impossible unless logicians cease using esoteric symbolic jargon *and* philosophers cease *a priori* to avoid symbolic texts. –

Ultimately, our hierarchy of types will be a *ramified hierarchy* of functions, see [Tichý 1988, 65-70]. Yet now we will define a *simple hierarchy*.

First we have to pre-theoretically motivate the choice of *basic* (*atomic*) *types.*

i) We certainly need the set **{T, F}**, where **T** is interpreted as the truth-value **True** and **F** as the truth-value **False**. (The meaning of *interpreted* in this context is succinctly explained in [Tichý 1988, 195-196].) One reason is that languages contain names of these objects, in English *yes* and *no*; further some sentences denote truth-values (mathematical sentences), some other propositions (empirical sentences), which can be true or false.

ii) A set called *the universe,* whose members will be called *individuals*, is another basic type; individuals are the simplest objects which can possess various contingent properties and are pairwise numerically distinct.

Remark: The types under i) and ii) correspond to Montague's types *t, e,* respectively.–

iii) The existence of *tenses* (see [Tichý 1980] for an excellent analysis of tenses in English) as well as the fact of the *temporal variability* of the values of intensions make it necessary to add the type whose members are *time moments ("times").* This type serves also as the type of numbers; TIL assumes that time is a continuum, so the type of time points is at the same time the type of *real numbers.*

iv) We already suggested that we need intensions. TIL employs a *possible-world semantics* (PWS); the last atomic type is the *logical space* (of the given natural language), whose members are *possible worlds* that are best thought of as (consistent maximal) sets of possible facts. That this characteristic does not lead to circularity is again best explained in [Tichý 1988, sections 36 and 38].

It is obvious that nominalists will object to this host of abstract entities. Never mind; what could be more interesting is a criticism of the *choice* of the basic types and/or the way they are characterised. One example is [Sundholm 2000], where each of the types above is subject to criticism. (This is however not the place where answers to the particular objections should be given.)

To mark the basic types under i)—iv) we use Greek characters ο, ι, τ, ω, respectively. The other types are generated as *sets of partial functions* over ο, ι, τ,ω.

Definition 1 (*types of order* 1)

i) The sets ο, ι, τ, ω are *types of order* 1 (they make up a kind of *base*).

ii) Let α, $\beta_1,\ldots,\beta_m$, $m \geq 1$, be *types of order* 1. Then the set ($\alpha\ \beta_1\ldots\beta_m$) of all partial functions whose arguments are tuples with members of the types $\beta_1,\ldots,\beta_m$, respectively, and values are members of α is a *type of order* 1.

iii) Only what is given by i), ii) is a *type of order* 1. –

Definition 2 (*α-objects*)

Let α be any type (of order 1, what follows will hold also for any higher order). Members of type α are called *α-objects.* –

Examples:

Arithmetic operations like addition, subtraction, etc., are (τττ)-objects.

Arithmetic sets are (οτ)-objects, binary relations like > are (οττ)-objects.

Indeed, any arithmetic operation of the kind above is a function that associates any pair of numbers with at most one number. (Partiality is obvious, e.g., in the case of division, where no pair with 0 in the second place is associated with any number.). As for sets/classes, it holds in general for any type α that a class of α-objects is an (οα)-object, viz. a function that associates members of the class with **T** and anti-members with **F**, analogously for relations.

Now which type would we associate with the object *the highest mountain*? It cannot be ι: this would mean that the respective definite description would denote an individual, but we have seen already that the objects *Mount Everest* and *the highest mountain* are two distinct objects. We have said that Mount Everest only happens to fulfil the condition that is encoded by the description, thus the object *the highest mountain* is just this condition that an individual has to fulfil. As such it is best modelled as a function whose type is ((ιτ)ω): given a possible world W this function associates with it a *chronology* of individuals, i.e., a function that (in W) associates every time point with at most one individual, viz. that one (if any) which is in W at the given time point the highest mountain. The dependence on possible worlds can be called *modal variability* (of the values of this function), the dependence on time points is then the *temporal variability*. –

Now we can define objects called *intensions*.

Definition 3 (*intensions*)

For any type α: the $((\alpha\tau)\omega)$-objects are *intensions.*

Intensions with constant α-value for any τ, ω are *trivial intensions.* –

When we speak about intensions without the qualification 'trivial' we mean non-trivial intensions. So we can say that *empirical expressions denote intensions.*

Abbreviation: For any type α, we will write $\alpha_{\tau\omega}$ instead of $((\alpha\tau)\omega$. –

Remark: If a type α is not of the form $(\beta\omega)$ for some β, any α-object will be called an *extension.* There are therefore objects which are neither intensions in the sense of Definition 3 nor extensions: their type is $(\alpha\omega)$, where α is not of the form $(\beta\tau)$. See examples in [Tichý 1980]. One important example is the type of the actual world, which is $(\omega\omega)$. –

Now we have to explain an important point. According to Parmenides' principle the sentence *The highest mountain is in Asia* is not about Mount Everest but about the highest mountain. Mount Everest is an ι-object while the highest mountain is obviously an $\iota_{\tau\omega}$-object, an "individual office" (as the founder of TIL termed this kind of intension) or individual role. Which type should be associated with *being in Asia*? As an empirical predicate this expression denotes a *property of individuals*, i.e., an $(o\iota)_{\tau\omega}$-object. Our sentence has to predicate this property to an individual but *the highest mountain* names no individual; the property to be in Asia cannot be predicated to an individual role. The solution of this problem will be given in 1.3.3. Informally, the sentence is true in those possible worlds and times where the individual occupying the individual office of the highest mountain is in Asia: this agrees well with our claim that empirical sentences do not denote truth-values but their 'roles', i.e., propositions — the latter take the value **T** in some worlds-times, **F** in other worlds-times and, as the case may be, are without any truth-value in the remaining worlds-times (in our case: in those ones, that is, where there is no highest mountain).

Now we can distinguish between *denotation* and (what we intend to call) *reference.*

a) In the case of non-empirical expressions there is no reason to make the distinction.
b) In the case of *empirical expressions* it holds that they *denote intensions*; their *reference* is the *value of the respective denotation* in the actual world+time ('absolute reference') or we can speak about their *reference w.r.t. the couple ⟨possible world, time point⟩*.

1.3.3 The idea of constructions

Intensions, as defined in TIL (and, in general, in PWS), are functions *qua* mappings, so they are *set-theoretical objects*. We have seen that concepts — if they are to play the role of meanings — can hardly be construed as set-theoretical objects. They should be rather *ways of identifying objects.*

Remark: For a Platonist these ways are objective. It would be interesting to compare some psychological or semi-psychological theories with this objectual conception; perhaps substituting in such mentalistic contexts "discovering concepts" for "concepts" and letting (objective) concepts be what is discovered could show usefulness of the Platonist view even for mentalists. (Cf. [Bartsch 1998], [Fodor 1998]) –

A parable illustrating the conception of concepts as *ways, paths* can be borrowed from Tichý 1988, 1], where it is used for *constructions*. Imagine travelling to some ('target') town X over some definite geographical points like towns or such like. Any such travel to X (and there are a host of them) can be described as an *itinerary*. Now whereas X, the target, does not contain the particular points of the given itinerary (and is fully independent of which itinerary has been chosen), the itinerary does, of course, contain these points. (Later we will see that this parable is not perfect, but now it is at least didactically useful.)

Going from the parable to a genuine example consider a simple arithmetic expression, say,

$$0 + 1.$$

There is no problem with denotation here: this expression denotes obviously the number 1. The same number is, of course, denoted by infinitely many expressions, e.g.

$$3 – 2 \text{ or } \textit{one} \text{ or } 1^2, \text{ etc.}$$

Using Church's terminology we can say that the *meanings/senses* expressed by these expressions are distinct *concepts* of one and the same *denotation*. (This is a variant of Frege's example with the medians.) Now we can ask: What is the *way*, the '*itinerary'* that leads to the object 1 in the particular cases?

An answer can be given by what I would call *linguistic deviation*. The way is given by the grammatical rules of the language used. From our viewpoint this is an unsatisfactory answer: the grammatical rules themselves are specific for the given language, and they are distinct for the language of arithmetic as used in our examples and for other arithmetical notational variants as well as for natural language (*zero plus one, three minus two* etc.). The obvious fact of mutual translatability (unproblematic in our examples) calls for an explanation. The explanation offered by TIL goes as follows: the factor shared by all the synonymous expressions (of various languages/notational systems) consists in 'encoding' *one and the same abstract procedure*. Informally this procedure (in the case of our examples) can be described as *identification of the denotations of particular meaningful expressions followed by applying an operation to these denotations.* In the case of the first example we identify the numbers 0 and 1 and the function +, and apply this function to the pair of these numbers. In the last example we identify the numbers 1 and 2 and the function *raising to a power*, and apply this function to the pair ⟨1,2⟩. A most important point is that this explanation can explain only if such procedures are viewed as extra-linguistic, objective procedures, so that the way to explicating meaning *independently* of synonymy and analyticity is open: Quine's criticism of Carnap in [Quine 1953] is refuted.

The idea of constructions consists in the project of defining a minimum class of basic procedures such that complex procedures could be composed (obeying unambiguous rules) from these basic ones.

The last version of realising this project can be found in [Tichý 1988]. Six basic procedures, partially inspired by the typed λ-calculus, are defined. For some reasons not important here (but see, e.g., [Materna 1998], 39, and [Zlatuška 1986]) I decided

a) to skip two of them,
b) to suggest some modifications.

Before I reproduce the definition modified according to a), I will try to formulate the motivation for the here accepted choice of basic procedures.

A. Area of objects of order 1.

In general, procedures have to operate on some input and return some output. Procedures called *constructions of order 1* operate on and return *objects of order 1*, where an object of order 1 is any α-object with α a type of order 1.

B. Variables.

Procedures have to be extra-linguistic entities. The usual definitions of variables assume that they are letters, characters, which can take values due to a value assignment or be bound by some 'operators'. Thus it is clear that either variables cannot be procedures/constructions, or else that they cannot be letters. TIL, inspired by λ-calculus, works with variables but these are construed as constructions *sui generis*, as atomic constructions which construct objects of the given type dependently on *valuation*; we say that they *v*-construct objects, where *v* is the parameter of valuations. The letters used for variables in TIL are, of course, *names of variables.* A thorough exposition of this conception of variables can be found in [Tichý 1988, 56-62].

Remark: In [Materna 1998, 50, Note 11] I suggest the possibility of a variant (which would be in a sense equivalent) theory of constructions; the inspiring factor would be Curry's and Feys' combinatory logic where no variables are needed. It seems that J.Peregrin in his [2000] would prefer this variant. –

C. Trivialisation.

The 'input' of constructions comes primarily from the area of objects of order 1. (We will see however that objects of higher orders are also possible inputs.) One way of getting these inputs is *via* variables. (But variables are at our disposal for any type, i.e., also for types of higher orders.) Anyway, another way is possible: imagine a construction that accepts an object (of any type) and returns the same object. Such a construction (called *trivialisation*) can be construed as the simplest non-atomic construction that constructs an object without using other constructions. (A *psychological counterpart* could be a situation where a child — for example — would be able to recognise a circle without any notion of such concepts as POINT, DIAMETER etc.) In λ-calculi there is no counterpart of this kind of construction; nevertheless, it will play a most important role in our later analyses.

D. Composition.

TIL is a system based on the notion of *function*. Therefore it is inspired by λ-calculi rather than by predicate logics. Church's ingenious idea of reducing procedures to *applying functions to arguments* and *creating functions via λ-abstraction* has been realised on the objectual level in TIL. The construction called *composition* corresponds to the former procedure. It consists in constructing a function and an argument and applying this function to the argument. The result is the value (*if any*) of this function on the given argument. The composition — unlike its result — contains all the steps necessary for obtaining the result, i.e., the (v-)construction of the function, the (v-)construction of the argument *and* the operation of applying the former to the latter. (The last member of this conjunction distinguishes a composition from a simple tuple — see the criticism of Cresswell in 1.2.2. and Bolzano's conception of concepts.)

E. Closure.

The construction called *closure* corresponds to λ-abstraction. A good intuitive exposition on the 'linguistic' level can be found in [Church 1956, Ch.0].

Now we can formulate the definition of constructions. Since *construction* is what is defined, it will be as usually italicised. Particular *kinds* of constructions will be emphasised *via* boldfaced letters.

Definition 4 (*constructions*)

i) ***Variables*** are *constructions.*

ii) Let X be an object of any type. Then ^{0}X is a *construction* called ***trivialisation***. ^{0}X constructs X without any change.

iii) Let X, $X_1,\ldots,X_m$, $m \geq 1$, be *constructions* which v-construct respectively $(\alpha\beta_1\ldots\beta_m)$-, β_1-,…,β_m-objects. $[XX_1\ldots X_m]$ is a *construction* called ***composition***. If X v-constructs a function F and F is defined on the tuple v-constructed by $X_1,\ldots,X_m$, then $[XX_1\ldots X_m]$ v-constructs the value of F on that tuple. Otherwise — i.e., if F is undefined on that tuple or some X_j fail to v-construct an object — $[XX_1\ldots X_m]$ is v-improper, i.e., does not v-construct anything.

iv) Let $x_1,\ldots,x_m$, $m \geq 1$, be pairwise distinct variables v-constructing respectively β_1-,…,β_m-objects and X a *construction* that v-constructs α-objects. Then $[\lambda x_1\ldots x_m X]$ is a *construction* called ***closure***. It v-constructs a following function F: Let $\langle b_1,\ldots,b_m\rangle$ be a tuple such that b_i is a β_i-object for $1 \le i \le m$. Let v' associate each x_i with b_i and be otherwise identical with v. Then if X is v'-improper, F is undefined on the tuple. Otherwise F returns what is v'-constructed by X.

v) *Constructions* are just what is defined by the points i) —iv). –

Definition 4a (*subconstructions*)

i) Let C be a construction.

ii) C is a *subconstruction of* C.

iii) If X is a construction, then X is a *subconstruction of* ^{0}X.

iv) If C is $[XX_1\ldots X_m]$, then X, X_1, …, X_m are *subconstructions of* C.

v) If C is $\lambda x_1\ldots x_m X$, then X is a *subconstruction of* C.

vi) If C_1 is a *subconstruction of* C_2 and C_2 is a *subconstruction of* C_3, then C_1 is a *subconstruction of* C_3.

vii) *Subconstructions* are just what is defined by the points i) —v). –

Remarks:

1) We stated already that composition and closure correspond to application and abstraction in λ-calculi, respectively. Now we must note some distinctions. First, and most important: The respective λ-terms are expressions of an artificial language. As such they contain brackets/parentheses, the λ-symbol etc. It must be clear by now that *the construction called composition does not contain brackets and the construction called closure does not contain brackets or the λ-symbol.* We have *defined* constructions and we have *used* a notational system for that purpose. Our way of 'linguistically' treating constructions is just as distinct from the constructions themselves as the expressions *elephant, town,* and *beauty* are distinct from elephants, towns and beauty. Thus we are entitled to write

$[^0+\ ^07\ ^05]$ *constructs* 12

or

'$[^0+\ ^07^05]$' *contains brackets*

but the following are meaningless:

$[^0+\ ^07^05]$ *contains brackets,*

'$[^0+\ ^07^05]$' *constructs* 12.

A second distinction can be stated: as a rule, λ-calculi work with total functions only. Then Schönfinkel's reducibility of n-ary functions to unary functions can be proved. As soon as partial functions are taken into account (as they are in TIL) this reduction is no longer unambiguous (see [Tichý 1982, 52-53]). Thus we have to work in general with n-ary functions for $n \ge 1$.

2) We can do without Tichý's constructions called *execution* and *double execution*. On the other hand, we can adduce an example of modifying the hierarchy of types and the respective system of constructions for the purposes of 'conceptual modelling': in the unmodified system of TIL there is no type of tuples. The modification I mean introduces a separate type of tuples and, of course, two further kinds of construction: one that *v*-constructs tuples, and the other one ('projection') that selects the *i*-th member of the given tuple. See [Duží 2000], [Zlatuška 1986]. In this system all functions are unary without Schönfinkel reduction: the arguments can be any tuples.

3) *Trivialisation* might be likened to a *constant* of formal languages. Still, there is an essential distinction (besides the fact that trivialisation is not an expression): Whereas a constant of a formal language denotes different objects under particular interpretations, the trivialisation of any entity X (^{0}X) always constructs X. Thus, e.g., ^{0}highest, ^{0}lowest always construct the empirical functions highest, lowest (both of the type $(\iota(o\iota))_{\tau\omega}$). If we wanted to express some common features of both functions we would use a *variable*, say, $c/*_1$, ranging over the type $(\iota(o\iota))_{\tau\omega}$. Thus our transparent approach is more precise and does not lose any expressive power. –

Now we will give a construction that constructs the object

the highest mountain,

assuming that the objects *the highest* and *mountain* are given (i.e., are not analysable in this context). All such tasks must begin with a *type-theoretical analysis*; the *synthesis* proper is the second step.

Types

the highest is what is denoted by the respective English expression; first we note that this expression is surely an empirical expression. Hence our object is an intension, i.e., an $\alpha_{\tau\omega}$-object. What does this function associate with a given world and time? It cannot be an individual — we could accept this option if the object were *the highest thing* or *the highest individual*, but being only *the highest* it needs an argument to return an individual (or to return nothing, being undefined on the given argument). So the form of the type α will be $(\iota\beta)$. What kind of argument can be expected? We can speak of such objects as *the highest tower, the highest tree, the highest flower* (and, of course, *the highest mountain*), but surely not about *the highest beer, the highest truth-value, the highest possible world, the highest present prime minister of Czech Republic*. We can easily guess why the last three examples are inadmissible: the arguments are not properties of individuals. As for the beer-example, we could construe (*being) beer* as a property (even of individuals) but even so the kind of property is somehow 'inappropriate'; it is a 'mass property'. Anyway, being a property of individuals seems to be a necessary condition for being an argument of *the highest.* (That some of these properties might be also inappropriate is not a *type-theoretical* objection.) Thus it seems that the type of *the highest* could be $(\iota\ (o\iota)_{\tau\omega})_{\tau\omega}$. Yet a closer inspection shows that this result is inadequate. True, the word following the expression *the highest* is always a name of a property but what happens ontologically is that this function returns an individual (if any) which is the highest *among the members of a class*: one cannot select the highest object by merely observing a property — one has to measure individuals against other individuals given as the members of a class. However, there is an essential connection between the given property and the relevant class. A property of individuals is an intension whose value in particular pairs ⟨world, time (point)⟩ is just a class (sometimes empty). Thus in every pair ⟨world, time⟩ there is a class which is the value of the property in this pair. Our solution of the type-theoretical problem is:

$$\text{H(ighest)}/\ (\iota\ (o\iota))_{\tau\omega},\ \text{M(ountain)}/\ (o\iota)_{\tau\omega}.$$

Remark: Notation:

Let X be an object. By X/α we say that X is an α-object.

Let X be a construction. By $X \rightarrow \alpha$ we say that the object (*v*-)constructed by X is an α-object.

This notational distinction is necessary, as we will see after the ramified hierarchy is defined.

The last part of the type-theoretical analysis consists in determining the type of the whole object, i.e., of the object *the highest mountain*. This question has already been answered: this type cannot be ι: being an intension (an individual role) our object is an $\iota_{\tau\omega}$-object.

Synthesis

The target construction has to construct an $\iota_{\tau\omega}$–object from $H/\ (\iota\ (o\iota))_{\tau\omega}$ and $M/\ (o\iota)_{\tau\omega}$. Further we know that H should be applied to the class of individuals that is the value of M in the given pair ⟨world, time⟩. *No particular world or time is given*, so we will need *variables v*-constructing ('ranging over') worlds and times. Once and for all we choose variables $w \rightarrow \omega$ and $t \rightarrow \tau$. To construct the $\iota_{\tau\omega}$-object (i.e., the $((\iota\tau)\omega)$-object) we use the closure of the form

$$\lambda w \lambda t\ X$$

(omitting brackets does not cause any misunderstanding), where X *v*-constructs an ι-object (i.e., $X \rightarrow \iota$). X is obviously a composition that realises the application mentioned above, so we have X:

$$[[[{}^0H w] t]\ [[[{}^0M w] t]]].$$

Remark: Notation. Where A is an $\alpha_{\tau\omega}$-object we write ${}^0A_{wt}$ instead of $[[{}^0A w] t]$. (More precisely, we should say "where **A** is a construction that constructs an intension A (type $\alpha_{\tau\omega}$) we write $\mathbf{A}_{wt}$ instead of $[[\mathbf{A} w] t]$. We will frequently use the simplification consisting in writing the specific construction ${}^0A$ instead of **A**.) –

The result is

$$\lambda w \lambda t\ [{}^0\mathrm{H}_{wt}\ {}^0\mathrm{M}_{wt}].$$

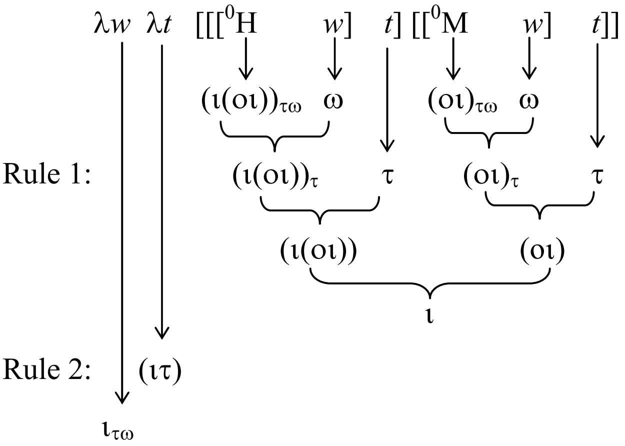
The resulting construction can be *checked* for type-theoretical compatibility. The algorithm of checking is simple: it is based on two rules:

Checking algorithm:

Rule 1: Let X (v-)construct an $(\alpha\ \beta_1\ldots\beta_m)$-object and let $\mathrm{Y}_1,\ldots,\mathrm{Y}_m$ (v-)construct β_1-,…,β_m-objects respectively. Then $[\mathrm{X}\ \mathrm{Y}_1\ldots\mathrm{Y}_m]$ (v-)constructs an α-object.

Rule 2: Let $x_1,\ldots,x_m$ (v-)construct β_1-,…,β_m-objects, respectively, and let X (v-)construct an α-object. Then $\lambda x_1,_{,,}x_m$ X (v-)constructs an $(\alpha\ \beta_1\ldots\beta_m)$-object. –

Applying the rules to our case:



It could be objected that the claim that the construction $\lambda w \lambda t\ [{}^0\mathrm{H}_{wt}\ {}^0\mathrm{M}_{wt}]$ is what is expressed by the expression *the highest mountain* presupposes that the intensions *the highest* and *mountain* are constructed just by trivialisations, whereas they may be constructed in another way. This objection is justified; see the *Remark* above. We should write the schema

$$\lambda w \lambda t\ [\mathbf{H}_{wt}\ \mathbf{M}_{wt}]$$

instead. Then a set of possible constructions falls under this schema. But as we already suggested, we will in similar cases write one representative of those possible constructions, viz. that one which — simplifying, as we will see — assumes that a *simple concept* (see Definition 11) is what is expressed by a simple *expression.* Such a construction is then really the trivialisation of the object denoted by the respective simple expression. –

Constructions can construct new constructions and among the objects we should be able to treat are classes and relations of constructions. Let us adduce two examples:

A. Let x_1 be a numerical variable. We write $x_1 \rightarrow \tau$ but we cannot say which type the variable itself belongs to, so x_1/?.

B. We will define closed constructions. To which type does the class of closed constructions belong?

In order to answer this kind of questions (and so be prepared to build up a rather general theory of concepts) we have to extend the hierarchy of types to get a *ramified hierarchy*. The next definition satisfies this requirement. It is a slightly modified definition from [Tichý 1988] or [Materna 1998]. The definition proceeds stepwise: first *types of order 1* are defined, second, the notion of *constructions of order n* is introduced, and third, *types of order n* + 1 are defined.

Definition 5 (*types of order n + 1*)

$\mathbf{T_1}$ *Types of order 1*: See Definition 1.

$\mathbf{C}_n$ *Constructions of order n*: Let α be a *type of order n*.

i) For any variable ξ: If $\xi \rightarrow \alpha$, then ξ is *a construction of order n.*

ii) Let X be an α-object. Then ^{0}X is *a construction of order n.*

iii) Let X, X_1, …, X_m be *constructions of order n*. Then $[XX_1 \ldots X_m]$ is *a construction of order n.*

iv) Let $x_1,\ldots,x_m$, X be *constructions of order n.* Then $[\lambda x_1 \ldots x_m\ X]$ is *a construction of order n.*

$\mathbf{T}_{n+1}$ Types *of order n + 1*: Let $*_n$ be the collection of all *constructions of order n*.

i) $*_n$ and all types of order *n* are *types of order n* + 1.

ii) Let α, $\beta_1,\ldots,\beta_m$ be *types of order n* + 1. Then $(\alpha\ \beta_1 \ldots \beta_m)$ (see Definition 1) is *a type of order n* + 1.

iii) *Types of order n* + 1 are only what is determined by i), ii). –

Remark: 'all types of order *n*' in $\mathbf{T}_{n+1}$ i) enables us to handle the cases when the types of the components of composition and closure (points $\mathbf{C}_n$ iii),iv)) are not of the same order. Then what counts according to $\mathbf{T}_{n+1}$ i) is the highest order. –

Now the examples above can be solved:

Ad a): If $x_1 \rightarrow \tau$, then $x_1 / *_1$ (see point $\mathbf{C}_n$ i)). Moreover, since the type of x_1 is $*_1$, it is a type of order 2 (see $\mathbf{T}_{n+1}$ i)) so that $^0x_1$ is a construction of order 2 and its type ($*_2$) is of order 3, etc.

Ad b): The class of all closed constructions of order 1 (of order 2, …) is an ($o*_1$)-object (an ($o*_2$)-object, …).

We will see that the ramified hierarchy as defined above is necessary for a general theory of concepts. Independently thereof its application offers a very good tool for handling propositional and notional attitudes (see, e.g., [Materna 1997a], [Duží 1999], [Duží 2003b].)

1.3.4 Concepts as closed constructions

I shall now briefly reproduce the arguments from [Materna 1998] to show that at least one good explication of the term *concept* can be given in terms of constructions as defined above. The arguments presuppose that the concepts we intend to treat are non-mental 'procedures' that make it possible to identify objects.

First we will compare some features of informally characterised concepts with some properties of constructions as defined above.

concepts	**constructions**
abstract	abstract
extra-linguistic	extra-linguistic
identify objects	*v*-construct objects
can be empty	can be *v*-improper

We can see that there could be a problem with the last two rows where the constructional properties analogous to the conceptual features are connected with a *parameter*, viz. with a *valuation.* Let us therefore test our intuitions connected with using the term 'concept' against the possibility of 'parametrised' identification of objects.

Remark: Before attempting at defining concepts let us use capital letters to denote concepts.–

Let THE FATHER OF be the concept expressed by the English expression *the father of*. Clearly this concept identifies an empirical function, type $(\iota\iota)_{\tau\omega}$. On the other hand, consider the expression *the father of x*. Could we say that this expression expresses a concept, i.e., THE FATHER OF *x*? No definite object is identified, since which individual role/ office is identified depends on the value of *x* ($\rightarrow \iota$). Replacing *x* by a constant, say, a proper name or a definite description we get rid of the parameter and we can surely say that what we get is a concept, for example THE FATHER OF ALBERT EINSTEIN or THE FATHER OF THE RICHEST MAN IN PRAGUE etc. In both cases the concept identifies a definite object, viz. an individual role (type $\iota_{\tau\omega}$) whose value depends, of course, on the respective state of the world.

So we are ready to formulate a first, preliminary definition of concepts; they are constructions which do not contain free variables. To make the definition exact we need first to define *free* and *bound* variables. We define therefore *free* and *bound occurrences of variables*; a free/bound variable can be defined as usually in terms of free/bound occurrences.

Remark: The following definition is not trivial. Unlike in the ordinary systems we have two kinds of boundness which 'behave' in a distinct way: λ-boundness does and ^{0}boundness ('boundness in virtue of trivialisation') does not obey the α-rule of the λ-calculi. –

Definition 6 (*free, ^{0}bound, λ-bound variables*)

i) Let C be construction containing an occurrence of the variable ξ.

ii) If C is ξ, then *the occurrence of ξ in C is **free in** C.*

iii) If C is ^{0}X, then *any occurrence of ξ in C is **^{0}bound in** C.*

iv) If C is $\lambda x_1 \ldots x_m$ X, then if ξ is one of $x_{1,\ldots,}x_m$, *any occurrence of ξ is **λ-bound in** C unless it is ^{0}bound in X;* if ξ is distinct from any of $x_1,\ldots,x_m$, then *any occurrence of ξ is **free in** C unless it is λ-bound or ^{0}bound in X.*

v) If C is [$XX_1\ldots X_m$], then *any occurrence of ξ free, ^{0}bound, λ-bound in X, $X_1,\ldots,X_m$* is, respectively, ***free, ^{0}bound, λ-bound in** C.* –

Remark: In virtue of the *objectual* character of constructions the term *occurrence* has a non-standard meaning. To illustrate compare two cases:

a) The *construction* λ*x* [^{0}> *x* ^{0}0], i.e., the procedure itself, contains *one* occurrence of *x*: that one which *v*-constructs the first argument of >.

b) The *expression* 'λ*x* [^{0}> *x* ^{0}0]' contains *two* occurrences of *the name* '*x*'.

The reason is that the 'λ*x*' is only our tool for defining the kind of construction. Abstracting over τ concerns only the one occurrence of *x*. –

The distinction between ^{0}bound ("trivialisation-bound") and λ-bound (occurrences of) variables can be easily demonstrated by following examples.

Consider the construction ($x \rightarrow \tau$)

$$\lambda x\ [^0\!> x\ ^0 0].$$

This construction constructs the class of positive numbers. Correctly substituting *y* for *x* we get

$$\lambda y\ [^0\!> y\ ^0 0],$$

which is an equivalent construction. In terms of λ-calculi an application of the α-rule guarantees the equivalence.

Compare therewith another pair of constructions seemingly parallel to the preceding case:

$$^0[\lambda x\ [^0\!> x\ ^0 0]],$$

$$^0[\lambda y\ [^0\!> y\ ^0 0]].$$

This time the constructions are not equivalent. *Each of them constructs another construction.* True, the constructions they construct are equivalent, but they are distinct. In the first case *x, y* were λ-bound, in the second case they were ^{0}bound (see points ii), iv) of the definition). We can say that the λ-bound variables are *used*, whereas the ^{0}bound variables are *mentioned.*

Definition 7 (*closed constructions*)
A construction C is *closed* iff no occurrence of a variable is free in C. –

Now our preliminary definition of concepts (we will speak of *concepts**) can be formulated.

Definition 8 (*concept**)
A concept (of order n)* is a closed construction (of order *n*). –

1.4 Concepts

1.4.1 Concepts identify objects

It has been already suggested that conceptual identification corresponds to constructional 'construction'. We should further know what is meant by the word 'object'. Having introduced the ramified hierarchy we can now say that *an object is a member of a type (of any order).* We say, therefore, that concepts identify

a) *1st order objects*, i.e., α-objects where α is a type of order 1, in particular *1st order extensions* and *intensions,*
b) *higher order objects,* i.e., α-objects where α is a type of order *n, n* >1. Since concepts, as explicated here, are constructions and the latter are higher order objects (see Definition 5), concepts can identify other concepts.

The identification can however break down. We know that there are *empty concepts.*

1.4.1.1 'Empty concepts'

Some concepts are *empty*. Examples: THE GREATEST PRIME, NUMBER(S) DIVISIBLE BY 0, THE PRESENT KING OF FRANCE, A HOBBIT. We can show that actually there are more kinds of emptiness. Bolzano, for example, has clearly distinguished between analytically empty concepts (our first two examples—Bolzano uses ROUND SQUARE) and empirically empty concepts (our last two examples—Bolzano uses A GOLDEN MOUNTAIN). For us there are at least three kinds. Let us demonstrate the respective distinctions.

a) THE GREATEST PRIME. Combining the concept THE GREATEST with the concept PRIME (NUMBER) we simply get *nothing*. The latter concept identifies an infinite class no member of which is the greatest one. The function identified by THE GREATEST is a partial and not total function. This kind of empty concept could be called *strictly empty* (see [Materna 1998]).

b) PRIMES DIVISIBLE BY 3 AND 5, NUMBERS DIVISIBLE BY 0.

Surprisingly we can show that concepts of this kind are not 'strictly empty': they do identify an object, viz., an empty class. We can show this as follows:
Take the second example. The construction of the class of (natural) numbers divisible by 0 is

$$\lambda x\ [{}^0\exists\ \lambda y\ [{}^0{=}\ y\ [{}^0{:}\ x\ {}^0 0]]].$$

The class *v*-constructed by

$$\lambda y\ [{}^0{=}\ y\ [{}^0{:}\ x\ {}^0 0]]].$$

is 'degenerate' or 'quasi-empty' for any valuation in the sense that it is undefined for any argument; so it is not non-empty and

$$[{}^0\exists\ \lambda y\ [{}^0{=}\ y\ [{}^0{:}\ x\ {}^0 0]]].$$

gives False for any valuation. Thus the class constructed by

$$\lambda x\ [{}^0\exists\ \lambda y\ [{}^0{=}\ y\ [{}^0{:}\ x\ {}^0 0]]].$$

is empty.

This distinction is more important than it could seem. We can say something about an empty class and be right or wrong, whereas nothing can be predicated when there is no object at all. In other words, classes — and therefore also empty and 'quasi-empty' classes — are objects *sui generis* but there is no 'empty particular' ('empty number', 'empty individual' etc.).

I shall call this kind of empty concept *quasi-empty* (see [Materna 1998]).

c) THE PRESENT KING OF FRANCE, A HOBBIT. Both these concepts can be called *empirically empty.* That there is just now no King of France is not a logically necessary fact, as well as that there are no hobbits. Instead of saying that existence of the King of France or of hobbits is logically possible we can say that there are pairs ⟨world, time moment⟩ where an individual plays the role of the King of France (in this case we can even claim that in the actual world there were such times when there was such an individual) and that in some non-actual possible worlds a non-empty class of hobbits realises (at some time points) the property described by Tolkien.

As soon as we explicate concepts as concepts*, i.e., as closed constructions, we can replace our intuitions concerning conceptual emptiness with precise definitions. So we have:

Definition 9 (*empty concepts**)

i) A concept* C is *strictly empty* iff C is an improper construction.

ii) A concept* C is *quasi-empty* iff C constructs an empty (or a 'quasi-empty') class / relation.

iii) A concept* C is *empirically empty at (time point) t* iff C constructs an intension such that its value at t in the actual world is either missing or an empty (or: quasi-empty) class. –

Remark: We can speak of the actual world but the actual world — as (informally) the collection of all true facts — cannot be identified as one definite world (i.e., as an ω-object): rather it is the identical function, type (ωω). Indeed, not knowing all true facts we cannot know which world is the actual world. On the other hand, given a possible world W, which world is actual in W? Surely W itself.

Therefore an expression containing some reference to actuality is equivalent to the expression which skips this reference. 'It rains' is equivalent to 'It actually rains': no information is given by 'actual(ly)' and suchlike. Or: when we say that p is true we properly speaking say that among the worlds in which p is true the actual world is. We could omit 'the actual world' from point iii) above. See [Tichý 1972]. –

Applying Definition 9 to our examples: THE GREATEST identifies the function, say, G/ (τ(οτ)), PRIME identifies the class, say, P/ (οτ). The respective concept* is

$$[{}^0G\ {}^0P].$$

By the way, even on this somewhat simplified analysis (see 2.2) one phenomenon can be well explained: we understand such expressions as *the greatest prime* although they do not denote anything; the reason is that we possess the respective concept, i.e., we know the procedure which would lead us to the number described *if there were such a number*. The procedure (given by the construction above) shows that there cannot be such a number.

Our second example: Simplifying again we can suppose that *the numbers* denotes real numbers, those which can be *v*-constructed by numerical variables; DIVISIBLE identifies the relation D/ (οττ). We have

$$\lambda x\ [{}^0D\ x\ {}^00].$$

Our definitions make it clear that *a closure cannot be (v-)improper*. Our construction is a closure, so it *has to construct* some function. This function is the characteristic function of the 'quasi-empty' class of numbers. In general it holds that (*quasi-*)*empty concepts are either trivialisations of an empty (quasi-empty) class or closures.*

Concerning our empirical examples, the concept THE (PRESENT) KING OF FRANCE obviously identifies an 'office': an $\iota_{\tau\omega}$-object. Let the type of France be α, King is an empirical function of type $(\iota\alpha)_{\tau\omega}$. The office is constructed by composing the simple concepts ^{0}King and ^{0}France:

$$\lambda w \lambda t\ [{}^0\mathrm{King}_{wt}\ {}^0\mathrm{France}].$$

^{0}King constructs an intension; the empirical emptiness consists in the fact that the value of this intension in the actual world nowadays at the argument France is missing. Similarly in the case of HOBBIT, where the concept* constructs a property whose value in the actual world is an empty class.

Summarising this section we can state that *with the only exception of strictly empty concepts every concept identifies an object—in the worst case an empty (or 'quasi-empty') class-relation.*

Remark: Our definitions solve the old problems known from Russell's analysis of the denoting relation and his polemics with Meinong. Already in 1905 Russell writes:

> [*the present king of France* is a] complex concept denoting nothing. The phrase intends to point out an individual, but fails to do so: it does not point out an unreal individual, but no individual at all
>
> [quoted by [Coffa 1991, 106]]

Yes, in the case of empirically empty concepts the respective procedure 'intends to point out' an object but the state of the given world + time makes it impossible to find the value of the respective intension.

Or: in *The Principles of Mathematics* Russell tries to prove that "The golden mountain is not" is false or meaningless, for "whatever the golden mountain may be, it certainly is". (See [Coffa 1991, 107].) We need not work out a neomeinongian solution (like Zalta) to this (slightly enigmatic) dilemma. Our answer to the question "Does the golden mountain exist?" is unambiguously negative, which we can show as follows:

Let E be an existence predicate, type $(o\ \iota_{\tau\omega})_{\tau\omega}$. It denotes the following property of individual roles: In a pair $\langle$W, T$\rangle$ the individual role *I* has this property iff *I* is occupied by an individual in $\langle$W,T$\rangle$. The sentence "The golden mountain exists" gets the following analysis (GM/ $\iota_{\tau\omega}$):

$$\lambda w \lambda t\ [{}^0\mathrm{E}_{wt}\ {}^0\mathrm{GM}]$$

This construction constructs the proposition that is true in such worlds + times where the role GM is occupied by an individual. Since in the actual world + time there is no such individual the proposition is false (in the actual world + time).

(As Coffa in [Coffa 1991, 107] says:

> In general, the presence or absence of a denotation...has nothing whatever to do with whether the statement in question is a mere noise or expresses a meaningful proposition.

We would say 'reference' instead of 'denotation', of course.)

1.4.1.2 Mathematical concepts

In this section I do not intend to offer a deep analysis of mathematical concepts in general. More interesting points are postponed to later sections. Here only the question of which kind of objects are identified by mathematical concepts (in comparison with empirical concepts) is to be explained.

All the same we must already distinguish between uninterpreted systems on the one hand and their interpretations on the other hand. The principle we follow here reads:

Uninterpreted systems are schemes only: concepts$^{(*)}$ do not contain symbols, only constructions of objects given by an ***interpretation.***.

In the case of, e.g., formalised arithmetic of natural numbers the respective concepts construct objects given by the 'intended interpretation'. If a non-standard interpretation takes place, then we get other objects, and so other concepts too.

It does not follow from the TIL approach that the *base* of a type-theoretical system (see Definition 1) must be just the set $\{o, \iota, \tau, \omega\}$. We can build up the hierarchy sufficient for a Peano-like arithmetic on the set $\{o, \nu\}$ where ν would be the set of all natural numbers. The concept* of the successor (say, S) would be then ${}^{0}S$, $S/(\nu\nu)$. (See also [Palomäki 2001].) No base for mathematical systems would need the types ι, ω. The concepts* over such bases construct either particular numbers (type τ or ν) or functions over the respective base or truth-values (in the case of mathematical sentences).

Remark: No explication is perfect. Whereas our claim — in harmony with [Church 1956] — that the meaning of an expression is a concept of what the expression denotes seems intuitive when applied to the expression *a prime* (understanding this expression we possess a concept of the class of prime numbers) the intuition fails in the case of *sentences*: It is surely a little bit strange to say that the meaning of the sentence

Two is the only even prime

is a concept of the truth-value **T**. This particular fact should not, however, prevent us from accepting Church's proposal. We will see that already when a sentence is empirical our linguistic intuition comes back to us again. –

To adduce an example of a mathematical concept: THE SUCCESSOR OF THE SMALLEST PRIME. We have $S/(\nu\nu)$, $Sm/(\nu(o\nu))$, $P/(o\nu)$.
Our concept* is (type-theoretical checking added):

$$[{}^{0}S \quad [{}^{0}Sm \quad {}^{0}P]].$$

$$\underbrace{(\nu\nu) \quad \underbrace{(\nu(o\nu)) \quad (o\nu)}_{\nu}}_{\nu}$$

The expression *the successor of the smallest prime* expresses this concept* and denotes the object constructed by the latter, viz. the number 3.

Any impression that all mathematical concepts are 1st order concepts is mistaken. We can speak about equations, about substitutions, about proofs etc. etc. Such discourse concerns *constructions* rather than what is or what can be constructed. See [Tichý 1988, 70-76] for an excellent exposition of such a ‘higher order discourse’. As for the logico-philosophical importance of this approach to mathematical texts see [Tichý 1995] or a quotation from his 1988, p.76:

> The arithmetician is interested in a certain class of mathematical constructions and proves theorems which tell us what such constructions construct. To state such theorems, he has to *use* notational devices enabling him to refer to those constructions. But to study how these notational devices work is a task for the grammarian of arithmetical discourse, not for the arithmetician himself. There is nothing untoward, of course, about an arithmetician’s taking interest in the particular notation he uses. But while he indulges this interest he is not doing arithmetic any more than a zoologist is doing zoology when his attention strays temporarily from beasts to the grammar of the language he speaks.

1.4.1.3 Empirical concepts

Empirical concepts(*) are procedures which ensure that Parmenides’ Principle (see 1.3.2) is obeyed. We most frequently contravene this principle when we assume that an empirical expression is not about an intension but rather about the object that happens to be the value of that intension in the actual world. (I remind the Reader of our example with the highest mountain.) Empirical concepts* are either *trivialisations of intensions* (see however 2.1) or constructions of the form $\lambda w \lambda t$ X, where X is an arbitrary open construction with *w,t* as the only free variables. In both cases it is obvious that there is no way of constructing the value of the respective intension in the actual world (and time), as explained in the Remark following Definition 9.

Another consequence is that empirical concepts(*) are never strictly empty or quasi-empty. As an example let us consider the concept THE TOWNS WHERE THE NUMBER OF INHABITANTS IS DIVISIBLE BY ZERO. Types: T(owns)/ $(o\iota)_{\tau\omega}$, I(nhabitants of)/ $((o\iota)\iota)_{\tau\omega}$, N(umber of)/ $(\tau(o\iota))$, D(ivisible by)/ $(o\tau\tau)$, Z(ero)/ τ, $x \to \iota$. The concept* is

$$\lambda w \lambda t\ \lambda x\ [{}^0\wedge\ [{}^0T_{wt}x]\ [{}^0D\ [{}^0N\ [{}^0I_{wt}x]]\ {}^0Z]].$$

For every world-time the construction

$$[{}^0D\ [{}^0N\ [{}^0I_{wt}x]]\ {}^0Z]$$

is (v-)improper. Thus the object identified by the concept (constructed by the concept*) is a very strange property of individuals: its value is in every world-time a ('non-standard') class, viz. a function that is undefined for any individual, so a quasi-empty ('degenerate') class. Still another strange (but not infrequent) phenomenon can be seen here: the concept seems to be an empirical concept but it is *de facto* a non-empirical (in this case not just a mathematical) concept, since the intension identified by it is a trivial, constant intension (see Definition 3).

Remark: This last example shows that our classification of concepts is not very fine-grained. We should divide concepts into empirical and non-empirical concepts and use a subdivision for non-empirical concepts, distinguishing mathematical and 'the other' non-empirical concepts. We can adduce many examples of the members of the last group. Take, e.g., the concepts INDIVIDUALS SUCH THAT IF THEY ARE MAMMALS, THEN THEY ARE VERTEBRATES (identifies the universe; but MAMMALS WHICH ARE VERTEBRATES is an empirical concept that identifies the property *(being a) mammal*!), BEING A TABLE OR NOT BEING A TABLE (but see 1.4.3.4) etc... –

Being strange or useless does not mean, however, being 'non-existent'. We can suppose that there are millions of concepts that are at the given moment entirely useless (or even strange in the above sense). To adduce a Bolzanian example, we can certainly understand the expression *a bush with exactly 235 leaves*, so that there is a concept that identifies the respective property. Nobody needs such a concept but we cannot say that there is no such concept. (By the way, we can imagine a fairy tale where finding such a bush is a condition for liberating a princess.)

Remark: The distinction between empirical and non-empirical concepts is relevant for logic. When we use the notion of *intension* in the PWS sense to distinguish concepts that identify intensions and are thus empirical we possess a logical tool essentially important for, e.g., the analysis of the use of a concept *de re* vs. *de dicto* (see for example [Tichý 1978b], [Duží 2004]). A small example can serve as an illustration. We can ask: does the expression *yellow* express another concept than the expression *yellowness*? (By the way, this is a problem Bolzano tried to solve within his system. See [Bolzano 1837, §60]) One surely admits that YELLOWNESS identifies a property of individuals. But what about YELLOW? We can easily solve this problem for we explicitly use variables *w, t*: Let us analyse two contexts (let X be a construction which constructs an individual A):

A is yellow.

Yellowness is a colour.

To be a colour is to belong to the class of colours, i.e., of properties of individuals. So we have $C/\ (o\ (o\iota)_{\tau\omega})$. We claim that Y(ellow) as well as Y(ellowness) is simply a property of individuals, $Y/\ (o\iota)_{\tau\omega}$. Our analyses of both the contexts above result in:

$$\lambda w \lambda t\, [{}^0\mathrm{Y}_{wt}\ \mathrm{X}],$$

$$[{}^0\mathrm{C}\ {}^0\mathrm{Y}].$$

We can see that *yellow* expresses the same concept$^{(*)}$ as *yellowness*, viz. ^{0}Y (but see 2.2): the two distinct expressions serve only to make the distinction between *de re* (the first context, where the proposition constructed is true or false depending on whether A does or does not belong to the class which is the value of Y in the given world-time) and *de dicto* (the second context which is simply true independently of worlds-times; the respective sentence is an analytic sentence, where what is spoken of is the property itself, not the particular classes as its 'populations'— therefore no 'intensional descent' is present).

This is only a particular, not very intricate example exhibiting the importance of logically capturing the distinction between empirical and non-empirical concepts; for imagine that our question concerning the possible distinction of the concepts YELLOW and YELLOWNESS should be answered without having at one's disposal the variables *w, t* or even without taking into account the notions of intension and of extension. Never ending more or less sophisticated discussions could be expected. But such examples notwithstanding you find mostly such formulations as "the necessity to introduce possible worlds into logic arises whenever (in the worse cases "only if") we want to analyse modal contexts". But we have seen that our example did not concern any modal context. –

Unlike mathematical concepts Church's proposal to let also sentences express concepts is not at all counterintuitive. One could, of course, object that normally we would not say that a concept could be *true* or *false* and that we would have to admit it if also sentences (empirical sentences!) expressed concepts. This objection can be however easily refuted: Consider an empirical sentence, say,

The mayor of Moscow is corrupt.

This sentence, as well as the proposition denoted by it, is, of course, true in some pairs $\langle$W,T$\rangle$ and false in others, but what about the *concept*$^{(*)}$ expressed by it?

A concept should *identify an object*. We already know which kind of object is identified by empirical concepts: it is always an intension. The concept expressed by an empirical sentence identifies a *proposition*. So it cannot be true or false, for *identifying a proposition is not the same as verifying (falsifying) a proposition.* We understand the sentence about the mayor of Moscow, i.e., we possess the concept of the respective proposition (= truth conditions of the sentence), so we are theoretically able to verify (falsify) it, but whereas the process of identification is a (relatively) *a priori* process given by the linguistic convention, the process of verifying/falsifying (an empirical sentence/an empirical proposition) is an *empirical, a posteriori* process. Thus the principle according to which concepts are neither true nor false holds even in the case that the concept is expressed by an empirical sentence.

Remark: What if the sentence is a mathematical sentence? In 1.4.1.2 we have seen (in a Remark) that our linguistic intuition resists admitting that, e.g., a true mathematical sentence expresses a concept of the truth-value **T**. All the same, even if we accept this Churchian idea, we can say that the concept *identifies* **T** but we would hesitate to say that it *is true*. –

1.4.2 Concepts are abstract procedures

1.4.2.1 Constructions as abstract procedures: too fine-grained a construal

Our aim is to make it possible to handle concepts as *abstract procedures*. Let procedures be characterised or defined no matter how; they always have to consist of some 'steps', (or 'instructions', at least one step, that is) and there have to be some atomic, indecomposable building stones. We have also to know the *identity* or *equality* conditions for procedures. Fletcher [1998] defines these conditions (for his (intuitionist) constructions) as follows (p.52):

> $x = y$ iff x and y are built out of the same atoms using the combination rules in the same way.

In our system *variables* play the role of atomic constructions. Applying the above definition we get, e.g., that

$$\lambda x_1\ [{}^0\!> x_1\ {}^0 0]$$

is distinct from (albeit equivalent to)

$$\lambda x_2\ [{}^0\!> x_2\ {}^0 0].$$

The distinction stems from the fact that distinct variables are used.

Our question is: Is this result compatible with the way we intuitively understand what a *procedure* is? And even: is it compatible with Fletcher's definition above? Let us try to verbally describe the procedure connected with the construction $[\lambda x_1\ [{}^0\!> x_1\ {}^0 0]]$.

*We identify the relation > and apply its characteristic function to all pairs ⟨m, 0⟩, where m is a real number, returning **T** in the cases where the pair belongs to the relation >.*

This description does not mention the variable x_1, nor is it distinct from the description associated with any construction λx_k […] for k any natural number. Our conclusion therefore is:

Constructions (as defined in Definition 4) are more fine-grained than procedures.

Yet this is no asset of constructions when compared with procedures. When procedures are indifferent to bound variables we have to ask: *Do we need bound variables?* In [Materna 1998] as well as in [Peregrin 2000] the possibility of building constructions *via* a theory inspired by Curry's combinators rather than by λ-calculi is suggested. There are, however, other options. In the next two sections an alternative (introduced in [Materna 1998]) is explained, in the APPENDIX another (today I would say: a better, and later the preferred) alternative is described.

1.4.2.2 Quasi-identity of closed constructions

First we define two relations that may obtain between two closed constructions.

Definition 10 (*α-equivalence*)
A closed construction C is *α-equivalent to* a closed construction C' iff C' arises from C by correctly replacing n ($n \geq 0$) occurrences of a λ-bound variable ξ by a variable η. –

Remark: Remembering the non-standard meaning of *occurrence* in TIL (see the Remark that comments on Definition 6) we have to represent the result of such a correct replacement by writing η instead of ξ also in the λ-part of our representation of closure, although we have seen that there are no occurrences of ξ in this part. –

Claim: *α–equivalence is reflexive, symmetric and transitive.* –

Proof:
Symmetricity and transitivity are obvious. Reflexivity is guaranteed by admitting $n = 0$. –

Thus the constructions

$$\lambda x_1 [{}^0{\geq}\, x_1\ {}^0 0]$$

and

$$\lambda x_2 [{}^0{\geq}\, x_2\ {}^0 0]$$

are α-equivalent.

Definition 11 (*simple concepts*)
A concept* of the form ^{0}X, where X is a non-construction (or at most a variable) is called *a simple concept**. –

Definition 12 (*η-equivalence*) (in [Materna 1998] called β-equivalence)
Let us accept following abbreviations:

Instead of $(\alpha\beta_1\ldots\beta_n)$ we will write $(\alpha\beta_n)$.

Instead of $\lambda x_1\ldots x_n\,[\mathrm{X}\, x_1\ldots x_n]$ we will write $\lambda x_n[\mathrm{X}x_n]$.

Then: Let C be a simple concept* constructing an $(((\alpha\beta_{m1})\gamma_{m2})\ldots\delta_{mn})$-object, and let $x_{mn},\ldots,y_{m2}, z_{m1}$ be variables ranging over $\delta_{mn},\ldots,\gamma_{m2}, \beta_{m1,}$ respectively.

C is *η-equivalent to* each of the concepts*

$\lambda x_{mn}\,[\mathrm{C}x_{mn}],\ \ldots,\ \lambda x_{mn}\ldots\lambda y_{m2}\,[[\mathrm{C}x_{mn}]\ldots y_{m2}],\ \lambda x_{mn}\ldots\lambda y_{m2}\lambda z_{m1}\,[[[\mathrm{C}x_{mn}]\ldots y_{m2}]z_{m1}]$,

and all these concepts* including C are pairwise *η-equivalent.* –

Claim: *η-equivalence is reflexive, symmetric and transitive.* –

Proof: A trivial consequence of Definitions 4. and 12.–

Thus following constructions are η-equivalent:

$$ {}^0\text{Cat},\ \lambda w\,[{}^0\text{Cat}\ w],\ \ \lambda w\lambda t\ {}^0\text{Cat}_{wt},\ \ \lambda w\lambda t\lambda x\,[{}^0\text{Cat}_{wt}x]. $$

Or (Bel(ieve)/ $(o\iota{*_1})_{\tau\omega}$, $x \to \iota$, $c \to *_1$):

$$ {}^0\text{Bel},\ \lambda w\,[{}^0\text{Bel}\ w],\ \lambda w\lambda t\ {}^0\text{Bel}_{wt},\ \ \lambda w\lambda t\lambda xc\,[{}^0\text{Bel}_{wt}\ x\ c]. $$

Now we can state that

> *(closed) constructions that are α- or η-equivalent differ in such a way that their difference is not accompanied by a difference between procedures; if C is α- or η-equivalent to C', then the respective procedure is the same (see 1.4.2.1).*

This statement justifies our *transition from concepts* to concepts.*

Definition 13 (*quasi-identity*)

Closed constructions C, C' are *quasi-identical* (*QUID*) iff there are constructions $D_1,\dots,D_m$ such that D_1 = C, D_m = C', and for any D_i, D_{i+1}, $1 \le i \le m$-1, it holds that D_i is α- or β-equivalent to D_{i+1}. –

Remark: Clearly, for any *n,* the type of $\text{QUID}^{(n)}$ is $(o\ *_n\ *_n)$. –

Definition 14 (*concept generated by a construction*)

Let C be a concept* of order *n*. Let *d* range over $*_n$. The *concept* generated *by* C (abbreviation: C̲) is constructed by $\lambda d\,[{}^0\text{QUID}^{(n)}\ d\ {}^0\text{C}]$. –

Remark: Thus the type of C̲ is $(o*_n)$. –

Some *consequences*:

1) Any concept* C *unambiguously* generates the concept C̲.
2) All members of C̲ identify (i.e., construct) one and the same object (in an almost the same way), or are strictly empty.
3) C̲, being a *class*, is not a construction.

Comments, illustrations:

Ad 1): According to our definitions, C̲ is the class of all closed constructions (= concepts*) that are 'QUID-related'. We can easily see that the $\text{QUID}^{(n)}$ relation is an equivalence for any *n*.

Ad 2): The members of C̲ are, of course, distinct. Yet from the viewpoint of explicating *concept* the differences are not essential. Expressions of a natural language do not distinguish α- or η-equivalent concepts* (because these concepts* identify one and the same object in an 'almost identical' way). The choice of bound variables is not important for the *way* of identifying an object.

Ad 3): This is a controversial point. We have always emphasised that concepts should not be set-theoretical entities. Concepts* are not, of course, since they are constructions, but concepts are classes (of constructions, but all the same classes), i.e., set-theoretical entities. An apology thereof can be found in [Materna 1998], here only briefly:

A concept is a specific class: not every ($o*_n$)-object is, of course, a concept. The specific character of a class-concept is given by the points 1) and 2) above. We can say that each member of the class-concept defines the *procedure* that is an *explicans* for *concept*. But when we *use* this procedure we can use *any member of the given concept.* See 1.4.2.3.

All the same, we will accept a more adequate definition, which preserves the constructional character of concepts: see APPENDIX to 1.4.2.

Examples:

Let C be ^{0}Cat. The infinite class $\underline{^0\text{Cat}}$ is

$$\{ {}^0\text{Cat}, \lambda w\, [{}^0\text{Cat}\ w], \lambda w \lambda t\ {}^0\text{Cat}_{wt}, \lambda w \lambda t\ \lambda x\, [{}^0\text{Cat}_{wt}\ x], \lambda w_1\, [{}^0\text{Cat}\ w_1], \lambda w_1 \lambda t\ \lambda x[{}^0\text{Cat}_{w1t}\ x], \lambda w_1 \lambda t_1 {}^0\text{Cat}_{w1t1}, \ldots \}$$

The class $\underline{^0\text{Suc}}$, where Suc(cessor)/ (νν) (see 1.4.1.2) is

$$\{ {}^0\text{Suc}, \lambda x\, [{}^0\text{Suc}\ x], \lambda x_1\, [{}^0\text{Suc}\ x_1], \lambda x_2\, [{}^0\text{Suc}\ x_2], \ldots \}.$$

The singleton $\underline{^0 0}$ is $\{ {}^0 0 \}$.

The singleton $\underline{^0[\lambda x\, [^0\leq x\ ^0 0]]}$ is $\{ {}^0[\lambda x\, [{}^0\leq\ x\ {}^0 0]] \}$.

(Notice that x is ^{0}bound here so that it cannot be replaced by other variables.)

Now any concept is unambiguously generated by any of its members. So we can ask: if the object identified by a concept $\underline{\text{C}}$ is unambiguously given by the concept* C, do we need the category CONCEPT at all? The next section shows that there is a context where we would probably agree that this new category must be exploited. (See, however, Appendix to 1.4.2.)

1.4.2.3 Using and mentioning concepts

(See also [Materna 1998, 5.5].)

The distinction between *use* and *mention* is well known in the area of *expressions*. When we say

Cats are predators

we use the expression 'cats', whereas when we say

'cats' is the plural of 'cat'

we mention the expression 'cats'.

But we can observe that in mentioning the expression 'cats' we *use* another expression, viz., 'cats', which is indicated by the inverted commas. Moreover, the distinction above is not accompanied by the distinction between using and mentioning a concept: in both cases a concept is *used*: in the first example the concept in question is a concept of the property *being a cat*, in the second example the concept used is the concept of the expression 'cats'. In neither expression above is a concept mentioned!

If we mention a concept we speak about this concept. If we use a concept we speak about the object identified by this concept.

It is no wonder that natural languages (usually) do not have at their disposal a systematic grammatical means that would signal that we speak about a concept rather than about, say, the property identified by the concept. Only when we theorise about concepts do we use some linguistic means to explicitly distinguish between the two cases. Even then the reader need not understand that an essential distinction is indicated. A good example can be found in Bolzano [1837 II.] where he distinguishes (§148, p.89) even between equivalent *Sätze an sich,* if they differ by containing two equivalent concepts ("Vorstellungen", *an sich*, of course):

> allein um Sätze als von einander verschieden anzuerkennen, genügt es, dass sie nur aus verschiedenen Vorstellungen bestehen, wenn sie auch einerlei Gegenstand betreffen.
>
> (to recognise sentences as mutually distinct it is sufficient when they consist of distinct representations, even if they concern one and the same object.)

Bar-Hillel in his [1950] believes that unless we reconstruct Bolzano's formulations in terms of a distinction between language and meta-language we get a contradiction. But Bolzano speaks of distinctions between *concepts*, not between *expressions*.

We will see that even such a fine-grained problem as distinguishing between using and mentioning *concepts* can be solved without resorting to introducing an extra meta-language. What we are after is not primarily a *linguistic* problem: abstract procedures (concepts*) are not linguistic entities, just as mathematical objects are not.

Consider following four sentences:

A) Charles is highly intelligent.
B) Being highly intelligent is a desirable property.
C) BEING HIGHLY INTELLIGENT is a psychological concept
D) The construction '$\lambda w \lambda t\ \lambda x\ [{}^0\text{Highly\_Intelligent}_{wt}\ x]$' contains the variable *x*.

We will show that in A) and B) the concept HIGHLY INTELLIGENT is *used*, in C) it is mentioned and in D) neither used nor mentioned.

In A), as well as in B), we speak about *properties*, not about concepts. In A) the property in question is predicated of Charles, and the truth-value of the sentence is dependent on the population of the property in the given world-time, so (the concept of) the property is in the supposition *de re*. In B), the property itself — as a function — is subject to predication:

the truth-value of the sentence is this time independent of the population of the property in particular worlds-times. The supposition of it (or: of the respective concept) is *de dicto*. (See also [Tichý 1978b], [Duží 2004].)

In C) we speak about *the concept itself*: it is said to be a member of the set of psychological concepts. *Nothing is said about the property identified by the concept.*

D) is a peculiar case. Here nothing is said about the property but neither is anything said about the concept: it is fully irrelevant to speak about variables contained in a concept. The only object of predication in D) is a particular construction.

Now we support our verbal characteristics by (simplified) analyses of A)—D).

Types: Ch/ ι, HI/ $(o\iota)_{\tau\omega}$, DP/ $(o\,(o\iota)_{\tau\omega})_{\tau\omega}$, PC/ $(o\,(o{*}_1))$, Co/ $(o\;{*}_1\;{*}_1)$, $x \rightarrow \iota$, $c \rightarrow {*}_1$

(Commentary: D(esirable)P(roperty) is a property of properties, P(sychological)C(oncept) is a class of concepts (here: of order 1), Co(ntain) is a relation (in-extension) between constructions in general and variables (here both of order 1).)

A'. $\lambda w \lambda t\, [^0\mathrm{HI}_{wt}\; ^0\mathrm{Ch}]$
B'. $\lambda w \lambda t\, [^0\mathrm{DP}_{wt}\; ^0\mathrm{HI}]$
C'. $[^0\mathrm{PC}\; \lambda c\, [^0\mathrm{Quid}\; c\; ^{00}\mathrm{HI}]]$
D'. $[^0\mathrm{Co}\; ^0[\lambda w \lambda t\, \lambda x\, [^0\mathrm{HI}_{wt}\, x]]\; ^0x]$

(Cf. a similar example in [Materna 1998], p.103.)

Observe that the distinction *de re* vs. *de dicto* is well visible: In A' the property HI is applied to *w* and (then) to *t*, which makes the truth-value of the constructed proposition be dependent on the population of HI in the given world-time, unlike in B'.

The example together with its analysis justifies our definition of concepts as sets *sui generis*. There is an analogy to Frege's claim that a concept word used in the subject position denotes an object rather than a concept (see [Frege 1892]); this is the case of C), where something is predicated of the concept itself and this concept is represented as a *class*. A general principle holds: *the object we speak about has to be constructed*. So the concept we speak about has to be constructed.

Yet another general principle holds too: *Speaking about an object we always use some concept.* In our case C' this means that in *mentioning* the concept $\underline{^0\mathrm{HI}}$ we *use* the concept *of this concept*, viz. the concept

$$\{\, [\lambda c\, [^0\mathrm{Quid}\; c\; ^{00}\mathrm{HI}]],\, \lambda c_1[^0\mathrm{Quid}\; c_1\; ^{00}\mathrm{HI}]],\, \ldots \,\}$$

Indeed, any member of this last concept, e.g.,

$$[\lambda c_{23}\, [^0\mathrm{Quid}\; c_{23}\; ^{00}\mathrm{HI}]].$$

can do the job.

APPENDIX TO 1.4.2

The difficulties connected with defining concepts procedurally (concept*) vs. set-theoretically (concept) can be avoided in a perhaps more natural way, viz. *via* a normalisation. This choice has been made in [Horák 2001]. His key definitions (I modified their numbering by adding 'H') are:

> **HDefinition 11** (*concept normal form*). Let us suppose a fixed ordering of all types, i.e. let ξ_i be the *i*-th type over the TIL objectual base and let V^{ξ}_{ij} be the *j*-th variable of the type ξ_i (i.e., ranging over ξ_i, P.M.) for two natural numbers *i* and *j*.
>
> An *α-normal form* of a construction *C* is the construction $NF^{\alpha}(C)$ (we would write $[{}^0NF^{\alpha}\ C]$, P.M.) that ensues from construction *C* in the following way — the structure of the construction is exactly the same except that every free or λ-bound variable is consistently renamed to a first unused variable of the corresponding type (we parse the construction from left to right).
>
> A *β-normal form*+ of a construction *C* is the construction $NF^{\beta}(C)$, where for n > 0 there exist constructions $D_1, \ldots, D_n$ such that $D_1 = C$ and $D_n = NF^{\alpha}(C)$ and for each $i = 1, \ldots, n-1$ every D_{i+1} is a β-reduction+ of D_i and D_n is not β-reducible+ any more.
>
> A *normal form* of a construction C is the construction NF(C)such that $NF(C) =_{df} NF^{\alpha}(NF^{\beta}(C))$ –

+Here "η-normal form" and "η-reduction" should stand, since the β-equivalence is based on η-reduction. P.M. –

> **HDefinition 12** (concept). Let *C* be any CONCEPT* and let *D* be the CONCEPT* constructed by NF(*C*). We call *D* a *concept* and we say that *C points* to the concept *D*. –

[Horák 2001, 58, 60]

Case C in our example (BEING HIGHLY INTELLIGENT is a psychological concept) would be analysed in the following way: $PC/(o{*_1})$, $HI/(o\iota)_{\tau\omega}$, ${}^0HI/{*_1}$, $NF/({*_1}{*_1})$;

(C') $[{}^0PC\ [{}^0NF\ {}^{00}HI]]$. –

1.4.3 Concepts and expressions

1.4.3.1 A basic misunderstanding: confusing semantics with general linguistics

Perhaps it would be better to use the term *logical analysis of natural language* (LANL, see [Materna 1998]) than *semantics* (of natural language), for what is called *semantics* is more often used in the sense of *linguistic semantics*. Linguistic semantics can be construed as a part of *general linguistics*, which means, among other things, that our investigations are *empirical*: they construe natural language(s) as a *natural phenomenon*, i.e., the link between an expression and its meaning/denotation is a *contingent* link — which it is, of course. Semantics in the sense of LANL presupposes *that the respective linguistic convention is already given*, so that the *expression–meaning–denotation* link is a (relative) *a priori* link. (We have seen, however, that the *expression–reference* link is not *a priori* for LANL.) To illustrate these claims, from the viewpoint of 'empirical semantics' it is a contingent fact (connected, therefore, with a piece of information about English) that the expression

the highest mountain

denotes just what it denotes (the individual role of being the highest mountain) rather than, e.g., the property being a hungry dog; the same holds w.r.t. the meaning of that expression. Applying, on the other hand, a LANL view to the above expression we presuppose that the respective convention has already determined both meaning and denotation (but not the reference, of course) thereof. Looking at [Frege 1892a] we can see that Frege's *Sinn* is construed as an entity determined *a priori*. (As for his *Bedeutung*, this is not certain when we take into account his unfortunate *Morgenstern–Abendstern* example where *Bedeutung* is reference rather than denotation, in contrast to his earlier example with medians.)

1.4.3.2 Given a convention, what is a priori?

Summing up, the respective linguistic convention determines — at the given *temporal slice*, see 3.1—*via* its lexical part the meanings/denotations of particular 'basic' simple expressions, and—*via* its grammatical (syntactical) part the meanings/denotations of complex expressions.

This global characterisation does not mean that the convention determines the meanings *and*—independently—the denotations: The denotation (in our sense) is unambiguously determined as soon as the meaning is. At least this should follow from our identifying meanings with constructions: setting aside (temporarily) the complicating factors to be mentioned in 1.4.3.4 and 1.4.3.6 we can state that the unambiguous determination of denotation by meaning corresponds to the fact that what is (v-)constructed depends at most on the valuation v; no empirical factor can influence the outcome of the construction of an object. Since the object denoted by an expression E is this very outcome, our claim is proved.

We have seen that if E is an empirical expression, then our claim still holds: what is constructed is in this case an intension. No *a priori* way leads, however, from this intension to its value in the actual world + time. Thus *reference* is not *a priori* determined.

1.4.3.3 Meanings are concepts

In his [1956, p.6] Church, deviating from the Fregean notion of concept, proposed the following 'quasi-Fregean' scheme:

Expression E
↓
Sense of E = a Concept of
↓
Denotation of E

We have accepted this scheme by taking concepts as closed constructions or the sets of quasi-identical (closed) constructions (see 1.4.2.2). The problem of defining meaning (= sense) independently of the other members of the Quinean 'circle' (i.e., of *analyticity* and *synonymy*, see [Quine 1953]) can be solved in this way.

Remarks:

a) Observe that our solution is universal, unlike any solution that would identify meanings with intensions: mathematical expressions express meanings/concepts although they are not connected with any intension.

b) Setting again aside the complicating sections 1.4.3.4, 1.4.3.6 we can now formulate the following claim:

   Meanings are concepts.

   Now what about the 'inverse' claim, viz.

   ****Concepts are meanings*** ?

   This latter claim is, first of all, imprecise. Meanings are meanings ***of***. Concepts have been defined as non-functions; they are not concepts *of*. In a way, the former claim is also imprecise in this sense, but this impreciseness is innocuous: we can always say *Meanings of any (meaningful) expressions are concepts,* using the traditional notation

   $$\forall e\ (\mathrm{E}(e) \supset \mathrm{C}(\mathrm{M}(e))).$$

   (E … the class of expressions, C … the class of concepts, M … the meaning of.)

   Such a 'correction' of the latter claim poses, however, some problems. The most intuitive option would be *Concepts are meanings of some expression.,* i.e.,

   $$\forall c\ (\mathrm{C}(c) \supset \exists e\ (\mathrm{E}(e) \wedge c = \mathrm{M}(e))).$$

But in this case an unacceptable conclusion would follow: concepts would have a temporal dimension, for concepts would come into being dependently on existence of respective expressions. Our (and in this respect the Bolzanian) conception takes concepts to be entities not localisable temporally or spatially; thus we would say (using the imperfect notation of predicate logic)

$$\exists c\ (\mathrm{C}(c) \wedge \neg\exists e\ (\mathrm{E}(e) \wedge c = \mathrm{M}(e))). –$$

(There are concepts that are not expressed by any expression of our actual language.)

1.4.3.4 Homonymy, synonymy, vagueness

A. Homonymy

Saying — as we did — that *to be the meaning of* is a function is imprecise. The rules governing natural languages admit — unlike those governing artificial languages — frequent 'exceptions'. Homonymy (or polysemy, or ambiguity, see [Stechow, Wunderlich 1991, 99]: the terminological distinctions are not always relevant here) is a phenomenon that puts a limit on the assumed functionality. In some cases one and the same expression possesses more than one meaning. Two subcases are important (see [*ibidem,* 98]):

Lexical homonymy.

Accepting the simplifying (and essentially false) assumption that simple expressions express simple concepts (see Definition 11 and 2.1) we can define lexical homonymy as follows:

Definition 15 (*Lexical homonymy)*
A simple expression is *lexically homonymous* iff it expresses at least two concepts. –

Structural homonymy.

Let E be a structured (i.e., not simple) expression. Surprisingly enough, the next definition is practically identical with the preceding one.

Definition 15' (*Structural homonymy*)
A structured expression is *structurally homonymous* iff it expresses at least two concepts. –

Thus we have

Definition 16 (*Homonymy*)
An expression is *homonymous* (*ambiguous*) iff it expresses at least two concepts. –

Examples:
Some standard illustrations of Definition 15 suggest themselves, for example, *Bank*. One is tempted, of course, to say that the ‘right’ meaning is determined by the respective context: robbing a bank is not admiring the bank of the Thames, but this solution is wrong. The word *bank* possesses — in virtue of the presupposed linguistic convention for English — both the meanings; the context does not change this fact, it only *selects* the meaning that is the right one *therein.* The simple concepts connected with *bank* are, say, $^0\text{bank}_1$, $^0\text{bank}_2$.

The denotations are in this case also distinct; both are properties, type $(o\iota)_{\tau\omega}$.

The things are however not that simple. We will see that the assumption that a simple lexical unit of a natural language always expresses a simple concept is wrong (see 2.1). To see this from a ‘practical’ viewpoint let us consider the expression *prime (number)*. There are two non-equivalent concepts expressed by this term. One is

$$\lambda x\ \forall y\ [^0\supset [^0\text{Div}\ x\ y]\ [^0\vee [^0= x\ y]\ [^0= y\ ^01]]],$$

while the other is

$$\lambda x\ [^0= [^0\text{Card}\ \lambda y\ [^0\text{Div}\ x\ y]]\ ^02].$$

(Div/ $(o\tau\tau)$, divisibility (‘is divisible by’), Card/ $(\tau(o\tau))$, the cardinality of.)

The respective denotations are also distinct: the former concept identifies a class whose member is the number 1, the latter concept excludes 1 from “the class of primes”.

The moral is: True, *prime* as a simple expression expresses two concepts, say, $^0\text{prime}_1$, $^0\text{prime}_2$, but actually this means that the respective linguistic convention associates *prime* with two *complex* concepts. (See 2.1. It will be clear that in most cases — including the above example with *bank* — the same consideration can be applied.)

The structurally homonymous expressions are homonymous due to the distinction between the *grammatical structure* of the given language and the (“international”) structure of *constructions.* Not to adduce standard examples we can offer a more sophisticated illustration. (See [Duží, Materna 2001].) Consider the sentence

Charles believes that Venus is bigger than Mars.

There are strong reasons for taking propositional attitudes (believing, knowing, doubting etc.) to be relations(-in-intension) between individuals and constructions. From this viewpoint the meaning of the sentence can be represented by the construction (Ch/ ι, V/ ι, M/ ι, Big/ $(o\iota\iota)_{\tau\omega}$, Bel/ $(o\iota*_1)_{\tau\omega}$)

$$\lambda w\lambda t\ [^0\text{Bel}_{wt}\ ^0\text{Ch}\ ^0[\lambda w\lambda t\ [^0\text{Big}_{wt}\ ^0\text{V}\ ^0\text{M}]]].$$

Yet whereas logic cannot guarantee that Charles also believes that Mars is smaller than Venus (which is actually blocked by this 'constructional' analysis), the above sentence can be read in another way, viz. as Charles' attitude to the state-of-affairs, in other words, as his attitude to the respective proposition. In this case believing gets the type Bel'/ $(o\iota o_{\tau\omega})_{\tau\omega}$, and the meaning will be

$$\lambda w \lambda t\, [{}^0\mathrm{Bel'}_{wt}\ {}^0\mathrm{Ch}\ [\lambda w \lambda t\, [{}^0\mathrm{Big}_{wt}\ {}^0\mathrm{V}\ {}^0\mathrm{M}]]].$$

This time the conclusion that Charles believes that Mars is smaller than Venus does follow since the proposition that Venus is bigger than Mars is *the same as* the proposition that Mars is smaller than Venus. (Thus we can understand, why Peter, referring to Charles' opinion, can rightly say that according to Charles Mars is smaller than Venus.)

Thus our sentence possesses two distinct meanings (i.e., the respective constructions are distinct) and, moreover, two distinct denotations, since the respective *propositions* are distinct: in some worlds one of them is true while the other one is false.

Our conceptual framework makes it possible to consider a strange kind of homonymy: for an expression to be homonymous it suffices (according to Definition 16) to possess two distinct *meanings*. We take meanings/concepts to be constructions. Now it is possible that two distinct meanings identify one and the same object; the respective constructions can be *equivalent*.

Definition 17 (*equivalence of concepts*)
Let C, C' be closed constructions, i.e., concepts*. C is *equivalent to* C' iff C constructs the same object as C' or C and C' are of the same order and both improper. –

(Equivalence of *concepts* (as sets of concepts* or 'normalised concepts*') is easily definable in terms of Definition 17.)

So we can say that some expressions may be homonymous *without denoting distinct objects.* To exemplify this possibility, consider the expression

Blowing up banks is a destructive activity.

The two meanings of this sentence differ in that one of them contains the concept$^{(*)}$ ${}^0\mathrm{bank}_1$ and the other one the concept$^{(*)}$ ${}^0\mathrm{bank}_2$. On the other hand, the proposition denoted by the sentence is the same: 'Interpreting' it in the first or in the second way we get the same truth-conditions (assuming, as it is natural, that blowing up *anything* is a destructive activity). See *weak homonymy*, [Duží 2004].

Remark: The above analyses show that the TIL approach makes it possible to use more fine-grained tools than most of the standard approaches. Compare these analyses, e.g., with what is said about the key category *Bedeutung* in [Stechow, Wunderlich 1991]: Besides some pragmatic use of this term the *Wahrheitsbedingungen-Semantik* is (critically) referred to, and

Carnap's idea of *intensional isomorphism*, as well as Cresswell's *structured meaning* and Barwise-Perry's *situational semantics* are mentioned; Montague's approach is frequently used. This is an informative book, rich in content, but there is no place therein for systematically developing one consistent idea that would make it possible to solve most of the 'logical puzzles' stemming from unsatisfactory semantic analysis. In my opinion, TIL–being an open system — offers such an idea. Therefore you can say — rightly in a sense — that the present study is a *dogmatic* study (similarly, however, like any study based on one conception, for example any Montagovian study). –

B. Synonymy

A very fine-grained analysis of synonymy can be found in [Materna 1998, 124-127]. Here I only recapitulate the results.

Definition 18 (*synonymy*)
An expression E is *synonymous with* an expression E' iff E expresses the same concept as E'. –

Definition 19 (*equivalence of expressions*)
An expression E is *equivalent to* an expression E' iff E denotes the same object as E'. An expression E is *weakly equivalent to* an expression E' iff E is equivalent to but not synonymous with E'. –

Remark: Compare the above definition with Definition 17. –

Definition 20 (*coincident expressions*)
Expressions E and E' are *coincident* iff they denote distinct intensions whose value in the actual world + time is the same. –

Two principles characteristic of TIL are responsible for the fact that we are able to articulate the important distinctions given by the definitions 18-20:

I) *Meanings/concepts are abstract procedures linking expressions with their denotations.*

II) *Parmenides Principle* (see 1.3.2) *together with the claim that empirical expressions denote intensions.*

Without I) we would hardly be able to distinguish between synonymy and weak equivalence. At most we would say something like *synonymous expressions may possess distinct syntactical structure,* which is, of course, not a statement based on a *semantic* analysis. Besides, we would probably say that all mathematically true sentences and all mathematically false sentences are synonymous. We can say that all distinct true mathematical sentences as well as all distinct false mathematical sentences are only weakly equivalent.

Without II) weak equivalence and coincidence of empirical expressions would not be distinguishable. Let us return to the unfortunate interpretation of Fregean example with *morning star* vs. *evening star.* Our present approach to solving his problem can be summed up as follows: *morning star* is not (weakly) equivalent to *evening star*—these two expressions are just *coincident.* By the way, a generalisation of this example is highly instructive: for Frege all true empirical sentences (just as all false empirical sentences) would be (weakly) equivalent. We would say that they are only coincident. All these distinctions are given by II alone. For Frege *morning star* as well as *evening star* denotes Venus; for us they denote two distinct intensions (individual roles). For Frege every sentence denotes a truth-value; for us every empirical sentence denotes a proposition. Venus and truth-values are *references* rather than *denotations* of the respective expressions.

Our definitions make it clear that (and why) real synonymy is such a rare phenomenon. As for a true lexical synonymy, i.e., where we could presuppose that both (or, in general: all the) expressions express one and the same *simple* concept, why multiply expressions for one and the same (simple) procedure? (And indeed, can you find many examples? Well, what about consulting some *dictionary of synonyms*? Forget it; what the linguists take to be a pair of synonymous expressions almost never satisfies Definition 18.) As for synonymy of complex expressions, Definition 18 is very strict: no two such expressions can be synonymous which differ in some semantic feature. Thus the only distinction between synonymous expressions has to consist in some purely syntactical, semantically unanalysable difference. Perhaps the following two sentences can serve as an illustration (see [Materna 1998, 125])

Charles believes that his wife is clever. Charles believes his wife to be clever.

Definitions are a special case. Consider the classical 'equational definition' or 'equational explication'. The schema of such a definition/explication is

$$Dfd = \Phi(E_1,\ldots,E_n),$$

where *Dfd,* 'definiendum', is a simple expression and the right side of the equation is a complex expression containing only such subexpressions $E_1,\ldots,E_n$ whose meanings are supposed to be already given. From the semantic viewpoint the most important thing is that the meaning of the simple *Dfd* is identified with the meaning possessed by the right hand side. Thus such a definition is no statement; it is a stipulation. Now we can ask: is the *Dfd* synonymous with the right hand side? If we accepted the view that the *Dfd*, being a simple expression, expresses a simple concept, our answer would be *No*: a simple concept cannot be synonymous with a complex concept by Definition 18. Yet the case of definitions /explications is very clear: the *Dfd gets* its meaning due to the respective definition: it *does not possess another meaning.* Thus the meaning of the *Dfd* is not a simple concept but the complex concept connected with the right hand side of the definition. And therefore we can say that *as soon as the meaning of a simple expression is — in the given language **L** — explicitly given by such a definition/explication, the respective Dfd and the right side of the definition are synonyms (in **L**)*.

C. Vagueness

From the viewpoint of any theory of concepts the well-known phenomenon of *vagueness* is highly interesting. The semantically interesting question is (cf. [Materna 1998]):

> *Where should be the source of vagueness be looked for: is it the expression, the respective concept, or even the respective denotation?*

It would be probably very strange to suppose that objects denoted would be themselves vague. I cannot imagine a rational ontology that would underlie such an assumption. What we call *objects* is mediated by concepts. We ask therefore: *How do concepts contribute to vagueness?*

It will suffice to consider concepts*, i.e., closed constructions.

a) *Variables.* We can hardly make variables guilty of causing vagueness. (Especially when we know that in principle things can be done without variables, e.g., with Curry's combinators.)

b) *Trivialisation.* Consider simple concepts. According to Definition 11 a simple concept(*) ^{0}X is an abstract procedure that identifies (a variable or) the object-non-construction X without using any other procedure. Assuming that *heap* expresses a vague simple concept we get ^{0}heap. Now according to Definition 4 the object heap is identified without any change. We know, however, that at least in this frequently used example the linguistic convention is just 'vague' in the sense that it is not exactly binding on the users of the given language. Not only will distinct users dissent when deciding whether the given object is or is not a heap: one and the same user can have different opinions at different times. As Peirce says, "the speakers' habits of language [are] indeterminate" (see [Stechow, Wunderlich 1991, 251]). In Pinkal's article in [*ibidem,* 250-269] we can find various theories trying to deal with this unpleasant phenomenon; it is not our task here to thoroughly analyse vagueness but some interesting points are relevant for our study, in particular a comparison between homonymy/ambiguity and vagueness, and the approach of fuzzy logic. Let us therefore continue analysing the heap example. Here we set aside the interesting problem of how to accommodate — as the case may be — our theory of concepts to let it cover fuzzy sets. Instead we will articulate an idea (suggested already in [Materna 1998]) that would make vagueness a special kind of ambiguity. Pinkal in [Stechow, Wunderlich 1991, 264] quotes from Kit Fine's article in *Synthese 1975*:

> Vague and ambiguous sentences are subject to similar truth-conditions; a vague sentence is true if true for all complete precisifications; an ambiguous sentence is

true if true for all disambiguations. … to assert an ambiguous sentence is to assert, severally, each of its disambiguations. … precisifications are extended from a common basis and according to common constraints; to assert a vague sentence is to assert, generally, its precisifications.

So how do ambiguous and vague sentences (but in general, vague *expressions*) differ? For Fine precisifications are connected with a *common basis* and *common constraints.* The minimum number of grains (although not exactly registered) could be such a common basis for *heap*. Nothing like this is necessary in the case of ambiguous expressions (consider *bank*). Yet in a sense we could speak about homonymy (ambiguity): see [Materna 1998,128,129; (H_i/ $(o\iota)_{\tau\omega}$ is a property which in a pair ⟨possible world, time point⟩ determines the class of objects containing in the heap-like manner at least *i* grains)]:

> Now we can imagine thousands of such properties H_i differing by the number of grains admitted as the cardinal number of those sets which are 'taken into account' by the given H_i. … The expression *heap* will then be *homonymous* in a special way: it will cover a whole set of concepts, each of which will identify one of H_i. … one feature which distinguishes this kind of homonymy from the other kinds, could be formulated as follows: The concepts associated with the given expression are in a sense *similar*: there is no analogy with *bank, idealist,* etc. If A does not admit that the given object is a heap while for B it is (since A connects with *heap* another concept…than B) A will all the same admit that the controversial object is similar to a heap.

A consequence thereof is:

> …only the fact that the particular concepts are very similar *inter se*, together with the fact that there are nearly as many 'sublanguages' … as users of a given language cause the impression that the given expression possesses an enigmatic property '*vagueness*' rather than that it is homonymous.

Therefore, if we accept this view, we should write $^0heap_1$, $^0heap_2$, …, but this would be without any practical sense.

c) *Composition, Closure.* These constructions have been defined in such a way that the following claim is obvious:

Let *C,* C_1, …, C_n *be simple concepts expressed respectively by expressions E,* E_1,…, E_n. *Assuming that neither of these expressions is vague, neither the composition* $[CC_1…C_n]$ *nor any closure, say,* $\lambda x_1…x_m$ *C, can be a source of vagueness.*

Thus the so called vagueness is no special property of expressions, concepts or denotations. It is only a kind of a property of expressions called *ambiguity*.

Remark: It would be a mistake to believe that vagueness can be found only in such cases that seem to be paradoxical (*heap, bald*). For example, Black has shown (in his classic [Black 1937], the example with chairs in a 'logical museum') that all empirical expressions can be considered to be vague. Further, there are many kinds of vagueness, see [Stechow, Wunderlich 1991]; a special kind concerns individual descriptions, such as *the most famous composer* etc. Here we wanted to show only that our theory of concepts needed not be threatened by the excentric (albeit universal) phenomenon of vagueness.

1.4.3.5 Compositionality

There are many definitions of compositionality (cf. the recent analysis in [Sandu, Hintikka 2001]), many interpretations of "Frege's principle". In [Stechow, Wunderlich 1991, 107] we read:

> Wenn α ein zusammengesetzter Ausdruck ist, der mithilfe der syntaktischen Operation *F* aus den Ausdrücken $\alpha_1,\ldots,\alpha_n$ gewonnen ist, dann ist $b(\alpha) = G(b(\alpha_1),\ldots,b(\alpha_n))$, wobei *G* die *F* entsprechende semantische Operation ist.

(*b* is 'Bewertung', i.e., the semantic evaluation of a syntactically given expression; the quotation is intended to define *b* together with the point concerning simple expressions.)

The standard (Tarskian) definitions are based on the idea of *homomorphism of a syntactic algebra into a semantic algebra.* Here we would only like to show that the TIL theory of constructions offers a deep insight into this problem.

First, consider two natural languages L and L'. Suppose that a Montagovian-like analyst defines such a homomorphism for L, say, **H**, and a homomorphism for L', say, **H'**. Consider an expression *E* of L and its correct translation *E'* in L'. When we say that *E'* is a *correct translation* of *E* we mean that *E* shares with *E'* the same *meaning*. Now we have seen that **H** ensures the compositionality for L while **H'** does so for L'. At the same time, we know that in general the operations *F* and *G* on the one hand and operations *F'* and *G'* on the other hand are distinct. The respective syntactic algebras as well the respective semantic algebras are distinct. So where should we search for the common meaning of *E* and *E'?* To illustrate (in a simplifying manner) the problem, let us compare the English sentence

Charles is hungry.

with the Czech translation

Karel má hlad.

The operation *F* transforms the syntactic atoms into the English sentence so that we have

F (Charles, to be, hungry),

whereas *F'* does so for the Czech sentence:

F' (Karel, mít, hlad).

Compositionality is ensured by

$$G\ (b\ (\text{Ch}),\ b\ (\text{to be}),\ b\ (\text{hungry}))$$

and

$$G'\ (b'\ (\text{Karel}),\ \text{b'}\ (\text{mít}),\ b'\ (\text{hlad})).$$

We can see that *F* and *F'* lead in the Chomskian (and, partially, post-Chomskian) literature to *trees*. This idea belongs to the family of attempts to define meanings as structured entities, see, e.g., [Cresswell 1975], [Cresswell 1985], [Lewis 1972], in a sense also [Bealer 1982]. Lewis' formulation is a paradigmatic one (p. 182):

> It is natural…to identify meanings with semantically interpreted phrase markers minus their terminal nodes: finite ordered trees having at each node a category and an appropriate intension. If we associate a meaning of this sort with an expression, we are given the category and intension of the expression; and if the expression is compound, we are given also the categories and intensions of its constituent parts, their constituent parts, their constituent parts, their constituent parts, and so on down.

Thus compositionality should be automatically guaranteed, at least in the sense (ii) of [Sandu, Hintikka 2001, 49-50].

But our question is still unanswered. Having, say, the two trees, both guaranteeing compositionality for 'semantic analysis' of each of the two sentences (in general, expressions) does not give their meanings. We could speak about the meanings of the L-expressions and meanings of the L'-expressions (in particular, about the meaning of the English sentence and about the meaning of its Czech translation), but — if we accept some Lewisian definition of meaning — *the two meanings-trees are not identical*. (See [Materna 2002].)

Concepts (as essentially constructions) are, of course, *international.* Taking — for the sake of simplicity — Charles/Karel to be an individual and Hungry / $(o\iota)_{\tau\omega}$ a property of individuals and, further, accepting, for the time being, the hypothesis that simple expressions express (in the given case) simple concepts we get the construction

$$\lambda w \lambda t\ [{}^0\text{Hungry}_{wt}\ {}^0\text{Charles}]$$

as the common meaning of both sentences above. This construction/concept[(*)] fulfils the most general definition of compositionality: *The meaning of both the sentences is unambiguously determined by the meanings of their semantically autonomous parts.*

What could be called *the (logical) analysis of natural language* should be based on another philosophy. Tichý has articulated the main features of such an approach in a posthumous article [1996a].

Some points that characterise that approach are adduced in *Foreword* to [Tichý 1996a, 43], and the posthumous (probably unfinished) manuscript intends to build up a *meaning driven grammar* (of English). Here are some quotations:

> …the grammarian must also determine what those meanings are.
>
> A purely syntactic generator of well-founded expressions is in principle impossible because the well-formedness of a compound expression often depends not only on whether its components are well-formed but also on what they mean. Syntax and semantics must go hand in hand.
>
> A natural language is a *code* and an important part of the grammarian's task is to decipher that code. This task is not discharged by translating vernacular expressions into an invented 'ideal' notation which is based on substantially different coding principles.
>
> *The notion of a code presupposes that prior to, and independently of, the code itself there is a range of items to be encoded in it. Hence…meanings cannot be conceived of as products of the language itself. They must be seen as logical rather than linguistic structures, amenable to investigation quite apart from their verbal embodiments in any particular language. To investigate logical constructions in this way is the task of logic. The linguist's brief is to investigate how logical constructions are encoded in various vernaculars.*
>
> (Compare with ([Shapiro 1997, p.137].)

Finally, showing that the idea of 'autonomous syntax' is untenable Tichý adduces the standard 'phrase marker' analysis of the sentence *Ali slowly works* and asks how a 'pure syntactician' would define particular categories used in the phrase-markerese. In particular, as for the category 'sister' (*slowly* in our example is a sister of *works*) he says:

> Why is it, for example, that 'slowly' is a sister of 'works' but not of 'Fred'? The syntactician cannot explain it by pointing out the obvious fact that 'slowly' stands for an activity modifier, i.e. for a mapping which takes activities to activities, and that the activity named by the **VP** 'slowly works' is the value of that mapping at the argument named by 'works'. … For that would be transgressing the boundaries of autonomous syntax. No pre-theoretical meaning seems to attach to the term 'sister' either. (p. 46)

It is just this illusion about 'autonomous syntax', which can lead to prematurely stating that such and such kind of expression does not obey the compositionality principle. As a typical example we can adduce Stechow's analysis in [Stechow, Wunderlich1991, 112]: here we read that an expression whose logical form is

$$(\forall x)\ \mathrm{P}x$$

does not possess a compositional interpretation. The *b* (Bewertung) of such an expression would have to be calculated from $b(\forall x)$ and $b(\mathrm{P}x)$. Yet we see that the form of the *construction* underlying such an expression is

$$[^0\forall\ [\lambda x\ ^0\mathrm{P}x]]$$

so that the meaning of that expression can be calculated from the meanings of, say, *Every* ($^0\forall$) and of, say, *real irrational number,* which denotes a *class* of irrational real numbers, so that we have $\forall/\ (o(o\tau))$, $x \rightarrow \tau$, Ir/ $(o\tau)$, and the (false) sentence is analysed as follows:

$$[^0\forall\ [\lambda x\ [^0\mathrm{Ir}\ x]]],$$

which makes it possible to compose *b*(*Every real number is irrational*) from *b*(*Every*) and *b(real irrational number*): from $^0\forall$ and $[\lambda x\ [^0\mathrm{Ir}\ x]]$.

True, Stechow shows (*ibidem*) that Tarskian approach to semantics can safeguard compositionality (that this can be done universally is shown — at least for the case ii) — in [Sandu, Hintikka 2001]) but he at the same time criticises this way out from a philosophical point of view:

> Man steckt den nichtkompositionalen Teil der Semantik in die Ontologie. … In gewisser Weise verschleiert diese Formulierung also, dass die Semantik der Variablenbindung nicht kompositional zu behandeln ist.

Yet from our vantage point this objection is hard to understand. First, no semantics is independent of ontology: it *is* a link between language and ontology. Second: The problems with variables arise, of course, if variables are taken to be letters which are handled according some syntactic rules. This conception leads to analysing quantifiers *qua* 'operators': In the above example we seem to distinguish — besides 'P(x)'— two components: the 'operator' $\forall$ and the variable x. The semantics of such 'operators' is well-known from the standard courses of predicate logic: the 'operator' does not possess a self-contained interpretation, it is only an 'improper symbol'; only the whole context, '$\forall x$ A', where 'A' is a well-formed formula, gets the interpretation. In TIL the quantifiers (e.g., '$\forall$') are semantically self-contained expressions, type-theoretically polymorph (scheme of one of the possibilities: $(o(o\alpha))$). They are what can be generally called *predicates* applicable to classes.

Compositionality is a great theme for the Finnish school, in particular for Hintikka's GTS. Feeling that a confrontation with GTS and, e.g., IF-logic is an extra topic, maybe much relevant for the theory of concepts, we would like to postpone such a confrontation to another opportunity. For the present we assume that semantics based on our TIL-shaped theory of concepts fulfils the condition of compositionality.

1.4.3.6 Pragmatic factors. Open constructions

In *On What There Is* (in [1953]) Quine writes:

> If we are allergic to meanings as such, we can speak directly of *utterances* as significant or insignificant, and as synonymous or heteronymous one with another.

Here, in a nutshell, is the main thesis of the neopragmatists' conception of semantics. As I try to show in [Materna 1998, 116], all this post-analytic fashion with Quine's criticism of Carnap's attempt to define intensional semantics is based on a *category mistake.* Let us briefly recapitulate the core of this anti-Quinean claim:

Meanings are what makes *expressions* of a language meaningful, i.e., what makes it possible to understand those expressions. *Expressions* are abstract vehicles of meanings: we cannot spatio-temporally localise particular expressions (unlike expressions-tokens). What *semantics* (even in the linguistic sense, but especially as LANL, see 1.4.3.1) has to study is the connection between (abstract) *expressions* and what they are about (denotational semantics) and in which way this link expression—object is realised (semantics of sense/meaning).

In contrast therewith, *utterances* are spatio-temporal *events*: they are *concrete*, not abstract. To study utterances means to study *contexts of utterances*. These contexts *co-determine* what we are talking about in some cooperation with the meanings of the expressions. This is what *pragmatics* has to study. Therefore Montague in his *Pragmatics* (in [1974, 95-118]) has among his *indices* not only possible worlds and times but also context-dependent ones.

Remark: Usually the temporal expressions like *now, today,* etc. are also taken to be 'indexicals', context-dependent, pragmatic. Since the type τ is among the basic types in TIL, such expressions are analysed as context-independent. As for *now,* the argument goes as follows: this expression denotes a function, type $(\tau\tau)$: if applied to any time point t it returns the same time point. Thus using *now* does not contribute to the informative content of the sentence, being only an identity function. No context dependency can be stated. Example (simplified): Compare the sentences

Charles is hungry.

Charles is now hungry.

Types: Ch/ ι, H/ $(o\iota)_{\tau\omega}$, N/ $(\tau\tau)$. We get

$$\lambda w\lambda t\ [^0\mathrm{H}_{wt}{}^0\mathrm{Ch}],$$

$$\lambda w\lambda t\ [[[^0\mathrm{H}w][^0\mathrm{N}t]]\ ^0\mathrm{Ch}].$$

Since N is an identical function, $[{}^0\mathrm{N}t]$ is equivalent with t. Thus both sentences are (weakly) equivalent (Definition 19). –

The Quinean 'semantic' theories *replace* meaning as what concerns expressions by 'meaning' as an attribute of utterances. That an analysis of utterances can enrich study of semantics is obvious; what is less obvious (and is *wrong* from our point of view) is that all that can be said about meanings is given by analysing the *use* of expressions, i.e., utterances. The process of learning meanings is surely a process consisting in imitating the behaviour of our teachers when they perform particular speech acts: yet this does not mean that meaning itself is reduced to particular acts of teachers' behaviour. The teacher presents his/her knowledge of the given language by behaving so and so, but (s)he reproduces this convention. This convention has already determined that the English words *swan, black, some* mean what they mean, and the syntactic part of the convention has it that *Some swans are black* denotes such and such truth conditions and means such and such procedure / construction. The sentence possesses this meaning and denotation even if it is never uttered. And if some context of utterances reveals a metaphoric sense of the respective utterance of that sentence, we are surely aware of the fact that without this original meaning no metaphor would be possible.

But while some expressions are context-independent in that they are semantically self-contained an important class of expressions cannot be said to possess this property. For the members of this class it cannot hold that their meanings are concepts. They do not determine what they are about: they need *a pragmatic input*, given by an *utterance of the expression in a given situation.*

Remark: The situation of utterance of an expression E can be imitated by a preceding part of a text, which makes it clear that, e.g., the occurrence of a pronoun in E has to be read anaphorically. We do not analyse this case here. –

The most known representatives of such 'not-self-contained' expressions are pronouns — but we will see that also proper names ('genuine' proper names?) behave similarly — and, of course, expressions containing pronouns. Since the time of Frege's *Gedanke* endless discussions try to offer a satisfactory analysis of (the pronoun) *I*, and Kaplan's well-known analyses of indexicals and demonstratives (see [Kaplan 1978]) have become a classic in this respect. Since these problems do possess relevance to any theory of concepts, we will present some remarks without laying a claim to having found a definitive solution.

In his *Pragmatics* (see [Davidson, Harman 1972, 380-397], but see also very interesting considerations in [Childers, Svoboda 2003, especially p.188]) Stalnaker says that (one part of) pragmatics concerns

> the features of speech context which help determine which proposition is expressed by a given sentence. (p.383)

Now imagine that somebody says:

I am hungry.

What corresponds to this *utterance* as an expression / sentence is the *sentence*

I am hungry.

Let our speech context be such that the utterance has been made by a) Richard Montague, b) Abraham Lincoln. The same sentence denotes (mind you, *denotes*, not expresses, in our terminology at least) *another proposition in the case a) than in the case b).* Kaplan's characteristic of his approach to explaining such phenomena goes as follows ('*Dthat*', in [Yourgrau 1990, 11-33]):

> [s]ome or all of the denoting phrases used in an utterance should not be considered part of the content of what is said but should rather be thought of as contextual factors which help us interpret the actual physical utterance as having a certain content. (p.19)

Kaplan speaks of a *content of an utterance*. On the other hand, his *sense* is the sense of an *expression* (see, e.g., [Kaplan 1978]) and possesses two components: content and *character*. The latter is then a function *from contexts to contents*. So the above *utterance* would be in case a) associated with the proposition that Richard Montague is hungry, in the second case with the proposition that Abraham Lincoln is hungry, for Kaplan considers *content* to be an intension, in the case of sentences a proposition, indeed.

Let us return to the sentence I am hungry. As a *sentence*, i.e., an *expression,* it should encode a construction. Let H(ungry)/ $(o\iota)_{\tau\omega}$ be the respective property; what about I ? Does it encode Montague? Lincoln? Obviously nothing like that. What I encodes is something like a 'peg'. The role of such a peg is played by *variables*. So let $x \rightarrow \iota$ be such a variable. Our construction would be

$$\lambda w \lambda t\, [{}^0\mathrm{H}_{wt}\, x]\ .$$

Thus what could be called *meaning* of such expressions would be an *open* construction rather than *closed*. (See [Materna 1998, 7.1].)

Yet there are more problems with this approach. First, if any other individual variable is substituted for *x*, it will do the same job. This problem could be handled analogically as the problem of concepts* vs. concepts. But second, the sentence You are hungry could be analysed in the same way, so that the clear (semantic? pragmatic?) distinction between I and you is lost. Thus it is not sufficient to use 'normal' variables: their pragmatic nature has to be respected, which can be realised by 'typing' them by various pragmatic indices (e.g., using a Montagovian style). Thus our constructions could be 'pragmatised' as follows:

$$\lambda w \lambda t\, [{}^0H_{wt}\, x_{speaker}],$$

$$\lambda w \lambda t [{}^0H_{wt}\, x_{addressee}],$$

etc. One can add further indices, of course (such as $_{place}$ and like). For linguistic analyses see [Stechow, Wunderlich 1991, IV]).

These 'pragmatic variables' make up a 'bridge' between semantics and pragmatics. Indexical expressions express constructions/concepts and denote objects only 'potentially'. The valuations of their free variables are given by any event when the expression is uttered. Then an object replaces every free variable and we get — in this 'pegwise' manner — again a concept.

For Kaplan, however, this is not enough. His way to the well-known *dthat* is given by the following generalisation:

> If pointing can be taken as a form of describing, why not take describing as a form of pointing? ([Yourgrau 1990, 24])

This point is incompatible with our viewpoint. (Empirical) descriptions — unlike pronouns etc. — are semantically self-contained. Let us take over Kaplan's example:

The spy is suspicious.

Kaplan compares two *utterances* of this sentence (his numbering):

(17) Dthat ['the spy'] is suspicious.

and

(3) The spy is suspicious.

The *contents* of these two utterances, i.e., the respective propositions, are distinct: For (17):

> The *relevant individual* is determined in the world in which the utterance takes place, and then the same individual is checked for suspicion in all other worlds ([Yourgrau 1990, 28])

for (3)

> [w]e determine a (possibly) new relevant individual in each world. (*Ibidem*)

Kaplan rightly sees that the utterance (3) can be ambiguous in that it could be intended as the utterance (17). We could say that also the *sentence* The spy is suspicious is ambiguous. It can encode an open construction, say,

(17') $\lambda w \lambda t\, [{}^0Susp_{wt}\, [x_{property \rightarrow individual}\, {}^0spy]]$

where x ranges over functions from properties to individuals, which would correspond to those cases of utterance which Kaplan endows with 'dthat', but it can also encode a *closed* construction

(3') $\lambda w \lambda t\ [{}^0\text{Susp}_{wt}\ {}^0\text{The\_spy}_{wt}]$,

where The_spy/ $\iota_{\tau\omega}$. (Clearly, the proposition constructed in this way will mostly lack any truth-value: show me a world/time where there would be just one spy. This is, however, unimportant in the present context.)

I am deeply convinced that descriptions are not "forms of pointing": *the area of pointings is the area of functions from utterances to objects; it is a typically pragmatic area. The propositional concept realised by (17') is dependent on particular situations, events. The propositional concept given by (3') is independent of particular utterances: it determines the proposition true in those worlds + times where the individual that is the only spy is suspicious, false where the only spy is not suspicious, and without any truth-value where there are no spies or more than one spy.*

A consequence of this approach is that Kaplan's *dthat*, applicable to *utterances*, cannot be applied as a logical construct applicable to analysing *expressions*. No semantic analysis can be done in terms of *dthat*; in trying to do so we would suppose that *dthat* determines the respective object as the value of some function in the *actual world*. We have already argued that this is impossible. (See the Remark following Definition 9.)

1.4.3.7 Summary

Our knowledge is realised *via* concepts. Our explication of the term CONCEPT, be it through Definition 14 or through HDefinition 12, makes it possible to handle some problems connected with the *process* of knowledge in a more rigorous (some will say "more formal") way than is customary. No analysis of *texts* can be expected, though; this shortage is innocuous since any stage of knowledge can be represented by a collection (set, if you like) of claims whose conceptual analysis is globally possible even within the framework of our approach. These claims can be formulated in such a way that no indexicals occur in them. Even if this were not the case, the preceding paragraph (1.4.3.6) shows that the situational context characteristic of using indexicals enables us to replace the respective open constructions by closed constructions (*via* 'pragmatic valuations') and so by concepts.

Thus we can move forward to our main topic, viz. to *conceptual systems*, presupposing that concepts are *abstract procedures* identifying *objects* in the broadest sense of the word.

Remark: The growth of knowledge is connected with the *asking of questions*. Becoming aware of something new can be construed as answering some question. Concepts are just tools for asking questions. In the case of mathematical concepts we ask: *What is constructed?* In the case of empirical concepts we ask: *What is the value of the intension (constructed by the concept) in the actual world?* Some examples (let the respective construction be a construction *pointing to a concept* according to HDefinition 120; types obvious):

CONCEPT	QUESTION
$\lambda x\ [^0\wedge\ [^0\mathrm{Prime}\ x][^0\mathrm{Even}\ x]]$	Which numbers are constructed? Or: Which primes are even?
$[^0{=}\ ^0\mathrm{Prime}\ ^0\mathrm{Odd}]$	Which truth-value is constructed? Or: Is it true that…?
$\lambda w\lambda t\ [^0\mathrm{Highest}_{wt}\ ^0\mathrm{Mountain}_{wt}]$	Which is the highest mountain?
$\lambda w\lambda t\ [^0\mathrm{Card}\ ^0\mathrm{Planet}_{wt}]$	How many planets are there?

(A similar suggestion can be found in [Materna 1998, 65].)

The respective English expressions for the above concepts are:

an even prime number, the class of primes is identical with the class of odd numbers, the highest mountain, the number of planets.

All these expressions are easily transformable into interrogative sentences corresponding to the above questions. See also [Horák 2001]. –

2. Conceptual Systems

2.1 The problem of 'simple concepts'

First of all, we have to specify the notion of *simple concept*. Definition 11 concerns only simple *concepts**. We can now easily formulate definition of *simple concepts* based on Definition 14:

Definition 14' (*simple concept*).
A concept is *simple* iff it has a member a simple concept*. –

It is, however, more advantageous to exploit the normalising definition H12: if a concept C is simple according to Definition 14', then Horák's NF applied to any member of C returns just that simple concept* (if any) which is a member of C. We can speak about a simple concept as a trivialisation of an object, which would be inappropriate if Definition 14 were applied.

Let us consider the more interesting case of Definition 11, viz. the case where the simple concept is a trivialisation of an object, not of a variable. As far as we examine concepts *in abstracto*, i.e., without any connection with expressions of a natural language, it seems that no great problem arises. Fully setting aside the way we use concepts *via* the respective 'linguistic codes' (expressions) we can presuppose in a radically platonic manner that, e.g., the concept ^{0}Prime_number *is* a procedure which identifies the set of primes without 'calling' another procedure, such as the procedure identifying the dividing function. We cannot imagine, let alone say what such a procedure would look like, but this does not disprove the abstract possibility of its existence. Thus, for example, according to our definitions, the concepts

$$^{0}\text{Prime\_number}$$

and

$$\lambda x\ \forall y\ [^{0}\supset [^{0}\text{Div}\ x\ y]\ [^{0}\vee [^{0}{=}\ x\ y]\ [^{0}{=}\ y\ ^{0}1]]],$$

are really two *distinct* concepts: a simple concept cannot be identical with a complex concept.

What then is 'the problem of simple concepts'?

We have already touched on this problem here and, in more detail, in [Materna 1998] and [Materna 2000]. To recapitulate, it is not at all obvious, and, indeed, simply false to say that expressions that are in the given language simple express simple concepts. Applying this claim to our example, to say that the English expression prime is not synonymous with the expression the number such that it is divisible just by 1 and by itself is at least doubtful: we understand the word prime just because we understand the second expression; it is a *definition* which endows the simple word with a meaning, i.e., which associates it with a concept. The simple concept does not work on the *word* prime, it works on the *object,* i.e., on the *set* of prime numbers. (Therefore, if sky-blue means the same as azure, there is only one simple concept here: we can write either ^{0}sky-blue or ^{0}azure, we are not confronted with *two equivalent constructions* but with *one and the same construction*!)

Remark: If Fodor's M(ental) R(epresentation)s are reinterpreted and viewed as *expressions of a natural language* instead of expressions in Mentalese then the problem of simple concepts is well formulated in [Fodor 1988, 40 …]. See, e.g.,

> [i]f concepts express properties, then it's not unreasonable to suppose that BACHELOR and UNMARRIED MAN express the *same* property.
>
> …the concession that being a bachelor and being an unmarried man are the same thing is meant to leave open the question whether BACHELOR and UNMARRIED MAN are the same concept.

Fodor speaks in the footnote about concepts as *definitions*. On this point see 2.2 and [Materna 1998, 144-145, Claims 19—19‘‘]. –

2.2. Simple concepts as primitive concepts. Conceptual systems

A 'non-mentalistic' reading of Fodor (if thinkable) makes it clear that he strives to solve problems similar to ours and that we would subscribe to, e.g., his criticism of 'Inferential Semantics' (see [Fodor 1998, in particular p.36]). Compare:

> Since a mental representation (*read: [our] concept* —P.M.) is individuated by its form and content (*read*: *by its structure and by what is constructed* —P.M.)…both of these are assumed to be determined by specifying the *inventory of primitive concepts (emphasis mine)* that the representation contains, together *with the operations by which it is assembled from them (emphasis mine).*
>
> (*Ibidem,* 28)

This reminds us of the quotation from Bolzano (1.2.2 above) where a concept is said to be the way to *combine* the elements of its content. Another parallel to Fodor's idea can be drawn: Fletcher's characteristics of (intuitionist) *constructions* (see 1.1.2). Complex concepts are the results of assembling from the simple concepts. But Fodor speaks of *primitive concepts*. How are the notions *simple concept* and *primitive concept* connected?

Assuming that simple concepts are defined as here (essentially Definition 11) we can answer:

A concept is simple iff it obeys Definition 11 (see, however, 2.1); *a concept is primitive with respect to a conceptual system.*

To make this explanation clear we have, of course, to say what a *conceptual system* is. In [Materna 1998] I have defined conceptual systems as follows:

Let $\{C_1,\ldots,C_m\}$ be a set of simple concepts (of any order) of the form ${}^0X$, where X is not a construction. Let $\{C_{m+1},\ldots\}$ be the set of all concepts distinct from $C_1,\ldots,C_m$ and such that all their subconstructions consist of either variables or some members of the first set. Then the set

$$\{C_1,\ldots,C_m\} \cup \{C_{m+1},\ldots\}$$

will be called a *conceptual system.* Let **CS** be a conceptual system. The first set will be called the set of *primitive concepts of* ***CS***, denoted $\mathbf{PC_{CS}}$, the second set is then the set of *derived concepts of* ***CS***, denoted $\mathbf{DC_{CS}}$.

However, sets are not systems. In this respect the definition is not precise. Systems can be construed as a kind of 'machinery' that produces objects from some initial set(s). Conceptual systems can be seen as systems in this sense.

Definition 21 (*Conceptual system*)
Any *conceptual system* consists of two subsystems, it is two-dimensional:

A. ***Types*** (= *Preconcepts*)

B. ***Concepts***

The system **A** produces the set T of *types of any order* from a set of *atomic types.*

The system **B** produces concepts from the set P of *primitive concepts* and the set V of *variables*.

A-dimension: The atomic types are members of the Base together with collections of constructions of any order ($*_n$, $n \geq 1$). The rules of **A** create more complex types from simpler ones: if T, $T_1,\ldots,T_n$ are types then $(TT_1\ldots T_n)$ is type, see Definition 5.

B-dimension: Let $P_1, \ldots, P_k$ be a finite set of simple concepts. Let $t_1,\ldots,t_k$ be types of order 1. The 'machinery' for **B** can be schematised as follows:

$$\langle\ \{P_1 \rightarrow t_1, \ldots, P_k \rightarrow t_k\}, \text{V, Trivialisation, Composition, Closure}\ \rangle,$$

where the rules creating complex concepts from the simpler ones obey HDef.

The whole scheme of a conceptual system **CS** is thus

$$\boldsymbol{\langle\ \langle\{T_1,\ldots,T_n\}, \text{Definition 5}\rangle\ \langle\ \{P_1 \rightarrow t_1, \ldots, P_k \rightarrow t_k\}, \text{V, Triv, Comp, Clos, HDef}\ \rangle\ \rangle. -}$$

Remarks:

Let Π be the set of expressions denoting $T_1,\ldots,T_n$. Let $\mathtt{T}$ be the set of expressions denoting the members of T. *$\mathtt{T}$ is decidable in the set of finite sequences of the members of Π and left and right parentheses.* T is thus well-defined.

The number of primitive concepts is finite. Further concepts can be produced by Triv, i.e., trivialisation. Simply iterating Triv does not lead, in general, to interesting outcomes; on the other hand, trivialisation of constructions is inevitable when propositional attitudes are analysed (see [Duží, Materna 2001]). Trivialisation, Composition and Closure unambiguously produce constructions using also members of V. The procedure that eliminates those constructions that contain some free occurrence of a variable is also unambiguous. Reducing all Quid-related constructions to the normalised representative is unambiguously given by HDef, i.e., HDefinition 11 and HDefinition 12.

The set of concepts generated by the given **CS** is well-defined. This can be seen when the notion *length* for constructions is defined (which can be achieved in various ways but is always inductively definable); then first the constructions of length 1 are produced (with the exception of variables) and for every length *d* there are finitely many possibilities of constructions of length *d* due to the fact that there are finitely many primitive concepts and that the HDef normalisation cuts the possibility of using infinitely many variables and infinitely many 'η-spreads'. –

It should be clear that on the *abstract level*, where no connection between concepts and expressions of a language is taken into account, we can imagine infinitely many conceptual systems whose importance for the analysis of expressions of a natural language ("NL expressions") is nil. It is, e.g., of no use at all to consider such exotic **CS**s as those whose $\mathbf{PC_{CS}}$ contain one member (try to imagine the **CS** such that its $\mathbf{PC_{CS}}$ is $\{^0\mathrm{Cat}\}$) or such concepts whose combinations cannot be taken advantage of from any rational viewpoint (e.g., $\{^0 2,\ ^0\mathrm{blue}\}$). Also, the way the constructions have been defined makes it clear that some members of a $\mathbf{PC_{CS}}$ should construct functions, otherwise combining its members would be totally uninteresting. Such requirements are *pragmatic constraints*, i.e., constraints concerning the *usefulness* of conceptual systems. To formulate such constraints presupposes, however, a general definition of conceptual systems; therefore Definition 21 is necessary.

Remark 1: Primitive concepts (members of the $\mathbf{PC_{CS}}$) establish contact with objects. A concept, being a procedure, is algorithmically structured; it means that it consists of sub-concepts (sub-procedures), but never of non-procedural objects. Only a concept of an object can be used as a constituent of a composed concept (member of the $\mathbf{DC_{CS}}$). The simplest way to identify an object is using its trivialisation. But a concept C can be not only *used* as a constituent of another concept, it can be also *mentioned* as an input object of another composed concept, using a concept C' of the concept C. Thus trivialisation is indispensable in the TIL theory. Actually, trivialisation together with the ramified theory of types provide a tool for an essential extension of any classical theory: any entity of any type of any order can be mentioned within the TIL theory without generating an inconsistency. This feature is in particular useful when analysing (propositional / notional) attitudes (see [Duží 2003b], [Duží, Jespersen, Müller 2004]). For instance, when calculating 2 + 5, we do not calculate the number 7; we are related to the *concept* of the number 7 (we are attempting to find out the number the concept 2 + 5 identifies). Thus the analysis of a sentence

Charles calculates 2 + 5

obtains as follows (Calculate / $(o\ \iota\ *_1)_{\tau\omega}$):

$$\lambda w \lambda t\ [^0\mathrm{Calculate}_{wt}\ ^0\mathrm{Charles}\ ^0[^0{+}\ ^02\ ^05]].$$

Due to the possibility of constructing concepts of concepts, and thus to *mention* concepts, we are also able to analyse problems with the semantics of propositional attitudes in a way that is inaccessible for other theories. To schematically illustrate this claim, consider a conceptual system whose $\mathbf{PC_{CS}}$ contains (perhaps among other items) the concepts

$$^0\mathrm{not},\ ^0\mathrm{everybody},\ ^0\mathrm{believe},\ ^0\mathrm{the\_Earth},\ ^0\mathrm{rotate};$$

types, respectively: (oo), $(o(o\iota))$, $(o\ \iota\ *_1)_{\tau\omega}$,, $\iota_{\tau\omega}$, $(o\iota)_{\tau\omega}$, abbreviations, respectively:

$$^0\neg,\ ^0\forall,\ ^0\mathrm{B},\ ^0\mathrm{E},\ ^0\mathrm{R}\ ;$$

then we will find in the respective $\mathbf{DC_{CS}}$ the concept

$$\lambda w \lambda t\ [^0\neg\ [^0\forall\ \lambda x\ [^0\mathrm{B}_{wt}\ x\ ^0[\lambda w \lambda t\ [^0\mathrm{R}_{wt}\ ^0\mathrm{E}_{wt}]]]]]$$

which can be the result of analysing the sentence

Not everybody believes that the Earth rotates.

This analysis corresponds to *explicitly* believing, the case of a believer being related to the *meaning* of the embedded clause. If, of course, among the members of the $\mathbf{PC_{CS}}$ were another attitude-concept, viz.,

$$^0\mathrm{believe'}$$

whose type would be $(o\ \iota\ o_{\tau\omega})_{\tau\omega}$ — *implicit* believing, we would find in the $\mathbf{DC_{CS}}$ the concept

$$\lambda w \lambda t\ [^0\neg\ [^0\forall\ \lambda x\ [^0\mathrm{B'}_{wt}\ x\ \lambda w \lambda t\ [^0\mathrm{R}_{wt}\ ^0\mathrm{E}_{wt}]]]]$$

underlying the other reading of the preceding sentence (see [Duží, Materna 2001]). –

Remark 2: Atomic types used in particular **CS**s play the role of what I would call ***preconcepts***. Thus in any system that uses the types ο, ι, τ, ω we have the following *preconcepts* at our disposal: *truth-values, individuals, time moments, real numbers, possible worlds*. These seem not to satisfy our definitions concerning *concepts* but the concepts proper could not be described without them. –

In the next four paragraphs we will consider conceptual systems to be isolated from languages. This purely theoretical view may seem too abstract and useless but to seem thus is the fate of all abstractions. If we avoided pure theories then, e.g., mathematics would be reduced to mere reckoning. Making premature syntheses is an illness well known from philosophy. Thus I would like to say to the Reader: *Be patient!*

2.3 Mathematical conceptual systems

We have seen that there is a systematic (and therefore not to be neglected) distinction between *extensions* and *intensions*. We have shown that intensions are denoted by empirical expressions, viz. such expressions which have to speak about objects of the real (actual) world. The most typical examples of the other kind of expressions are expressions of mathematics and logic. Using mathematical and logical expressions we do not intend to speak about real objects. No empirical identification/verification is needed when the denotation of a mathematical/logical expression is sought.

Remark: It is not just mathematical/logical expressions that denote extensions. The English word colour is not a mathematical or logical term but all the same denotes an extension, viz. a *class* of particular colours (which are, of course, intensions; type $(o(o\iota)_{\tau\omega})$). –

Mathematical conceptual systems have to contain only such primitive concepts which construct extensions. Taking mathematical discourse to be independent of everyday discourse, i.e., as concerning just mathematical objects, we need not take care of maintaining the base $\{o, \iota, \tau, \omega\}$: in 1.4.1.2 we have seen that, e.g., the semantics of such an important system as the arithmetic of natural numbers can work just with two types, o and ν. Consider now the **CS** with the following **PC** (the subscript $_{\mathbf{CS}}$ will be omitted):

$${}^0\mathrm{Z(ero)},\ {}^0\mathrm{Suc(cessor)}$$

Within this **PC** all natural numbers, but no claims, are conceptually given. Only ν is used as an atomic type.

Robinson arithmetic and Peano arithmetic (**RA**, **PA**) need o as the second atomic type. Their primitive concepts are

$${}^0\mathrm{Z},\ {}^0\mathrm{Suc},\ {}^0\neg,\ {}^0=,\ {}^0\leq,\ {}^0+,\ {}^0\times,\ {}^0\supset,\ {}^0\forall,\ {}^0\exists$$

types (viz., of the objects identified by the concepts), respectively, ν, (νν), (oo), (oνν), (oνν), (ννν), (ννν), (ooo), (o(oν)), (o(oν)).

(The respective *preconcepts* are *truth-value, natural number.*)

In **RA** and **PA** claims are conceptually given; these systems are neutral with respect to true and false claims. The **RA** as well as the **PA** contains such concepts (in the **DC**) as

$$[{}^0= [{}^0+\ {}^01\ {}^01]\ {}^05],$$

which construct **F**.

Thus not only concepts of **T** but also concepts of **F** are found in the mathematical **CS**s. The reason is, of course, that conceptual systems only enable us to ‘create’ new concepts from the basic ones: verifying/falsifying concepts that identify truth-values is another procedure.

Even (strictly) empty concepts (see Definition 9) are derivable in some **CS**s. Adding to $\mathbf{PC_{PA}}$ the concepts

$$^0\iota,\ ^0\geq,$$

where ι is the function *singulariser* of type (ν(ον)), "the only *x* such that...", that returns the only member of a singleton and is undefined on the other sets, and ≥ is of type (ονν), we find in the respective **DC**, e.g., the concept

$$[^0\iota\ \lambda x\ [^0\forall\ \lambda y\ [^0\geq x\ y]]]$$

("the greatest natural number") which is evidently strictly empty.

Note: We use the same symbol ι both for the type of individuals and the singulariser, since no confusion can arise.–

What about mathematical conceptual systems which are more formal than arithmetic of natural numbers, i.e., which may even lack the intended interpretation? (One of the simplest cases is the theory of groups.) Here we should read section 1.4.1.2. Concepts are one thing, schemes another one. *One and the same scheme (represented, say, by a formal expression) may cover more distinct concepts.*

This simple idea can be illustrated by a maximally simple example. Let us consider a formal 1st order theory with extra-logical axioms

1. $\forall x\ \exists y\ \mathrm{R}(x,y)$
2. $\forall x\ \neg\ \mathrm{R}(x,x)$
3. $\forall x \forall y\ (\mathrm{R}(x,y) \supset \neg \mathrm{R}(y,x))$
4. $\forall x \forall y \forall z\ (\mathrm{R}(x,y) \supset (\mathrm{R}(y,z) \supset \mathrm{R}(x,z)))$

Assuming that the symbols '¬' and '⊃' get the fix classical interpretation and that the system is 1st order can we say that the axioms 'implicitly define' the symbol 'R' or, better, the meaning of 'R'? As Tichý showed in his [1988] this would be a category mistake. There is nothing like *the* meaning of 'R'. We could only claim that a higher order relation (here: class of 1st order relations), say, Φ has been (explicitly!) defined in the following manner:

(Φ =)

$$\lambda r\ (\forall x \exists y\ r(x,y) \wedge \forall x\ \neg\ r(x,x\ ) \wedge \forall x \forall y\ (r(x,y) \supset \neg r(y,x)) \wedge \forall x \forall y \forall z\ (r(x,y) \supset (r(y,z) \supset r(x,z)))$$

To diminish the ambiguity connected with the absence of types in this example let us assume that the variables *r, x, y* range, respectively, over (ονν), ν, ν. Then Φ / (ο(ονν)), i.e., our theory defines the class of those binary relations between natural numbers which obey the above axioms. Obviously, the member of this class is <. The concept so defined is the closure underlying the λ-expression above (or its NF after Horák [2001]). Any **CS** whose **DC** contains this closure has to have among the members of its **PC** following concepts:

$$^0\forall,\ ^0\exists,\ ^0\wedge,\ ^0\neg,\ ^0\supset,$$

but we must also not forget that the *preconcept* **natural number** is presupposed: otherwise we would have to admit also some non-standard models, or, if the preconcept were, e.g., INTEGERS, to state that > also satisfies the scheme.

From our characterisation of preconcepts it follows that

changing preconcepts = *changing atomic types.*

Thus — returning to our axioms — we can generalise as follows: Let α be any atomic type (not necessarily ν, ι, τ, ο, ω). Then the scheme given by the *uninterpreted* axioms 1. —4. can be interpreted so that α is an infinite set and the concept expressed by 'R' under the given interpretation is a construction (or its NF) of an ordering relation over α (so we have R/(οαα)) holding in accordance with Axiom 1.

The standard model of **PA** can be conceived of (from the present viewpoint) as given by a conceptual system among whose preconcepts the concept ^{0}natural_number is. Non-standard models are conceptually founded in such **CS**s that work with *other preconcepts.*

2.4 Empirical conceptual systems

Empirical conceptual systems are easily definable: their **PCs** contain at least one concept of a (non-trivial) intension. Thus at least one concept of the form ^{0}X is a member of the respective **PC** where X / $\alpha_{\tau\omega}$.

Yet our Definition 21 says that the members of **DCs** are concepts whose simple subconstructions are *variables* or primitive concepts; thus it seems as if we could accept also such **CS**s as empirical systems where all the primitive concepts would construct extensions, and the members of **DC** would use variables *w, t*, so that empirical concepts would be 'created' within the respective **DC**. Theoretically, it is possible, yet to think that a really empirical concept could be derived in this way is an illusion:

We can try to show a formal example. Let K_1, K_2, K_3 be three cubes, K_i / ι. Consider a **CS** whose **PC** contains the concepts

$$^0K_1,\ ^0K_2,\ ^0K_3.$$

We could place in the respective **DC** concepts

$$\lambda w\lambda t\ ^0K_1,\ \lambda w\lambda t\ ^0K_2,\ \lambda w\lambda t\ ^0K_3.$$

This is, of course, not the same as getting a concept CUBE. Well-known psychological experiments with concept acquisition show, however, that if *n* in K_n is great enough the child begins to possess (understand) the concept ^{0}cube. Can we judge on the basis of these results that intensions can be given by **CS**s whose primitive concepts identify only extensions? So can, e.g., properties be given by classes? Not at all.

In any case, *there is no simple, direct link which would connect **CS**s as defined above with the way concepts come to be possessed by somebody.* Our definition concerns abstract objects, functions and constructions; it cannot be immediately used as an explanation of the psychological process of acquiring concepts, similarly as mathematical concepts cannot explain the process of learning mathematics.

Consider our last example and the members $\lambda w \lambda t\ ^0K_1$, $\lambda w \lambda t\ ^0K_2$, $\lambda w \lambda t\ ^0K_3$. When derived from the **PCs** (type ι) they should identify the *individual roles* which the particular cubes K_1, K_2, K_3 play. But there is no one determinate role played by K_1, nor by K_2, nor K_3. As particular concrete objects they may play in(de)finitely many roles. If they are given (in the **PC**) as individuals, then — according to our anti-essentialist conception of individuals — no empirical properties that these object possess can be taken registered. Maybe that, e.g., the colour of the cube is important for the selection of the 'right' role, maybe that its weight would do this selection, etc. etc.

The point is that the way in which the **DCs** have been defined should not admit ambiguous concepts as members, and *it does not*: the concepts $\lambda w \lambda t\ ^0K_1$, $\lambda w \lambda t\ ^0K_2$, $\lambda w \lambda t\ ^0K_3$ are definite concepts, they identify definite intensions but the latter are *trivial intensions*! Their value is for each of those cubes the same in all worlds at all times, viz. the respective individual, the respective cube. Such 'degenerated concepts' are certainly no empirical concepts.

Another illustration of this principle: if a concept of a particular class of individuals, say, C is a member of the **PC**, then the only property derivable in the **DC**, i.e., the property identified by $\lambda w \lambda t\ ^0C$, is the trivial property, whose value is C in all worlds at all time points.

In general:

Principle

If A/α, where α is the type of an extension, then the concept $\lambda w \lambda t\ ^0A$ is not an empirical concept. Empirical CSs must contain some primitive empirical concepts.

So we will consider only those **CS**s whose $\mathbf{PC_{CS}}$ contain some *primitive empirical concepts*, i.e., constructions of the form ^{0}C where C is an intension. We have seen that *in any empirical **CS** some empirical concepts must be primitive.* These are a kind of criterion which can be used without any other concept. If, e.g., ^{0}blue is a member of some **PC** then it means that there is some abstract procedure that identifies blue objects immediately, without exploiting any concepts used, e.g., by physics.

Empirical **CS**s are theoretical constructs. Various work in psychological research concerning concept acquisition show — as we already suggested—that people acquire 'their' (empirical) concepts in virtue of some experience with real, concrete objects. Interesting philosophical generalisations based thereupon can be found in the vast literature on this topic, see [Bartsch 1998]. Our **CS**-abstraction may be important for logically handling concepts independently of the way they have been acquired. Thus the gap between particular objects and empirical concepts inspires empirical scientists to find some empirical (cognitive) processes in terms of which the transition from particular things to empirical concepts could be explained (understood). Such a transition, however, is nothing that a logician should and could study. From the viewpoint of a *logical* theory of concepts empirical concepts are not derivable from such a collection of concepts which contains only non-empirical concepts. In other words, there is no *logical* transition from extensions to intensions: empirical concepts as concepts are independent of the way they have been possessed. How we acquire our concepts being acquainted with particular things — this question concerns a non-conceptual phase of our cognitive processes. Even so, some points of a *logical* analysis of concepts may be of interest for a cognitive scientist, or so I hope.

Remark: Consider, e.g., the process that leads a child to possessing the concept YELLOW. The child learns, roughly, to accept objects that are distinct but share the yellow colour, and to refuse objects which are not yellow. Distinguishing particular *properties* of the objects is a necessary condition of success. Does it mean that the child already possesses the concept PROPERTY (OF INDIVIDUALS)? Every teacher will say: No, the concept PROPERTY is a 'higher level concept'. Many interesting questions can be solved in this connection, see [Bartsch 1998], in particular Ch.1; when defining concepts *in abstracto* like we do here we only try to fix a logically definite notion, roughly, to take concepts to be abstract identification procedures that can be interesting from the viewpoint of deducing conclusions from premisses. The cognitive scientist, however, is free to exploit such an explication: at worst it is innocuous, at best it can make some of his/her claims more precise.

2.5. Properties and relations of conceptual systems

We now recapitulate definitions 38 – 43 from [Materna 1998]. The attributes of **CS**s defined are important but not very complicated, so we will not number the definitions. The notions introduced will be marked by italics.

1. We have said that concepts identify objects. Since concepts are essentially closed constructions our explication of this identification has been simple: 'to identify' means 'to construct'. We can speak also of particular **CS**s by saying that they identify objects. Thus a natural definition says that a **CS** *identifies* an object A iff it has a member that identifies (i.e., constructs) A.

2. We will call the set of objects identified by **PC** $\cup$ **DC** *the area of* the respective **CS**. *This notion of area is the most important for our later definitions.*

3. Some **CS**s are comparable in an obvious sense. The next notion applies to pairs of comparable **CS**s. A **CS**$_i$ is said to be (*strongly*) *weaker than* a **CS**$_j$ iff the area of **CS**$_i$ is a (proper) subset of the area of **CS**$_j$. (So the **CS** underlying Robinson's arithmetic is strongly weaker than the **CS** underlying Peano's arithmetic.)

4. Obviously, if **CS**$_i$ is weaker than **CS**$_j$ and vice versa then we say that **CS**$_i$ *is equivalent to* **CS**$_j$. The area of the former is then identical with the area of the latter.

5. **CS**$_i$ is a *(proper) part of* **CS**$_j$ iff **PC**$_{\mathbf{CS}i}$ is a (proper) subset of **PC**$_{\mathbf{CS}j}$.

It might at first sight seem that the relation sub 5 is identical with that sub 3 or, at least, that the former is a subrelation of the latter: having less primitive concepts at our disposal we can conceptually cover less objects. This claim is false. An extremely simple counterexample: compare two conceptual systems underlying (classical) propositional logic. **CS**$_1$ contains in its **PC**-part the concepts $^0\neg$ and $^0\vee$, the **PC**-part of **CS**$_2$ is $\{^0\neg, {}^0\vee, {}^0\wedge\}$. Artificial as this example is it shows that the area of both systems is the same, namely, the set of all truth-functions.

So we have to find the constraint that makes the claim false. A first try: observe that the **DC** of **CS**$_2$ identifies the same object as one member of its **PC**, viz. the conjunction. It seems that this point distinguishes **CS**$_2$ from **CS**$_1$, but we can immediately object that a member of **DC**$_{\mathbf{CS}1}$, viz., $\lambda p\ [^0\neg{}^0\neg{}^0\neg p]$, also identifies the same object as one member of **PC**$_{\mathbf{CS}1}$, viz. $^0\neg$. Now the distinction can be found: In the case of **CS**$_1$ the concept $\lambda p\ [^0\neg{}^0\neg{}^0\neg p]$ is *dependent on* $^0\neg$ in the following sense: $^0\neg$ is one of its *subconcepts*. In the case of **CS**$_2$ the respective member of **DC**, viz. $\lambda pq\ [^0\neg\ [^0\vee\ [^0\neg p]\ [^0\neg q]]]$ is *independent of* $^0\wedge$. We can define dependence of concepts as follows:

A concept C_i is *dependent on* a concept C_j iff a subconcept of C_j is a subconcept of C_i.

Further: A **CS** is *independent* iff no member C of **PC**$_{\mathbf{CS}}$ identifies the same object as a member of **DC**$_{\mathbf{CS}}$ independent of C.

Thus the following claim is provable [Materna 1998, 111, Claim 17]:

If ***CS***$_i$ *is a proper part of* ***CS***$_j$ *and* ***CS***$_j$ *is independent then* ***CS***$_i$ *is strongly weaker than* ***CS***$_j$.

(Proof in [Materna 1998, 111].)

From now on we will take into account only independent ***CS****s.*

Preparing Section 3 where the abstract notion of conceptual systems will be connected with (natural) language we can introduce some classes of **CS**s whose importance will be obvious from the pragmatic/semantic viewpoint.

A **CS** that could underlie a *theory* has to contain some concepts that make it possible to construct some claims. Thus concepts of truth functions, quantifiers and some mathematical concepts are necessarily members of such **CS**s. Further, as we already stated, any empirical theory is based on some primitive empirical concepts. Such **CS**s could be called *normal.* An important class of **CS**s can be called simply (*logico-*)*mathematical **CSs***. These contain no empirical concepts, of course.

Remark: An obvious connotation is connected with our way of defining **CS**s. The members of **PC**s seem to correspond to *primitive terms* of an axiomatic system, **DC**s seem to correspond to sets of expressions *definable* in terms of the primitives (of the respective axiomatic system). Although we do not deny some formal resemblance we have to stress the fundamental difference. The primitive as well as the defined expressions are *just expressions*. Their choice and character is fully relativised to a particular axiomatic system. The members of **CS**s are *not expressions*, they are *concepts* (as essentially abstract procedures) and their choice and character are fully independent of any particular axiomatic system. We could perhaps say that axiomatic systems realise a choice of a **CS**; moreover, the purpose of axiomatic systems differs from the purpose (excuse this pragmatic term) of **CS**s: *Axiomatic systems have to draw a borderline between true and false claims (in a particular area) whereas **CS**s only offer tools for identifying objects* in *some area.* –

2.6 Complex concepts as 'ontological definitions'

Before we begin to speak of 'ontological definitions' we must elucidate the general role played by concepts in our sense. We have defined concepts as abstract entities independent of any particular language; this does not mean that we are not interested in their usefulness in the sense that we *need to possess concepts*. Possessing concepts can be analysed either from the psychological or from the linguistic viewpoint. Here the connection between concepts and expressions will be primarily studied; after all, we accept Church's proposal to identify *meanings* with (possessing) concepts. Thus the role played by concepts is closely connected with the role played by *language.* The situation could be perhaps modelled as follows: concepts are abstract procedures that are *potential tools* whose realisation consists in attaching them to linguistic entities. (This attaching is not a mechanical procedure which would consist—if you like parodies—in having concepts on one side and language expressions on the other side and then connecting both sides.) What is important in this vague model is the question: *Tools for what?* So we returned to our original question: *What role do concepts play*?

Empirical concepts classify objects into various kinds (the giving of names in *Genesis* should be understood in this sense, see *Concluding Essay*). The respective procedures never identify concrete particular objects — we have seen that our definitions make it impossible to conceptually identify particular things. (See also [Bolzano 1837, §74, 333].) They identify intensions, i.e., some *criteria* that will be probably useful. (Diachronically: the obvious fact of the development and change of these criteria corresponds to the development and change of *languages.* See 3.2.) Non-empirical concepts identify what are frequently called *formal means*, i.e., ways of combining empirical concepts to get more complex concepts.

This is not the whole story, as we will see later, but it is a very essential point that enables us to answer our question.

Now we can better understand the following question: *What do we do when we **define something**?*

The important subquestion is: *What sorts of things do we define?* There are three options of` answering this subquestion:

a) We define expressions.

b) We define concepts.

c) We define objects.

I doubt option a). An expression is defined if at least its morphological, syntactic and semantic features are articulated. Such a (linguistic) definition is possible (and common in linguistics) but then the given expression is a kind of *object* to be studied; if we define, e.g., prime numbers then we do not define the expression *prime number* in the sense above. This expression occurs in such a definition *qua* expression rather than *qua* object.

A similar doubt can be raised about option b). In defining particular ***concepts*** or the category ***concept*** we define an object; the concept or the particular concepts in question are here *objects to be defined*.

Hence we would accept option c). Yet although our intuition in this respect may be strong, the formulation of c) is by far not as clear as it should be, for while expression and concept are one-place predicates (something is an expression, something is a concept) we cannot say the same about object. Even expressions or concepts can be objects, viz. '*with respect to*'. This at least is one possibility of interpreting object. To accept option c) under this interpretation means to claim that everything can be an object *of our interest* and that what is defined is just such an object. In other words, if expressions or concepts are *used* then we do not view them as objects of our interest; they are means of arriving at such objects. Expressions or concepts become objects of our interest only if they are *mentioned*.

Thus we can always say that what is defined are objects.

We are almost always told that definitions are *expressions* of a certain kind. In the case of classical, equational, definitions the so-called *definiens* is a complex expression whose meaning is given to the *definiendum*, i.e., to a simple expression which does not possess any meaning before and gets its meaning as a present from the definiens. Or so can be definitions interpreted in the spirit of *Principia Mathematica*:

> A definition is a declaration that a certain newly introduced symbol or combination of symbols is to mean the same as certain combination of symbols of which the meaning is already known.
>
> [Whitehead, Russell 1964, p.11]

Modern definitions are thus abbreviations: they are read 'from right to left' (if they were read 'from left to right'—the right side should explain the 'essence' of what the left side speaks about—then Popper's strong criticism of definitions—see [Popper 1986] — would be better understandable). But not surprisingly you cannot offer any *purely syntactical* definition of definitions. The way they behave on the linguistic level is given primarily by their semantics. Now I believe that this can be best explained just in terms of conceptual systems:

In any **CS** we have at our disposal some basic, simple, within the **CS** primitive concepts which are based on some preconcepts and identify some objects. (Each of them always identifies just one object since no simple concept can be strictly empty, which is a trivial consequence of Definitions 9 and 11.) The members of the respective **DC** are no longer simple; we can call them *complex concepts*. These also identify objects (this time they can be — even strictly — empty); they do so *via* combining the primitive concepts.

Definition 22 (*ontological definition)*
Any member of a $\mathbf{DC}_{\mathbf{CS}}$ *defines an object in the respective **CS*** unless it is strictly empty. –

Remark: Thus *identifying* is what any concept does independently of **CS**s, whereas *defining* is relativised to **CS**s. (On the other hand, for every complex not strictly empty concept C there exists at least one **CS** such that C is a member of it and defines an object in it). Compare [Rey 1998]:

> [i]f definitions are not to go on forever, there must be *primitive* concepts that are not defined but are grasped in some other way. –

Ontological definition is independent of language. For any type α and number n it is a function of the type $(\alpha\ *_n)$, which associates any closed construction of order n with the α-object constructed by it. Whereas a recursive definition in an axiomatic system is an expression, often considered to be a scheme with various possible interpretations, its conceptual counterpart corresponds to a definite interpretation. As an example consider the axiomatic definition

$$x + 0 = x$$

$$x + y' = (x + y)'$$

and compare with ontological definition in a **CS** whose **PC** is

$$\{^0Z, ^0S, ^0\iota, ^0=, ^0\wedge\}$$

(i.e., zero / ν, successor / (νν), the function singulariser / ((ννν) (ο(ννν))), identity / (ονν), conjunction / (οοο)) and whose **DC** therefore contains the concept ($f \rightarrow$ (ννν), $x,y \rightarrow$ ν)

$$[^0\iota\, \lambda f\ [^0\forall\ \lambda xy\ [^0\wedge\ [^0{=}\ [f\,x\ ^0Z]\ x]\ [^0{=}\ [f\,x\ [^0S y]]\ [^0S\ [f\,x\,y]]]]]].$$

This *concept* ontologically defines addition (assuming the *preconcept* ^{0}natural_number). Once more: we do not say that the artificial expression above defines addition: the abstract procedure/concept that is fixed, encoded by this expression does. Therefore one distinction between this ontological definition and the preceding recursive formula is that the latter is a scheme that has to be interpreted while the former — being a procedure rather than an expression — cannot be interpreted: concepts are what can be attached to an expression, i.e., what makes the expression be interpreted.

Remark: Imagine, for a moment, that our intuitions, being fallible, deceive us as to the character of natural numbers. Let us admit — for the sake of a thought experiment — that the standard interpretation of Peano arithmetic is wrong in the sense that there is some limit beyond which the natural numbers no longer obey Peano's axioms. Further, let us admit that in consequence thereof the addition operation behaves beyond this limit in way other than it should according to the standard interpretation of the recursive definition above. What would we say about our conceptual (ontological) definition

$$[^0\iota\, \lambda f\ [^0\forall\ \lambda xy\ [^0\wedge\ [^0{=}\ [f\,x\ ^0Z]\ x]\ [^0{=}\ [f\,x\ [^0S\ y]]\ [^0S\ [f\,x\,y]]]]]]?$$

Apparently we would have to choose another concept (to cope with these difficulties) but what should be stressed just now and remembered is that the concept above would not be influenced: it surely defines some function (unless it is strictly empty); what would change would be the way we have ('before') understood the *word* 'adding'—now a new concept would have to be attached to it. This example, admittedly very academic and improbable, signalises an important problem (to be handled later). –

Intermezzo: PARMENIDES PRINCIPLE

Concepts are abstract objects. (This point is shared by all non-mentalistic theories of concept, see, e.g., [Peacocke 1992, 99].) Only our abstraction makes us 'see' them. (Nominalistic version: Only our abstraction creates them.) Concepts that are not connected (as meanings) with some expression cannot be known. Expressions not connected with concepts cannot be understood (or, better to say: such quasi-expressions are actually only some 'dead forms' not deserving to be called an expression).

Remark: Some structuralist conceptions (see, e.g., [Peregrin 2001]) will reject our viewpoint (viz. that particular expressions are connected with abstract concepts) on the grounds that it is the outdated 'myth of the museum'. If however concepts are considered to be abstract *procedures* then such a label can hardly be applied. A holistic (Quine, Davidson) or nearly holistic (Dummett) standpoint is, of course, incompatible with our 'naïve' conception.–

Now it could seem that the task of logically analysing expressions of natural language would become a very easy task: it would suffice to attach concepts to particular subexpressions of an expression and get so the construction that would be *its (logical) analysis*. Actually the task of logically analysing an expression is very difficult. In what follows we will try to show that already the notion THE ANALYSIS OF is questionable and that the solution of the problems connected therewith is much more complex than we perhaps expected. (See [Duží, Materna 2003].)

First let us quote from [Frege 1884]:

> Ueberhaupt ist es unmöglich, von einem Gegenstande zu sprechen, ohne ihn irgendwie zu bezeichnen oder zu benennen.

This principle (which has been called *Parmenides Principle* by Tichý in an unpublished monograph) can be reformulated (and slightly modified) as follows:

PP: *An expression E talks about an object X iff E or some of its subexpressions denote X.*

There are two warnings involved in PP.

The first warning: *If you (via the respective expression) want to talk about something then you have to denote it.*

This side of PP is ignored as soon as our analysis *adds* something, like when we believe (due to our analysis) that the sentence *The highest mountain is in Asia* talks about Mount Everest.

The second warning (not explicitly contained in Frege's formulation quoted above) can be formulated as follows: *If an expression does not talk about an object then it does not denote it.* In other words: If an expression (or some of its subexpressions) denotes X, then it talks about X.

This warning is ignored if our analysis *omits* some object that is, actually, denoted by a given (sub)expression. Example: if our analysis suggests that the above exemplifying sentence talks about the highest mountain but does not talk about mountain, then it means that we ignore the subexpression *mountain* that surely denotes mountain.

The importance of this example will become clear later. Now we can try to define what a (logical) analysis of (an expression) is.

Definition 23 (*a (logical) analysis of)*
A construction C is *an analysis of* an expression E iff a) C constructs the object denoted by E and b) the possibly occurring closed subconstructions of C construct the objects denoted by those subexpressions of E to which these subconstructions have been attached. –

Remark: This definition guarantees that the analysis does not *add* a reference to an object that is not denoted by a subexpression of E. –

Definition 23 does not define *the* analysis of an expression; it defines – for every expression – the *class* of possible analyses of an expression. To illustrate, let us adduce possible analyses of our sentence

The highest mountain is in Asia.

We have H/ $(\iota\ (o\iota))_{\tau\omega}$, M(ountain)/ $(o\iota)_{\tau\omega}$, A (being in Asia)/ $(o\iota)_{\tau\omega}$. Let HMA be the proposition (so HMA/ $o_{\tau\omega}$) denoted by our sentence. Further let HM be the individual role the_highest_mountain. Then we can offer the following analyses:

1) ^{0}HMA

2) $\lambda w \lambda t\ [^{0}A_{wt}\ ^{0}HM_{wt}]$

3) $\lambda w \lambda t\ [^{0}A_{wt}\ [^{0}H_{wt}\ ^{0}M_{wt}]]$

4) $\lambda w \lambda t\ [^{0}A_{wt}\ \lambda w \lambda t\ [^{0}H_{wt}\ ^{0}M_{wt}]_{wt}]$

We certainly want to make at least an attempt at defining *the analysis of*. Intuitively, such a unique analysis should be *the best* analysis. So what criteria for choosing the best analysis are available? Quoting [Duží, Materna 2003]:

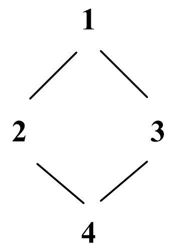
> There are two mutually dependent criteria leading to the 'ideal' choice.
>
> Criterion 1: *An analysis A is worse than an analysis B iff using A blocks some correct inferences made possible by B.*
>
> Criterion 2: *An analysis A of an expression E is worse than (>) an analysis B iff some semantically self-contained subexpression of E has not been analysed in A and—ceteris paribus—has been analysed in B.*

It would be hardly feasible to systematically evaluate particular analyses according to Criterion 1, although we will surely agree that it is a good criterion: any analysis is useful primarily according to which (correct) logical inferences it makes possible. Fortunately, Criterion 2 enables us to proceed systematically. Furthermore, it is highly intuitive to say that evaluating according to Criterion 2 we at the same time evaluate according to Criterion 1 because a finer analysis makes it in general possible to draw more conclusions.

Let us apply Criterion 2 to our mountainous example. It would seem that what makes an analysis better according to Criterion 2 is the number of subconstructions attached to particular subexpressions. Thus if *worse than* is denoted by > then our first evaluation would offer

1 > **2** > **3** > **4**.

Yet the number of subconstructions is not a sufficient condition for satisfying Criterion 2. This criterion makes it possible that two analyses are *incomparable* in the sense that an analysis A is not worse than an analysis B but the analysis B is not worse than analysis A. In our example analysis 2 ignores ^{0}H and ^{0}M but analysis 3 ignores (the concept) ^{0}HM. Thus we cannot apply Criterion 2, for *ceteris paribus* does not hold. So assuming that > has been redefined so as to become antisymmetric (≥) we have a *lattice*



Now we already can define *the analysis of* (quoting again [Duží, Materna 2003]):

Definition 24 (*the analysis of*)
A is the analysis of *E iff there is a one-to-one function* f *such that* f *associates every semantically self-contained subexpression S of E with a closed subconstruction* C *of A and* C *is an analysis of S.* –

Obviously we would like to prove that for every expression there exists such a one-to-one function. For ideal cases such a proof can be found in [Duží, Materna 2003]. Here we have to stress that some complications have to be taken into account, one of which is connected with conceptual systems. I will formulate some such complications as an *advocatus diaboli*:

a) *Which expressions are semantically self-contained?*

b) *At which stage of analysis should a disambiguating procedure be used?*

c) *In general, when analysing simple expressions we cannot attach simple concepts to them* (see 2.1).

Now we will try to cope with these complications.

Let us begin with a):

Even in our simple example there are other options of choosing semantically self-contained expressions. What about *being in*, or even *being, in*? We would get various distinct analyses, distinct also from our **1** through **4**. Some choice could be made by professional linguists but I suspect that, first, their criteria are (at least slightly) distinct from those accepted by logicians and, second, they would probably also end up with more options.

We will see that the a) problem can be taken to be a special case of the c) problem.

As for b), this problem concerns the fact that homonymy (see 1.4.3.4) is a frequent phenomenon. Theoretically, there are two kinds of solution: either the logician gets as the input a disambiguated expression – i.e., the linguists have performed the disambiguation (see, e.g., [Stechow, Wunderlich 1991, 102]) – or (in the case of syntactic ambiguity) an analysis (in the sense of Definition 23) is primarily performed and the result recommends the linguists the syntactical variants (‘readings’). From our viewpoint the second way is preferable because syntax cannot be determined in isolation from semantics. (See a good example in [Stechow, Wunderlich 1991, 221], where semantic considerations determine the syntactic analysis in a decisive way.) Also, Montague’s approach has shown the impossibility of doing syntax independently of semantics.

The key point is c). Compare the two sentences:

Some bachelors are homosexuals.

Some unmarried men are homosexuals.

Type-theoretically we have:

S(ome)/ $((o\ (o\iota))(o\iota))$, B(achelor), H(omosexual), M(an)/ $(o\iota)_{\tau\omega}$, U(nmarried)/ $((o\iota)(o\iota)_{\tau\omega})_{\tau\omega}$

Analyses:

$$\lambda w\lambda t\ [[{}^0S\ {}^0B_{wt}]\ [{}^0H_{wt}]]$$

$$\lambda w\lambda t\ [[{}^0S\ [{}^0U_{wt}\ {}^0M]]\ [{}^0H_{wt}]].$$

Assuming (theoretically) that the conceptual system at our disposal contains in its **PC** the concepts ${}^0S$, ${}^0B$, ${}^0H$ (but not ${}^0U$, ${}^0M$) we have to accept the first analysis. If the **CS** contains ${}^0U$, ${}^0M$ *instead of* ${}^0B$ then the second analysis (but not the first) has to be accepted. Besides, this second **CS** makes it possible to accept the analysis

$$\lambda w\lambda t\ [[{}^0S\ [\lambda w\lambda t\ [{}^0U_{wt}{}^0M]]_{wt}\ [{}^0H_{wt}]]].$$

Let the three adduced analyses be numbered **1, 2, 3**, respectively. One thing is clear: **2** ≥ **3**, since **2** shows that our second sentence talks about some, unmarried, man, and homosexual, while **3** talks about the same objects and, *moreover*, about the property unmarried_man. Now what about **1**? We could say that **1** talks about bachelor (unlike **2** and **3**) but it does not talk about unmarried and man – so is it incomparable with them? However, does it speak about unmarried_man? We have to admit that it *does*: the *object* bachelor is the same object as the object unmarried_man!

Returning to 2.6 we can see that our first **CS** simply *identifies* while the second **CS** *defines* the object unmarried_man (= bachelor). So the first **CS** is — within the respective part of its area— more coarse-grained as for classifying the objects than the second. *Ceteris paribus* the second **CS** is more fine-grained and, therefore, more useful, so preferable.

Now we can ask: is **3** *the* analysis of the second sentence? And: is **1** *the* analysis of the first sentence? Ignoring a remark that will be formulated in the end of this example we can say: accepting the **CS** that has been characterised in the connection with analysing the second sentence it holds: Yes, **3** is *the* analysis (but see the remark!). We could say analogically that under the given conditions **1** is *the* analysis (of the first sentence, with the same reservation). These evaluations are valid only if we assume that the **CS** that underlies the first sentence is distinct from the **CS** that underlies the second sentence (so that the better quality of the analyses **2** and **3** is, properly speaking, given by the better quality of the second **CS**).

Remark: All the analyses above, i.e.,

$$\lambda w \lambda t\ [[{}^0S\ {}^0B_{wt}][{}^0H_{wt}]]$$

$$\lambda w \lambda t\ [[{}^0S\ [{}^0U_{wt}{}^0M]][{}^0H_{wt}]]$$

$$\lambda w \lambda t\ [[{}^0S\ [\lambda w \lambda t\ [{}^0U_{wt}{}^0M]]_{wt}[{}^0H_{wt}]]]$$

can be made still better, which we have ignored for the sake of simplicity. Their improvement will show that our sentences can provably talk also about some_bachelors, for the first sentence, and about some_unmarried_men, for the second sentence. We show it for the first sentence, where the 'improved analysis' will be

$$\lambda w \lambda t\ [[\lambda w \lambda t\ [{}^0S\ {}^0B_{wt}]]_{wt}\ [{}^0H_{wt}]].\ -$$

The moral of our INTERMEZZO is that *if we want to get always just one (the best) analysis of an expression then this goal can be achieved only when a conceptual system (or a class of conceptual systems that share a respective part of their* ***PCS****) is given. So the analysis of a given expression is relative to a conceptual system.*

3. Languages and conceptual systems

3.1 Synchronic view (a temporal slice)

The synchronic view assumes that during a period (whose particular length is empirically determined and is not relevant for our analyses) a language is a relatively stable phenomenon. Under this assumption we can take for granted that the vocabulary and grammar of the given language is (during the period in question) fixed. In the ideal case then we could expect that there is a definite **PC** underlying the given stage of the language. Unless we accept Fodor's atomistic conception of concepts (for criticism thereof see [Jackendoff 1995], referred to in 3.2.4.2, Intermezzo) we could further state that all meaningful expressions of the language could be taken to express concepts that are unambiguously determined by that set of primitive concepts (**PC**).

This simple scheme does not work.

(If it did ideal dictionaries would avoid circular definitions; for more on this argument see in [Materna 1998, p.132].)

One reason is that no natural language is homogeneous; there are many sublanguages of any language **L**. Setting aside the case of professional languages (in particular languages of science) we can see that even within one and the same national language there are many groups (or even individuals — see Kripke's Pierre) using some kind of *idiolect* whose lexicon and even sometimes syntax differs from the 'official' language. And if some speaker of English does not know the meaning of, say, *spin*, will he be said not to know English, or not to know physics? (See 3.2.4.2.b).)

Thus we can suppose that there are many distinct (albeit overlapping) conceptual systems that can be said to underlie a given natural language.

Let a particular **CS** be given and let $\mathbf{L_{CS}}$ be that fragment of **L** which is based on **CS**. We will show that $\mathbf{L_{CS}}$ can contain homonymous expressions (even with one **CS** as its background). This can be proved *via* an example. Let *prime (number)* be an expression of $\mathbf{L_{CS}}$. Let our **CS** contain the concept ^{0}D(ivisible), concepts of particular natural numbers, the concept of cardinality (of classes of natural numbers), the concept of identity (of natural numbers) and concepts of logical objects. It is well known that there are at least two concepts attached to *prime*:

$$\lambda x\ [\ {}^0\forall\lambda y\ [\ {}^0\supset [{}^0\mathrm{D}\ x\ y]\ [{}^0\vee\ [{}^0{=}\ y\ x]\ [{}^0{=}\ y\ {}^0 1]]]]$$

or

$$\lambda x\ [\ {}^0{=}\ [\ {}^0\mathrm{Card}\ \lambda y\ [{}^0\mathrm{D}\ x\ y]]\ {}^0 2].$$

(Writing ^{0}D, ^{0}1, ^{0}2 etc. we do not mean that the concepts of D, 1, 2 etc. are necessarily in the **PC**-part of the **CS**; excuse me, please, for such simplifications.)

The sentence *1 is a prime* is true in the first case and false in the second case. To emphasise that no contradiction is present we can write

$$\textit{1 is a prime}_1,$$
$$\textit{1 is not a prime}_2.$$

Imagine a **CS** that does not contain concepts that would make it possible to define cardinality. A language based on such a system could not even formulate the second sentence above.

The importance of connecting theories of language with the theory of conceptual systems is not very obvious if we restrict our attention to the synchronic view. Particular interesting insights can be expected but what seems to be much more promising is comparing distinct possible stages of a language and of a theory from the viewpoint of our theory of conceptual systems; this can be called

3.2 The Diachronic view

3.2.1 Definitional (conservative) model of development of language

Again let a **CS** be fixed and consider the language $\mathbf{L_{CS}}$. We can show that $\mathbf{L_{CS}}$ can develop even without any change of the **CS**; this form of development is not very interesting (being extremely conservative) but an investigation of its semantic background can yield some useful insights. Some consequences useful for clarifying some aspects of development of a *theory* (in particular, of a science in the Kuhnian 'normal' stage) can be discovered as well.

Our scheme can be only very artificial. The topic of our study is not concrete historical research: we are interested in a semantic analysis of model situations. So let us proceed:

We will define particular stages of $\mathbf{L_{CS}}$ inductively.

Definition 25 (*definitional development of a language)*
$\mathbf{L_{CS0}}$ (the ideal starting point) is a language with following features:
a) A 1-1 function connects its simple expressions with the members of $\mathbf{PC_{CS0}}$.
b) Its grammatical rules make it possible to encode compositions and closures over $\mathbf{PC_{CS0}}$.
$\mathbf{L_{CSn}}$ is identical with $\mathbf{L_{CSn\text{-}1}}$ with the only exception that $\mathbf{L_{CSn}}$ contains a finite set of equational stipulations of the form

$$E_i = \Phi_i\,(D_{i1},\ldots,D_{ik})$$

or a (finite) scheme of such stipulations, where E_i is a new simple expression (not occurring in any $\mathbf{L_{CSm}}$ for $n > m$) and Φ_i is a complex expression consisting of such expressions D_{ij} only that occur in $\mathbf{L_{CSn\text{-}1}}$. –

According to Definition 25 the later stages of this artificial development of $\mathbf{L_{CS}}$ differ from the earlier ones just by containing more simple expressions (and, of course, by containing expressions whose subexpressions are these new simple expressions).

The expressions $E_i = \Phi_i\ (D_{i1},\dots,D_{ik})$ are called (equational) *definitions*. We can immediately see that they are codes of ontological definitions (see 2.6): the right side (*definiens*) is an expression whose analysis is one of the members of the $\mathbf{DC_{CS}}$. The left side (*definiendum*) is the respective abbreviation of the right side. Since its meaning is given exclusively by the right side *our language begins to contain more and more simple expressions that do not express simple concepts.*

It is surely not the case that a development of a language **L** could be reduced to this kind of development that leaves the underlying **CS** unchanged. A truly interesting development is creative in the following sense: *A later stage of a creative development of **L** is based on a **CS'** such that its area (see 2.5, point 2) is greater than the area of that **CS** on which the earlier stage of **L** was based* (i.e., ***CS** is strongly weaker than **CS'***).

Before we proceed to examining some points concerning this creativity of the development of languages let us make some notational conventions:

The subset of a $\mathbf{PC_{CS}}$ which contains concepts that enable us to derive *logical* tools (let it be concepts of predicate logic of 1st order, or concepts of higher order, modal, intensional logics) will be called $\mathbf{LOG_{CS}}$. Analogically, that subset of a $\mathbf{PC_{CS}}$ that contains concepts enabling us to use some portion of *mathematics* will be called $\mathbf{MATH_{CS}}$. Finally, the subset of $\mathbf{DC_{CS}}$ that contains empirical concepts will be called $\mathbf{EMP_{CS}}$.

Definition 26 (*inessential extension of (the area of) a **CS***)

(*The area of*) ***CS'** is an inessential extension of (the area of) **CS*** ($\mathbf{IEE_{CS'CS}}$) iff $\mathbf{EMP_{CS}} \subset \mathbf{EMP_{CS'}}$ and all members of $\mathbf{EMP_{CS'}} - \mathbf{EMP_{CS}}$ are constructions using only members of $\mathbf{EMP_{CS}}$ and $\mathbf{LOG_{CS'}} \cup \mathbf{MATH_{CS'}}$. –

Remark: The intuition underlying Definition 26 can be exemplified as follows: Suppose that $\mathbf{LOG_{CS'}} \cup \mathbf{MATH_{CS'}}$ does and $\mathbf{LOG_{CS}} \cup \mathbf{MATH_{CS}}$ does not contain concepts that would make it possible to derive conjunction. Let **CS** contain the empirical concepts ^{0}Cat and ^{0}Wild or concepts that make it possible to define cats and wild objects. Then $\mathbf{EMP_{CS'}} - \mathbf{EMP_{CS}}$ will contain, e.g., the concept

$$\lambda w \lambda t\ \lambda x\ [^0\wedge\ [^0\mathrm{Wild}_{wt}\, x]\ [^0\mathrm{Cat}_{wt}\, x]].$$

True, **CS** did not enable us to identify entities that are at the same time wild and cats, but there was no necessity to do it *via* extending **CS** by some other *empirical* concepts. Only such tool-like concepts as known from logic and mathematics were needed. –

Clearly, the notion $\mathbf{IEE_{CS'CS}}$ cannot be applied to non-empirical conceptual systems.

The notion of *an essential extension of the area of an empirical **CS*** can be derived, of course, from Definition 26: To be an *essential* extension $\mathbf{PC_{CS'}}$ has to contain some *new empirical primitive* concepts. The abbreviation will be $\mathbf{EE_{CS'CS}}$.

3.2.2 Conceptual systems and problem sets

Remark: The notion of *problem* is closely connected with the analysis of questions. As for the way TIL analyses questions see, e.g., [Tichý 1978], [Duží, Materna 2002]. –

Notational changes used sometimes to simplify reading:

- Binary connectives and identity will be written using infix notation (which we are more used to in these cases).
- Quantifiers will be denoted in the usual way.
- The trivialisation symbol ^{0} will not be used in the case of connectives, identity and quantifiers.

Thus instead of

$$\lambda w \lambda t\ [^0\supset [^0\exists\ \lambda x[^0\mathrm{Planet}_{wt}\, x]\ [^0\neg\ [^0=[^0\mathrm{Numberof}\ ^0\mathrm{Planet}_{wt}]\ ^00]]]]$$

we will write

$$\lambda w \lambda t\ [\ \exists x\ [^0\mathrm{Planet}_{wt}\, x] \supset \neg[^0\mathrm{Numberof}\ ^0\mathrm{Planet}_{wt}] = {}^00]. \quad –$$

One of the possible criteria of *creativity* of an extension of a **CS** has been defined in terms of the *area* of the given **CS**. Another criterion, perhaps more important in some contexts, can be defined in terms of *problems that can be posed by a* ***CS***. Therefore we will now explicate the notion of a *problem.*

Definition 27 (*problems*) See Definition 6.

i) Concepts that do not contain λ-bound variables and are not simple concepts are *singular problems.*

ii) Concepts that contain λ-bound variables are *general problems.*

iii) *Problems* are *singular* or *general.* –

From this vantage point concepts can be viewed as problems. The only exception is any simple concept.

Remark: A very important fact implied by Definition 27 is that we can have non-identical but *equivalent* problems, in the case, indeed, when the respective constructions/concepts are distinct equivalent constructions (concepts). –

Now we will explain our motivation. First of all, from Definition 27 it follows that *no empirical concept is a singular problem.* (Indeed, unless an empirical concept is simple – in which case it has not been defined as a problem – it constructs intensions and contains therefore λ-bound variables *w, t*.)

Some examples will support our intuitions. (The ‘solutions’ are preliminary.)

NON-EMPIRICAL PROBLEMS

Problems		Solutions
singular	$[{}^0{+}\ {}^0 3\ {}^0 5]$	8
	$[{}^0{>}\ [{}^0{+}\ {}^0 3\ {}^0 5]\ [{}^0\surd\ {}^0 49]]$	T
general	$\lambda x\ [{}^0{>}\ x\ {}^0 0]$	to compute the class of positive numbers
	Fermat's Last Theorem	T
	$\lambda pq\ [[p \wedge q] \supset p]$	$\{ \langle T,T \rangle, \langle T,F \rangle, \langle F,T \rangle, \langle F,F \rangle \}$
	$\forall p \forall q\ [[p \wedge q] \supset p]$	T

Here we can see that another convention could be used: solutions of singular problems should be particulars (individuals, numbers, truth-values), solutions of general problems should be functions. In this sense general problems would be similar — if not identical — to what are usually called *mass problems* (and what is or is not algorithmically solvable). In some contexts this criterion can be useful, in particular if we consider mathematical problems only; in the case of empirical problems we need another kind of intuition. Consider, e.g., the case of Fermat's Last theorem. According to our criterion this hypothesis represents general problems; the second criterion would locate it among singular problems. True, the solution consists in the simple answer Yes/No, but to get this answer we have to *calculate* (in the broad sense of the word), i.e., to perform some (oh! how many…) intellectual steps, which should not be the case when solving singular problems. You can object that singular problems can also be connected with laborious, at least time-consuming, operations (take only a very long polynomial) but then at least no functions need to be calculated (they are at most used as simple concepts, cf. the use of the arithmetic functions in performing adding, subtracting etc.).

Another objection can be raised: According to Definition 27 the concept ${}^0{+}$ is not a problem whereas $\lambda xy\ [{}^0{+}\ xy]$ would be a general problem. But according to HDefinition 12 the latter construction points to a simple concept.

Remark: If the only occurrences of variables in a concept C are ^{0}bound, then Definition 27 says that C is a singular problem. Example: Let Cont_n be the relation that holds between a construction C of order n and a variable x of order n iff C contains some occurrence(s) of x. Then the concept

$$[{}^0\mathrm{Cont}_1\ {}^0[{}^0{>}\ x\ {}^0 0]\ {}^0 x]$$

is a singular problem. (It is the problem whether the construction $[{}^0{>}\ x\ {}^0 0]$ contains the variable x, and our intuition is that this is a very typical singular problem. Even the problem

$$[{}^0\mathrm{Cont}_1\ {}^0[\lambda x\ [{}^0{>}\ x\ {}^0 0]]\ {}^0 x]$$

is a singular problem, though there is a λ-abstraction as a subconstruction, but the variable x is here o-bound, see Definition 6.) –

Now we will adduce some very simple examples of *empirical problems*:

EMPIRICAL PROBLEMS

Empirical problems	**Verbal expression**	**Solution** (preliminary formulation)
$\lambda w \lambda t\ \lambda x\ [[{}^0\text{Mammal}_{wt}\, x] \wedge [{}^0\text{Liveinwater}_{wt}\, x]]$	Which mammals live in water?	Finding individuals who are mammals and live in water
$\lambda w \lambda t\ \exists x\ [[{}^0\text{Mammal}_{wt}\, x] \wedge [{}^0\text{Liveinwater}_{wt}\, x]]$	Do some mammals live in water?	Finding the truth-value of the proposition
$\lambda w \lambda t\ [{}^0\text{Capital}_{wt}\ {}^0\text{Poland}]$	What is the capital of Poland?	Finding the capital of Poland
$\lambda w \lambda t\ \lambda x\ [[{}^0\text{Mammal}_{wt}\, x] \wedge \neg[{}^0\text{Vertebrate}_{wt}\, x]]$	Which mammals are not vertebrates?	To find such mammals that are not vertebrates. (No result.)
$\forall w \forall t\ \forall x\ [[{}^0\text{Mammal}_{wt}\, x] \supset [{}^0\text{Vertebrate}_{wt}\, x]]$	Are all mammals vertebrates?	Yes

Comments:

a) The first example is useful for understanding what should be called the *solution* of empirical problems. Our formulation suggests that a problem is solved when the members of the class that is the value of the property constructed by the problem in the actual world + time are found. This formulation is obviously misleading: one of its consequences is that such a problem (that constructs an empirical property) could never be solved. Actually, when we say that this problem can be solved we bear in mind that any singular instance thereof can be — in principle — answered so that a *criterion of deciding* is found. So we could better characterise the situation of having solved that problem as follows: “Give me the actual state of the world and some individual, and I will be able to say whether it is a mammal that lives in water, or not.” The role of *experience* in solving empirical problems is obvious (“Give me the actual state of the world” – this is what cannot be calculated by the logical analysis). (The artificial character of the example is given by the fact that a realistic problem would be expressed rather by the interrogative sentence *Which kinds of mammal live in water?* The solution would consist in finding the particular kinds rather than particular individuals. The respective concept can be easily found in terms of a variable over properties of individuals and the set-theoretical inclusion.)

b) The simple answer to the second problem is the result of empirical investigation regulated by particular concepts that make up the problem. Which empirical procedures are used is not important in the present context: suffice it that without knowing the concepts encoded by the expression that expresses the given concept/problem it would be impossible to choose a procedure, since no problem would be recognised.

c) *The capital of Poland* is not a *name*: as an empirical description it expresses an empirical concept/problem. Compare with the proper name *Warsaw*. Accepting – as we cautiously do – that proper names express simple concepts we have the simple concept ^{0}Warsaw: no *problem* is present.

d) The last two examples are dubious. In both cases the answer is given *a priori*, no empirical investigation is necessary (in virtue of the mutual dependence of the concepts MAMMAL and VERTEBRATE). We have included both cases in the class of empirical problems in order to show that they only *seem* to be empirical: the particular subconcepts are, of course, empirical concepts, but they are not mutually independent. The first example shows a construction that constructs an *empty property*, i.e. a property whose value is an empty class of individuals in every world + time. The second example is a construction of the analytic proposition TRUE.

So do the last two concepts exemplify problems? To answer this question we need the following consideration.

As Tichý has suggested, e.g., in his [1979] a class of relations-in-extension between intensions can be defined as follows: Let *X be ^{0}X or any construction constructing X.

Definition 28 (*requisites*)
Let A, B be $\alpha_{\tau\omega}$- or $(o\alpha)_{\tau\omega}$-objects. Let Xr*x* abbreviate [X*x*] or [*x* = X], types derivable. B is said to be a *requisite* of A (denoted by [^{0}Req *B *A]) iff it holds that

$$\forall w \forall t\ [[{}^0\mathrm{E}_{wt}\ {}^*\mathrm{A}] \supset \forall x\ [{}^*\mathrm{A}_{wt}\mathrm{r}x \supset {}^*\mathrm{B}_{wt}\mathrm{r}x]].$$

(A generalisation for relational types and sequences of variables is easy.) –

Remark: E is the property 'existence'. Its type is $(o\alpha_{\tau\omega})_{\tau\omega}$, it is a property of an intension – 'being instantiated'. In predicating existence of an intension *I* in the pair ⟨W, T⟩ we say that *I* is occupied in ⟨W, T⟩ by an α-object (if I / $\alpha_{\tau\omega}$, α not being a type of a class) or by a non-empty class (if I / $(o\beta)_{\tau\omega}$). See [Tichý 1979]. The antecedent [^{0}E$_{wt}$ *A] is necessary if *A constructs a role (α not being a type of a class).

Examples:
Let USP/ $\iota_{\tau\omega}$ be the office of the President of the USA and USC/$(o\iota)_{\tau\omega}$ the property *to be a citizen of the USA*. It holds that the following construction constructs T:

$$[{}^0\mathrm{Req}\ {}^*\mathrm{USC}\ {}^*\mathrm{USP}].$$

(In our examples we will use trivialisation instead of *; the *-generalisation is clear.)

Our last example with mammals and vertebrates is a case of the requisite relation:

$$[^0\text{Req}\ ^0\text{Vertebrate}\ ^0\text{Mammal}].$$

Thus it is not an empirical problem, and at the same time it is a singular problem. Indeed, no empirical procedures are needed: we only have to know the respective portion of English.

The other example with mammals, i.e.,

$$\lambda w \lambda t\ \lambda x\ [[^0\text{Mammal}_{wt}\, x] \wedge \neg[^0\text{Vertebrate}_{wt}\, x]],$$

is a little strange. According to Definition 27 it is a general empirical problem. Yet the answer, i.e., the empty class, can be obtained as a consequence of solving the singular problem given by the construction $[^0\text{Req}\ ^0\text{Vertebrate}\ ^0\text{Mammal}]$, which is a non-empirical problem. *But no solution of a non-empirical problem can be the only source of a solution to an empirical problem.* Thus this case is one of a seemingly empirical but actually a non-empirical problem. All concepts that construct *trivial intensions* (see Definition 3) are problems of this deceptive kind.

Now our intuitions concerning the notion of the *solution* of a problem should be made more precise. First let us return to non-empirical problems. There are two kinds of *general non-empirical problems* (*GNP):*

a) *GNP that construct some function.* They will be denoted by *GNPI.*

b) *GNP that apply some function to what some GNPI has constructed.* Let them be denoted by *GNPII.*

In the table NON-EMPIRICAL PROBLEMS the first and the third example (of general problems) are GNPI, the other two examples are GNPII.

Empirical problems will be denoted by *EP*. *Singular problems* are *SP*.

SOLUTION

Kind of problem	**Solution (explication)**
SP	What is constructed
GNPI for natural numbers	Algorithm (recursive functions), nothing otherwise
GNPII for natural numbers	The result of applying a function to the function given by the given GNPI
EP	An empirical method of finding in the actual world+time the value of the intension constructed by the problem or particular instances of this value.

The preceding definitions and explications make it possible to define *problem sets* connected with conceptual systems. This notion is important as soon as we are no longer content with the EE-criterion (see above) of creativity of development of conceptual systems.

Definition 29 (*problem set*)

Let **CS** be a purely logico-mathematical system, i.e. let **CS** contain no empirical concept. Then the *problem set of* ***CS*** ($\mathbf{PS_{CS}}$) is the set of all members of $\mathbf{DC_{CS}}$ that are of the kind GNP (I or II).

Let **CS** be an empirical system (containing $\mathbf{LOG_{CS}}$ as well as $\mathbf{MATH_{CS}}$ but also empirical concepts). Then the *problem set of* ***CS*** ($\mathbf{PS_{CS}}$) is the set of all members of $\mathbf{DC_{CS}}$ that are of the kind EP and are not 'spurious' (see the following Remark). –

Remark: What kind of problems are *spurious (futile)*? Newton-Smith in [Newton-Smith 1981, 187] adduces examples like "Why will sugar never dissolve in hot water?", "Why are swans green?" etc. From our viewpoint the concepts underlying such formulations are constructions that are either improper (in the case of logico-mathematical concepts) or construct functions that are undefined for actual world+time. In the examples above the value of the respective intension for a given world+time is some event or property that is the cause of sugar's not dissolving in hot water (of swans' being green). Here the presupposition, viz. that sugar does not dissolve in hot water (that swans are green) is false at least in the actual world+time. Therefore — since nothing is such a cause — the intensions constructed by the respective concepts lack any value in the actual world+time. –

Now we are able to define another *criterion of creativity* of the development of conceptual systems; we can call it the *expressive power* of a given system.

Definition 30 (*expressive power*)

The *expressive power of a* ***CS*** ($\mathbf{ExP_{CS}}$) is the $\mathbf{PS_{CS}}$. –

The way expressive power has been defined suggests that the role of an empirical conceptual system essentially differs from the role of a purely logico-mathematical conceptual system (see Definition 29!). In the latter only constructions and what is constructed counts whereas in the former the role of constructions is *instrumental*. This distinction can be clarified as follows: The problem set of a purely logico-mathematical **CS** consists of concepts that construct *functions* (and some of their features, see GNPII), and what is required is to compute these functions (see the table SOLUTION) and evaluate the respective features (*ibidem*). No problems concerning the objects of the real world can be posed; logico-mathematical conceptual systems produce tools. In contrast with such systems an empirical **CS** *uses* the tools offered (represented by the set $\mathbf{LOG_{CS}} \cup \mathbf{MATH_{CS}}$) to *pose* the problems concerning the real world, and since the *actual* values of those intensions constructed by these problems cannot be computed, the solutions which are required are *empirical* methods (see again the table SOLUTION).

*Thus we could characterise the Kuhnian 'normal phase' of a science as such process during which the underlying **CS** is the same and the only distinction between particular stages of this process consists in that some problems being posed by the **CS** are solved. The 'revolution' makes it possible not only to solve problems but also to pose new problems: new not in the sense in which some problems that can be posed within the old system are recognised, but in the sense that they could not have been posed in it at all.*

Remark: 'Normal' empirical systems are, of course, *mixed* in that they contain not only empirical concepts but also the part $\mathbf{LOG_{CS}} \cup \mathbf{MATH_{CS}}$. Something like 'pure empirical systems' would be of no use at all. –

Purely logico-mathematical **CS**s underlie formal languages of logic and mathematics. As for *formal* languages a reservation must be articulated: in general, a formal language uses what could be called *pseudo-constants*: any such pseudo-constant is a constant *with respect to* an interpretation (remember Frege's polemics with Hilbert!), which means that *a formal language is connected with more than one conceptual system*. As a classic example we can adduce the case of formal arithmetic of natural numbers, where only one of the conceptual systems is that of natural numbers (mathematical logicians speak about 'metamathematical natural numbers'): formal arithmetic allows for *non-standard* models where concepts produce other objects than standard natural numbers. To use the theory of conceptual systems in philosophy of mathematics would be a most interesting task; but it is too large, however, to be undertaken in the present study.

3.2.3 Creative extension

In the case of *empirical **CS**s* we can state that we have two competing criteria of creativity of extending a **CS**. One of them is definable in terms of $\mathbf{EE_{CS'\,CS}}$, i.e., we could call an extension **CS'** of a **CS** creative if $\mathbf{EE_{CS'\,CS}}$ were true (remember: essential extension); the other criterion is definable in terms of $\mathbf{ExP_{CS}}$ (remember: expressive power), i.e. an extension **CS'** of **CS** would be creative if $\mathbf{ExP_{CS}}$ were a proper subclass of $\mathbf{ExP_{CS'}}$. We can ask, of course, whether these two criteria are independent. Let us try now to answer this question.

First, let us suppose that $\mathbf{EE_{CS'CS}}$ holds. (We do not consider here theoretically possible cases of overlapping.) Then in the area of **CS'** (but not in the area of **CS**) there is a class O of some objects in the abstract sense (intensions, of course) and in $\mathbf{EMP_{CS'}}$ a class C of concepts such that the members of C belong to $\mathbf{EMP_{CS'}} - \mathbf{EMP_{CS}}$ and identify the members of O. But the members of C represent some new (empirical) problems that did not occur in **CS**. Thus we have

Claim: *If $\mathbf{EE_{CS'\,CS}}$ holds then $\mathbf{ExP_{CS}}$ is a proper subclass of $\mathbf{ExP_{CS'}}$.* –

(The same argument justifies

Claim': *If* $\mathbf{IEE}_{CS' CS}$ *holds then* $\mathbf{ExP}_{CS}$ *is a proper subclass of* $\mathbf{ExP}_{CS'}$*.* –)

Now suppose that $\mathbf{ExP}_{CS}$ is a proper subclass of $\mathbf{ExP}_{CS'}$. Then there is a class C of concepts that belong to $\mathbf{EMP}_{CS'}$ — $\mathbf{EMP}_{CS}$ and obviously construct some intensions. But two points prevent us from formulating the conversion of the claim above: First, the members of C could be equivalent to some members of $\mathbf{EMP}_{CS}$, in which case the area of **CS** would not change. Second, even in the other case the newly constructed intensions could make true $\mathbf{IEE}_{CS' CS}$ (remember: 'inessential extension') rather than $\mathbf{EE}_{CS' CS}$. In this sense the criterion based on $\mathbf{ExP}_{CS}$ is *weaker* than the other criterion.

On the other hand, the **EE**-criterion cannot be applied to logico-mathematical **CS**s, whereas the **ExP**-criterion can be applied to both kinds of **CS**.

Summing up, whereas the **ExP**-criterion is a satisfactory criterion of creativity in the case of logico-mathematical **CS**s, the **EE**-criterion seems to be the strongest criterion of creativity in the case of empirical **CS**s. Since it is *empirical CSs* that are of primary interest for us, the following definition stipulates creativity in terms of **EE**.

Definition 31 (*empirically creative extension*)
An extension **CS'** of a conceptual system **CS** is *empirically creative* iff $\mathbf{EE}_{CS'CS}$. –

We can see that if the creativity of empirical **CS**s were based on **ExP** it would not be necessarily connected with the relation $\mathbf{EE}_{CS' CS}$ (see Definition 26): in the last Remark in 3.2.1 we adduced as an example of $\mathbf{IEE}_{CS' CS}$ the case when a **CS** does not but a **CS**' does contain a concept of conjunction. **CS**' — unlike **CS** — contains the concept

$$\lambda w \lambda t\ \lambda x\ [^0\wedge\ [^0\mathrm{Wild}_{wt}\, x]\ [^0\mathrm{Cat}_{wt}\, x]].$$

Using the **ExP**-criterion we would then have to state that *ceteris paribus* **CS**' is a creative extension of **CS**. Artificial as this example is it all the same proves that the **ExP**-based creativity would be compatible with *inessential extension*, viz. in the situation when the new system arises due to adding only some non-empirical, i.e. logico-mathematical concepts to the original system. Yet this would be counterintuitive: in the case of **IEE** no new empirical research is needed to enlarge our 'knowledge base' — only some logico-mathematical member of the 'library of programs' is activated. On the other hand, one can ask: Is it not the case that an element of *creativity* is present even when 'only' logico-mathematical tools are activated? This problem (a rather terminological one) can be solved *via* following definitions:

Definition 31' (*mathematically creative extension*)
An extension **CS**' is a *mathematically creative* extension of **CS** iff $\mathbf{IEE}_{CS'CS}$. –

Definition 31'' (*creative extension*)
An extension **CS'** is a *creative* extension of **CS** iff **CS'** is an empirically or a mathematically creative extension of **CS**. –

All the definitions 31 – 31'' concern, of course, empirical **CS**s. If creative extension' should be defined for logico-mathematical systems, then the notion $\mathbf{ExP_{CS}}$ would obviously be relevant.

Remark: A phantastic question can be raised. Being confronted with the problem of the 'revolutionary' stage of a science (Kuhn *et alii*) as opposed to 'normal science' we can try to characterise the former as the case of creative extension of the respective conceptual system. Let us use the paradigmatic example of Newtonian vs. Einsteinian conceptual system. We would like to confirm the hypothesis that the latter is not only a creative extension of the former in the sense of Definition 31 but also an *essential extension* of it in the sense of Definition 26. But what if the Einsteinian system can be derived on the basis of the Newtonian system *only due to enrichment of the latter by some mathematical concepts* (like the tensor calculus etc.)? Then we would have to state that the Einsteinian system is only an *inessential extension* of the Newtonian system! Let this question be only a provocation; without a much deeper analysis involving details of both **CS**s our question cannot be answered, of course. Yet even if we could prove that the case of Newton vs. Einstein is not an instance of the relation $\mathrm{IEE}_{\mathbf{CS}`\,\mathbf{CS}}$ it would not mean that there are not some other interesting cases where what is interpreted as a revolutionary change would be actually an inessential extension. –

Returning to the problem of the development of *language* we can call a language L' a *creative extension of* the language L iff the **CS'** that underlies L' is a creative extension of the **CS** that underlies L.

3.2.4 New concepts

Our (simplified) model of the creative (as opposed to the 'conservative') development of language is connected with the assumption that in the creative case the new **CS** contains some *new concepts*. (In contrast to the conservative case where the ('encoding') new linguistic system contains new *expressions* whereas the underlying 'new' **CS** does *not* contain any new *concept,* i.e., is identical with the 'old' **CS**.) This holds (for the empirical **CS**s) for the case of *essential* as well as of *inessential* extension: in the latter case the new empirical concepts arise since some logico-mathematical concepts enlarge the class of possibilities of having new members of the respective $\mathbf{DC_{CS}}$. The case of *essential* extension is much more interesting.

We will try to analyse this case; since the role of the pure logico-mathematical **CS**s differs from the role of the empirical **CS**s (see the considerations following Definition 30) we will first investigate the problem of new concepts in the case of some number theoretic **CS**s.

Remark: Once again I would like to emphasise the fact that I would confuse distinct topics if I tried to exemplify my theoretical analyses by real, historical events. History, methodology and philosophy of science are rich disciplines with specific methods; let, e.g., historians of science consider the possibilities of documenting the theoretical analyses below. –

3.2.4.1 Arithmetic

It is possible using just positive integers to solve some mathematical, i.e., instrumental problems, in particular in connection with the concepts ADDITION, (limited) SUBTRACTION, MULTIPLICATION, (limited) DIVISION. Which are primitive and which are derived depends on the respective **CS**; for example whether it does or does not contain the 'descriptive operator' $^0\iota$ (THE_ONLY…SUCH_THAT).

Remark: 'Descriptive operator' is not a good term: this name denotes a linguistic expression. Let the concept $^0\iota$ be conceived of as what underlies this expression. (Tichý has called the function identified by $^0\iota$ as the 'singularizer'. Its type is schematically $(\alpha(o\alpha))$ – its value on α-singletons is α, on the other classes it is undefined.) –

The special number 0 had to be discovered. Then (limited) SUBTRACTION (as we know it from the theory of recursive functions) has become SUBTRACTION (of natural numbers). But theoretically, there is a **CS** with the *preconcept* (see 2.2, Remark 2) POSITIVE_INTEGER(S) and the concept $^0\iota$, so that an interesting situation comes about: in the respective **DC** we get the concept

$$[^0\iota\ \lambda x\ ^0\forall\ \lambda y\ [^0{=}\ x\ [^0{-}\ y\ y]]],$$

a concept, that is *strictly empty* if the preconcept in question is POSITIVE_INTEGER(S). But the challenge of existential commitment in the above construction can be accepted, and so we get the concept ZERO. Observe that the original **CS** made it possible to formulate a problem that is – from the later viewpoint – unsolvable within **CS** and at the same time to propose such an extension of itself that would make it possible to solve this very problem. Moreover, the preconcept of the new **CS** is no longer POSITIVE_INTEGER(S): it is now NATURAL_NUMBERS. Finally observe that ZERO need not be a *primitive concept* in any such **CS**.

Negative integers made it possible to define *subtraction of integers* (better: the need to subtract greater numbers from smaller ones has led to the introduction of the concept SUBTRACTION_OF_INTEGERS). The preconcept is now INTEGER(S).

(Limited) DIVISION also proved to be insufficient for solving some important problems. So a new kind of numbers had to be discovered: the concept RATIONAL_NUMBER(S) come about. Again, it can, but need not, be a primitive concept.

Nor do rational numbers suffice: the diagonal of a quadrilateral figure is incommensurable with it sides. Let us have *real numbers* and so the concept REAL_NUMBER(S). In our base (Definition 1) it is a preconcept, but not necessarily so. If some operations conceptually definable over the **CS** with RATIONAL_NUMBER(S) as a preconcept are conceptually redefined so that what was originally an improper construction (a strictly empty concept) now constructs an object (a number)—recall the *square root* operation — then a new **CS** with another preconcept arises.

Let us try to generalize from the simple examples. Let us begin with the scheme suggested above. Summing up we can describe it globally as follows:

Scheme [*of development of (number-theoretical) CSs*]

*The original **CS** with basic types (preconcepts) $\alpha_1,\ldots, \alpha_k$ and primitive concepts $C_1,\ldots,C_m$ determines some concepts that are strictly empty. Posing some object as the value of such a concept we change from **CS** to another system, say, **CS'**, where some α_i and maybe some C_j change (and become some β_i, D_j, respectively).*

How reliable is such a simple law of development? Not completely: our freedom of making such changes is limited. The following case serves as a counterexample: let a **CS** contain such preconcepts and concepts that make it possible to define prime numbers. In such a system we can easily derive the concept THE_GREATEST_PRIME. Now we can immediately see that the respective concept is strictly empty. But the above scheme cannot be applied: we simply cannot find a number as constructed by this concept and change the original **CS** in order to make the above concept non-empty. An even simpler example: we can want to enlarge the range of the division operation, leading to the transition from the **CS**s that capture integers to those that capture rational numbers. But we cannot enlarge this range so as to make it possible to divide by zero.

On the other hand, a seemingly similar case does exemplify the scheme: A **CS** that identifies real numbers and contains the concept THE_SQUARE_ROOT defines the concept THE_SQUARE_ROOT_OF_- 1, which is strictly empty; but unlike the case of zero division the transition has been realised and a new **CS** that is able to identify *complex numbers* has been discovered.

But what then is the criterion of applicability of the above scheme? Can such a *general* criterion be formulated at all?

The reliability of the Scheme is not absolute in the sense that it cannot be always applied. Yet on the other hand it seems that *whenever we can speak about new concepts (with respect to a **CS**) the Scheme has been applied.*

(This last claim obviously assumes that the respective **CS**s are *comparable*: in this sense the concepts, say, of a geometrical **CS** are not new with respect to an arithmetical **CS**.)

Now, what about *translation*? Imagine a person A who uses a **CS** that identifies natural numbers (as the only kind of numbers) and subtraction, and a person B who uses a **CS'** that identifies integers and subtraction. A fictitious dialogue:

A: I owed Charles 10 pounds but today I paid him 3 pounds. You see, I'm not good at calculating. How many pounds do I owe him now?

B: Well, you must subtract 3 from 10!

A (calculates): Yeah, still 7 pounds. But look, tomorrow I could pay him 8 pounds. Then…

B: Then Charles will owe you one pound. Great!

A: How do you know?

B: It's easy. Subtract 8 from 7!

A: Are you joking? You can't subtract larger from smaller!

B: Why not?

A: We learned: If you subtract *a* from *b* then *a* is *b* or smaller than *b*.

B: You can forget that. It doesn't hold any more.

A: One always changes something. Is it necessary?

B: But if you use subtraction in this new manner you can tell what happens when you tomorrow return Charles 8 pounds!

A: So I was always wrong when I used *my* subtraction?

B: Not at all, you were right when you subtracted smaller numbers from greater numbers. You were also able to subtract a number from itself. But you simply *did not know* what to do in the remaining case.

This schematic example shows *inter alia* that an important factor that causes difficulty in communication is that the *new concept* is — as a rule — connected with the *old expression.* So we have *subtraction* for SUBTRACTION_OF_NATURAL_NUMBERS, *subtraction* for SUBTRACTION_OF_INTEGERS, *subtraction* for SUBTRACTION_ OF_RATIONALS etc., and moreover the same mathematical sign is used. Thus the illusion arises that *concepts themselves developed.* Actually, concepts — as abstract objects — cannot develop, of course.

But there is some definable point of comparison between particular members of the sequence where 'new' concepts follow 'old' concepts.

The famous problem of translatability, maybe even of *incommensurability* can be identified already here in the simplest example. To sum up: we have one expression, *subtract*, and at least two concepts of two distinct functions, let us denote these functions SUB1, SUB2. Let the preconcept NATURAL_NUMBER be the type ν, the preconcept INTEGER be the type γ. The type of SUB1 is thus (ννν) whereas the type of SUB2 is (γγγ). We have two distinct concepts expressed by one and the same expression. Both concepts are similar as well as distinct. We easily detect points of difference, in particular SUB2 is but SUB1 is not total. But there are also similarities: Let **CS**1, **CS**2 be the systems with the preconcept ν, γ, respectively. Let C_i, $i \geq 0$, be concepts in **CS**1 that identify particular natural numbers and let D_i, $i \geq 0$, be concepts in **CS**2 that identify particular integers (each C_i, D_i identifying the number *i*). For every *i*, ${}^0C_i \neq {}^0D_i$ but at the same time $C_i = D_i$, i.e., C_i and D_i are *distinct equivalent concepts.* (See Definition 17!) Now let C_i (and therefore D_i) construct a number *i*, and C_j (and therefore D_j) construct a number *j*. If $i \geq j$ then applying SUB1 as well as SUB2 to $\langle i, j\rangle$ results at the same number. Otherwise, the partial SUB1 cannot be applied, unlike SUB2.

Now a question arises: Can we *compare* **CS**1 with **CS**2?

On the one hand, preconcepts are distinct and so are SUB1 and SUB2.

On the other hand, the area of **CS**1 is a subset of the area of **CS**2 and the $\mathbf{ExP}_{\mathbf{CS}1} \subset \mathbf{ExP}_{\mathbf{CS}2}$. (See Definition 30.) But then we can state that **CS**2 is a *creative extension* of **CS**1. (Here only the **ExP**-based criterion can be applied.) Indeed: every problem (here: every GNP) that can be solved in **CS**1 can be solved in **CS**2 but not *vice versa*: subtracting a greater number from a smaller one is *forbidden* in **CS**1 and realisable in **CS**2.

So what about a simulation of the (Kuhnian) incommensurability problem in this simple 'laboratory' case?

Let some people, say, **C** and **D** argue as follows:

C: Great progress was made in the transition from natural numbers to integers, wasn't it?

D: *Progress*? What do you mean? We simply got another system of concepts, but both systems are incomparable, I would say even incommensurable. The two systems simply speak about different things: different operations and different numbers.

C: Really? Take two numbers, say, 5 and 3. Are they natural numbers, or integers?

D: Well, it depends…

C: Suppose then that they are natural numbers. Subtract 3 from 5, you get 2, again a natural number. Now suppose that they are integers. Subtract 3 from 5, you get 2, again an integer.

D: This coincidence ends at that moment when you want to subtract 5 from 3!

C: Which supports my claim that the change to the second system is progressive: it makes it possible to solve the general problem of subtracting independently of whether the one number is or is not greater than the other one.

D: But wait, this problem is not a problem for the first system! You know that within the set of naturals no such problem can be formulated, subtracting greater numbers from smaller numbers is simply impossible, like dividing by zero!

C: And therefore we *enlarge* the class of numbers so that the general problem of subtracting can be solved.

D: But this is only a trick: you make some change to solve *another problem*, but you make believe that it is *the same problem* and that you found a solution to it.

Now let us interrupt **C** and **D** and take some standpoint to **D**'s objections. Assume that the system **CS**1 contains the usual *logical* concepts including $^0\iota$ (i.e., the function underlying the 'descriptive operator') and the concept ADDITION, let it be $^0+$. The concept of SUB1 need not be a primitive concept. It is the following construction:

$$\lambda xy\ [\ ^0\iota\lambda z\ [^0{=}\ [^0{+}\ z\ y]\ x]],$$

where $x, y, z \rightarrow \nu$. Clearly, SUB1 is not total (try to apply it to the pair $\langle 3,5 \rangle$). The above construction/concept is a GNP. Let it be denoted by P_1. The GNP consisting in subtracting any integers will be called P_2. P_2, constructing SUB2, will be

$$\lambda xy\ [\ ^0\iota\lambda z\ [^0{=}\ [^0{+}\ z\ y]\ x]],$$

where $x, y, z \rightarrow \gamma$. Clearly, SUB2 *is* total.

Now, **C** claims that **CS**2 is more progressive than **CS**1 because it can solve a problem that is unsolvable in the latter. **D** objects that **CS**2 simply poses (and then solves) *another problem* and that, therefore, 'progress' should not be mentioned.

To take up a standpoint we will try to analyse the question "*Is P_1 another problem than P_2*?" A simple comparison (see above) shows that *the only distinction between them consists in there being distinct preconcepts (i.e., basic types).* Thus **D** is right when he claims that the two systems pose distinct problems but he is wrong when he draws the conclusion that they (and thus also their solutions) do not share any features whatsoever so that they cannot be compared. We can state this result as follows:

CS2 is able to formulate (and so to make it possible to solve) the problem P_2 but since ν is a subset of γ **CS**2 offers solution also to P_1. In this sense **CS**2 is comparable with **CS**1; the former can do *more* than the latter.

To what extent can this example serve as a basis for generalisation? Can we dare to formulate the following hypothesis?

Hypothesis

*A **CS** gives rise to its creative extension **CS'** if some of its preconcepts are enlarged so that some not total functions identified in **CS** become total (in **CS'**) and (in virtue thereof) some improper constructions in **CS** change to proper constructions in **CS'**.* –

The plausibility of this hypothesis is questionable, of course, at most it holds for a kind of creative extension only. (The technical problem of making partial functions total has been discussed in [Duží 2003c, p.59].)

Another important problem has to be tackled: According to the **ExP**-based criterion, if a **CS**' is a creative extension of **CS** then it contains at least one new problem/concept, i.e., a problem not contained in **CS**. Now we have seen that **CS**2 contained a new concept, but **CS**1 also contained a concept that is not contained in **CS**2! Compare once again the two problems, viz. P_1

$$\lambda xy\ [\ {}^0\iota\lambda z\ [{}^0{=}\ [{}^0{+}\ z\ y]\ x]],$$

with variables ranging over ν, and P_2

$$\lambda xy\ [\ {}^0\iota\lambda z\ [{}^0{=}\ [{}^0{+}\ z\ y]\ x]],$$

with variables ranging over γ.

Distinct ranges of variables imply, of course, respective differences of types: In P_1 we have: =/ (ονν), +/ (ννν), ι/ (ν(ον)), in P_2: =/ (ογγ), +/ (γγγ), ι/ (γ(ογ)).

Our intuition says that P_2 is in a sense *new* with respect to P_1. We define:

Definition 32 (*new problem*)

Let P be a problem containing *n* (λ-bound) variables ranging respectively over types $\alpha_1,\ldots,\alpha_n$. Let P' be like P except that variables range respectively over $\beta_1,\ldots,\beta_n$ (with consequences for the types of particular subconstructions). We say that P' is *new with respect to* P ([^{0}New ^{0}P' ^{0}P]) if, for all *i*, $1 \le i \le n$, $\alpha_i \subset \beta_i$. –

From our definitions it follows that if [^{0}New ^{0}P' ^{0}P] constructs **T** then P is not identical with P'. Accepting all this let us return to our systems **CS**1 and **CS**2 as well as to the dialogue between **C** and **D**. Surely, **D** can say, **CS**1 and **CS**2 pose distinct problems, and rightly so, but our intuition says that **CS**2 not only poses and solves *other* problems than **CS**1 but if it does not 'forget' its predecessor it can in some sense pose the problems of **CS**1. In what sense?

Compare **CS**1 and **CS**1 ∪ **CS**2. The former cannot pose all problems that can be posed by the latter. The latter can pose (and solve) all problems of the former, and all problems from **CS**2 are new with respect to some problem from **CS**1. Thus the 'creative extension' from Definition 31 should be such a union:

$$\mathbf{CS'} = \mathbf{CS} \cup \mathbf{CS''},$$

where all the problems posed in **CS''** are new w.r.t. some problem from **CS**.

Now what would be the *general problem of subtracting* that **C** talks about in our dialogue? It would be obviously rather a *scheme* of problems:

$$\lambda xy\ [\ {}^0\iota\lambda z\ [{}^0{=}\ [{}^0{+}\ z\ y]\ x]],$$

where $x, y, z \to \alpha$ and α is any supertype of ν. (Such supertypes are, e.g., γ, τ.)

We can see that a) the above definition of new concepts is too narrow, even for mathematical **CS**s, and b) the 'creative development' based on accepting 'new problems' in the sense of that definition requires a *transition* from one type system to another one: there are no 'supertypes' within one and the same type system — the types cannot overlap within one type system. (True, considered from our meta-view we can meaningfully claim that the type ν as the set of natural numbers is a subset of the type τ as the set of real numbers.)

All these considerations (beginning, say, with Hypothesis) make it clear that the notion of creative extension is here (exhaustively?) determined by the change of *preconcepts*, i.e., of the basic types the variables range over. Besides, only purely mathematical (NB here only arithmetical) **CS**s are taken into account. This is not to claim that no other cases of what we would like to call *creative extension of mathematical systems* are thinkable. A general analysis of the ways various mathematical conceptual systems can be said to 'develop' would be a task for a separate study.

3.2.4.2 Empirical systems

Once more we have to emphasise that when analysing the following problems, we must be aware of the principal distinction between non-empirical and empirical **CS**s: the former produce tools, the latter contain non-empirical concepts as such tools. Evaluating the interconnections of empirical **CS**s and the questions of translatability, comparability, incommensurability etc. will be formulated and, as the case may be, answered exclusively from the viewpoint of their ability to pose and solve *empirical problems.* To be more precise, the empirical **CS**s alone can only *pose* the problems (*via* the respective empirical concepts); the *solution* is not *a priori* given (as it is the case in the non-empirical **CS**s); it is a matter of some *theories* based on the given **CS** and using the concepts when questioning reality.

We try to investigate empirical **CS**s in connection with *languages*. Here we have to distinguish two cases. First we will try to say something about *ordinary* (*colloquial*) *languages* in the sense to be defined below. Second, some problems with *languages of science* will be handled.

Tarski in [1956, 164–165] has stated the *universal character* of colloquial languages:

> [i]t could be claimed that 'if we can speak meaningfully about anything at all, we can also speak about it in colloquial language'. ... we must... admit into the language, in addition to its sentences and other expressions, also the names of these sentences and expressions, and sentences containing these names, as well as such expressions as 'true sentence', 'name', 'denote', etc. ... every language which is universal in the above sense, and for which the normal laws of logic hold, must be inconsistent.

The way Tarski characterizes universality of a colloquial language covers only one sense of universality. In this sense it is impossible to associate a language with one **CS**: the *primitive concepts of a **CS** that underlies a language* L *cannot identify the expressions of* L*;* another system, say, **CS**' is necessary, but such a **CS'** underlies another language, say, L', and the latter would play the role of a *metalanguage for* L. A consequence thereof would be, e.g., that whereas **CS** would underlie L as well as any (correct) translation of L there would be as many 'metasystems' **CS**', **CS**'',..., as there are translations of L. For Tarski, of course, L contains its 'metalanguage' L'. From the viewpoint of conceptual systems the semantic self-sufficiency of a natural language is simply impossible: a complete semantic theory for such a language cannot be expressed in the language itself.

But universality of a colloquial language is also connected with another phenomenon. The languages of sciences arise, after all, within the given natural language. True, we would perhaps hesitate to call the collection of colloquiality and professional jargons of mathematics, physics, chemistry etc., a 'colloquial' language, but it can't be helped: an English physicist is an English speaking physicist.

In this second sense a natural language is universal since any linguistic innovation that stems from a professional 'sublanguage' is a part of it. Our analysis can however make a natural 'cut': we will define *ordinary language* as that part of a ('universalistic') natural language L that does not contain the scientific 'sublanguages' of L. (When we say that somebody does or does not know a natural language we mean just ordinary language in this sense. For example, some applicant for a job claims that he fluently speaks English; being examined he confirms this claim *but* he does not know what the word *quark* means. All the same we can suppose — unless (!) his application concerns working in physics — that he really does fluently speak English, ordinary English, that is.)

3.2.4.2 a) Empirical systems that underlie ordinary languages

(It is useful to have a look at 2.4, in particular Principle.)

Borrowing terminology from Haeckel we can distinguish two viewpoints concerning the developmental analysis of languages: the *ontogenetic* and the *phylogenetic* one. The former concerns the way a particular person learns his mother tongue while the latter is interested in the way a given language can develop.

A. Ontogenesis

No psycholinguistic study can be expected here, only a rational reconstruction relevant from the viewpoint of logical (conceptual) analysis. Accepting, as we do, that every (stage of a) language is based on a conceptual system we should try to discover the **CS**s that underlie particular fragments of the given ordinary language as they are learned (first by children, later also by adults, in some cases, at least). Involved as the learning process is, one can use abstraction and ignore the empirical procedures accompanying every such process.

Remark: From the viewpoint of a cognitive science many highly interesting features of such process can be found in various studies. For one example only, valuable results are brought by R.Bartsch in her [1998]. –

Then following points may be relevant for our analysis:

Essentially we learn two interwoven things: *Vocabulary* and *Grammar*.

In the early stages of learning the simple expressions that a child is acquainted with express *simple concepts.* To understand such a simple empirical expression means to possess a simple concept, i.e., a simple procedure that identifies the respective object and does not need any other procedure (any other concept) to do so. This is not to say that the concept associated by the child with the expression *E* is identical with the concept that is *officially* (phylogenetically) associated with *E*. The first stages of the learned language are private, individual languages that gradually approximate the 'official' language as defined in the given stage of the language. One form of this evolution can be seen as follows: some simple expressions of the language to be learned have come into being due to definitions: they are abbreviations (see 3.2.1) and actually express complex concepts. The child can be acquainted even with such expressions but usually the concept associated by the child with the expression *E* will be simple and probably will not be equivalent to the concept officially associated with *E*.

Let us consider some artificial examples. Suppose a small child is taught — by being shown particular examples etc. — that such and such things are called *dog(s).* Whereas officially the word *dog* expresses either a simple concept ^{0}dog or some equivalent more complex concept, our child is a little too generalizing and associates with *dog* a concept that could be written

$$^0\text{dog\_or\_cat}.$$

this concept is equivalent to

$$\lambda w \lambda t\, \lambda x\, [^0\!\vee [^0\text{dog}_{wt}\, x]\, [^0\text{cat}_{wt}\, x]]$$

but it is the simple version, i.e., the child does not possess the concept CAT (nor, properly speaking, the concept DOG): let us suppose however that the child is able to distinguish the groups containing dogs and/or cats from other groups; he or she should be able to identify the property *being a dog or a cat*. Later our child learns separately the concepts DOG and CAT. The respective conceptual system will no longer contain the concept ^{0}dog_or_cat: instead it will contain either the primitives ^{0}dog and ^{0}cat, or such primitive concepts in terms of which the properties *dog* and *cat* can be defined.

Now our child is (simultaneously) taught grammar. It is grammar — together with some elements of vocabulary — which enables the child to say sentences like

Some dogs are cats.

and

No dogs are cats.

(Because of what grammar is from our viewpoint: a prescription for associating expressions with constructions.) Thus supposing that our **CS** contains ^{0}dog, ^{0}cat, ..., ^{0}some, ^{0}no, ..., and that the respective types are given — let it be for *some, no,* ((ο(οι))(οι))— we get as members of the DC part of our hypothetical **CS**

$$\{...\ \lambda w\lambda t\ [[^0\text{some}\ ^0\text{dog}_{wt}][^0\text{cat}_{wt}]],\ \lambda w\lambda t\ [[^0\text{no}\ ^0\text{dog}_{wt}][^0\text{cat}_{wt}]],...\}.$$

Remark: The type of *some* and *no* is explained as follows: *some* (*no*) associates with any class A of (here) individuals the class of those classes (of individuals) whose intersection with A is non-empty (empty). –

The two propositions constructed by these two concepts are, of course, incompatible, one being the negation of the other. They are *problems* to be solved. If the **CS** possessed by our child really contains (now already official) concepts DOG and CAT then our child is able to solve these problems. *Our* problem is whether this is a good example of *empirical* problems. Recall Definition 28; the proposition constructed by the first (second) construction above is surely false (true) in all worlds and times because the concepts DOG and CAT are not independent — the problem is only seemingly an empirical problem. Let us therefore suppose that the child can formulate the sentence

Some cats are black.

The respective concept (= problem) is this time empirical. There is no necessary link between the properties *cat* and *black*. Thus our child can use our **CS** to pose the problem (= to formulate the sentence above) — but to solve it our child has to investigate reality.

(These and following considerations do not mean that a learner is able to understand the notion of *construction*, of course. Just as one can wonder that he uses prose (see Molière) most people would be surprised when informed that the procedures encoded by the expressions normally used have been *rationally reconstructed* as constructions in our sense. But this is a general phenomenon connected with every theoretical analysis.)

There is a degree (vaguely delimited, indeed) of one's acquaintance of one's mother tongue that will justify the claim that the given person already knows this language. Summing up, our learner (child) has gradually acquired various conceptual systems that approximate and more and more resemble one of the conceptual systems that underlie the official language (in the given stage of development). From our (logical) viewpoint this process can be (discretely) described as a sequence of **CS**s where the later members are more similar to the language of adults than those earlier ones. In general, comparing the members of such a sequence, say, $\mathbf{CS}_i$ and $\mathbf{CS}_{i+1}$, the latter should be a creative extension of the former: the child using the latter is able to pose more problems than the child using only the former.

Remark: This is, of course, a simplification also in the sense that the sets of concepts acquired in the earlier stages are not necessarily subsets of the sets of concepts acquired in the later stages. –

As for the role of Grammar, its expressions very often express concepts from the LOG ∪ MATH part of the given **CS**. The child A that possesses the same **CS** as the child B with the only exception that A's **CS** does not contain some concept that underlies a syntactic word (like *and* or *if*) is not able to formulate (pose) some problems that B is able to. This does not mean that B is also able to *solve* these problems, but *ceteris paribus* B has got a better start for solving them.

An artificial example: Let the concepts contained in $\mathbf{CS}_1$ and $\mathbf{CS}_2$ make it possible to formulate the sentence

There is lightning.

and the sentence

Thunder can be heard.

Let $\mathbf{CS}_2$ contain – unlike $\mathbf{CS}_1$ – the concept WHENEVER (whenever/ $(o\ o_{\tau\omega}\ o_{\tau\omega})_\omega$). This concept is empirical and can be represented as

$$\lambda w \lambda pq\ [^0\forall \lambda t\ [^0\supset p_{wt}\ q_{wt}]].$$

The person who uses $\mathbf{CS}_1$ cannot pose, let alone solve the problem represented by the sentence

Whenever there is lightning thunder can be heard.

unlike the person using $\mathbf{CS}_2$. This is an ordinary example of a creative extension of a conceptual system due to a new empirical concept enriching the old **CS**. (Recalling the Principle from 2.4 we can say that our $\mathbf{CS}_2$ either contained the concept ^{0}whenever, or that its **PC** part contained some primitive(s) in terms of which the concept WHENEVER could be composed in the **DC** part.)

An analogy can now be stated in the case of a non-empirical syntactic *word*, i.e., such a word that expresses a concept from the LOG part of the given **CS**. Suppose that some $\mathbf{CS}_1$ does not and some $\mathbf{CS}_2$ does contain the concept of disjunction, either as ${}^0\vee$ or as $\lambda pq\ [{}^0\neg\ [{}^0\wedge\ [{}^0\neg p]\ [{}^0\neg q]]]$ or in some equivalent way. If this is the only distinction between $\mathbf{CS}_1$ and $\mathbf{CS}_2$ then the possessor of $\mathbf{CS}_2$ can — unlike the possessor of $\mathbf{CS}_1$ — formulate the problem given by the sentence

Today it is Monday or Tuesday.

(Observe, however, that the possessor of $\mathbf{CS}_1$ cannot have both the concept of conjunction and the concept of negation.)

Recalling Definition 26 we can state that the area of $\mathbf{CS}_2$ is only *inessentially extended* with respect to $\mathbf{CS}_1$.

It is obvious that the real process of learning the mother tongue is extremely complicated. Ambiguities are adopted from the very beginning. Consider the case with learning the concept(s) underlying the word *and*: One of these concepts is a construction of the classical truth function, type (οοο), the other one is an empirical concept, type $((οο_{τω}ο_{τω})_{ω})$. The first case is represented by the sentence

It rains and I watch television.

while the second case can be represented, e.g., by

My father returned home and went to bed.

Another piece of evidence of the complexity connected with learning grammar: tenses. A rational reconstruction of this process presupposes some kind of temporal logic; in TIL we have an excellent study [Tichý 1980], where what the grammar of English prescribes is legitimised by a thorough semantic analysis in terms of constructions, i.e., by a *conceptual* analysis in our sense.

Some interesting questions (parallelised in *Phylogenesis*) arise when we ask *What happens when we learn mathematics*?

We can, e.g., ask whether an extension of the $\mathbf{MATH}_{\mathbf{CS}}$ makes it possible to pose only new *mathematical* problems (which would make the notion of *inessential* extension more intuitive), or whether also some new *empirical* problems could be posed. The answer, based on our conception of **CS**s, will be in harmony with our pretheoretical intuition. An example will suffice:

Let $\mathbf{CS}_i$ arise from $\mathbf{CS}_{i-1}$ by adding a concept of *cardinality (of a class of individuals)* to the MATH part of $\mathbf{CS}_i$. One possibility is to add a *primitive* ^{0}card constructing the cardinality function, type (τ(οι)) (or (ν(οι)), if you like. There are, of course, infinitely many new mathematical problems concerning cardinalities of particular classes of individuals, but besides, new *empirical* problems can be now posed. For an illustration compare I. and II. below:

I. The user of $\mathbf{CS}_i$ possesses concepts of particular major planets of our Solar system (so ^{0}Mercury, ^{0}Venus, ^{0}Earth, ..., ^{0}Pluto) and poses (surely also solves) the problem

$$[^0\text{card}\ \lambda x\ [^0\vee\ [^0\vee\ ...\ [^0\vee\ [^0{=}\ x\ ^0\text{Mercury}]\ [^0{=}\ x\ ^0\text{Venus}]]...[^0{=}\ x\ ^0\text{Pluto}]]...]$$

that underlies the question

How many members of the class {Mercury,...,Pluto} are there?

This problem is not empirical, of course.

II. The user possesses an empirical concept of *planet*, for example ^{0}planet, where *planet* is a property of individuals, type $(o\iota)_{\tau\omega}$. (Here we bear in our mind again the major planets of our Solar system.) Then the problem formulated by the sentence

How many planets are there?

is the concept

$$\lambda w \lambda t\ [^0\text{card}\ ^0\text{planet}_{wt}].$$

Clearly, this concept is an *empirical problem* and could not be posed if no concept of cardinality (here ^{0}card) were at one's disposal.

(Observe that the problem sub I. is solved without any reference to actual world: it is a purely mathematical (albeit very elementary) problem. To solve problem sub II. one has first to identify the *class of planets in the actual world*, which cannot be done without experience. All the same, the *empirical* problem of the number of planets cannot be posed, let alone solved, unless some *mathematical* concept of cardinality is at our disposal. A clear IEE case.)

Now let our learner person finish her learning process and suppose that she has mastered her mother tongue; we will now make some points concerning the phylogenetic development of a(n ordinary) language.

B. Phylogenesis

First of all let us recall the argument in 3.1 according to which we cannot suppose that all meaningful expressions of a natural language could be derived from some base whose members would express primitive concepts. That such a unique **PC** is absent holds naturally not only for the synchronic analysis; in investigating possible relations between **CS**s that underlie various stages of development of a natural language we cannot suppose the existence of such a unique **PC** for a given stage. All the same some interesting points can be stated independently of that simplifying assumption.

There are principally two ways an ordinary language can develop:

a) *autonomous* development; new expressions are added due to some colloquial practice.

b) *heteronymous* development; new expressions are borrowed from non-colloquial practice, i.e., from professional vocabularies (sciences, technology).

Ad a): Autonomous development.

From the viewpoint of our theory of **CS**s case a) is rather simple. The simplest subcase is the *definitional* one. Some property identified by a complex concept becomes more and more frequently discussed so that a simple expression replaces the complex one. For an example, before the 1940s *Quisling* served as a personal name only. Since a man named *Quisling* collaborated with the Nazis (in his high political position) a new expression – this time the name of a *property* – has enriched various languages (and not just Danish): *quisling*. The respective concept is a complex one (A COLLABORATOR or so) so that the simple expression cannot be understood without the respective explication. The enriched language has got a new expression but this innovation has not been accompanied by an extension of any of the **CS**s that underlie the respective language. So it would not be adequate to extend such a **CS** by adding a *primitive* concept ^{0}quisling.

The second subcase, the non-definitional one, is much rarer. What we are after is an illustration of the case when a(n ordinary) language autonomously accepts a new simple expression that expresses a demonstrably *simple concept*.

One kind of example could perhaps exploit the phenomenon of *metaphor*. Some expression (mostly denoting a property) gets a new figurative meaning. A thorough and interesting analysis of this phenomenon can be found in [Bartsch 1998]. We borrow one of her examples (p.115); the word *dachshund* can be used by a mother who is disgusted by the behaviour of her child (and says "*I don't want a dachshund around me"*) or by somebody who observes a short man with a long back and O-form legs (and says: "*Look, what a dachshund*"). Bartsch sums up:

> By these examples, we have now generated a polysemic complex for the term *dachshund* that consists of three concepts:
>
> 1. the concept expressed by *dachshund* used under the prominent or default perspective of natural kind identification,
> 2. the concept expressed by *dachshund* under the behaviour perspective, also expressible by *someone who always follows his caretaker around*, and
> 3. the concept expressed by *dachshund* under body form perspective, also expressible by *someone with a long back and very short legs in O-form.*

From the viewpoint of our theory of **CS**s we can analyse this example as follows:

A key notion of our analysis is the notion of *requisite* (Definition 28). Construing Bartsch's "prominent or default perspective" as what determines the official meaning of the expressions of the respective (ordinary) language we have to analyse the other perspectives, e.g., those ones sub points 2. and 3. First of all, the result of applying the "behaviour perspective" is the property *always following one's caretaker around*, while the result of applying the "body form perspective" is the property *possessing a long back and very short legs in O-form*. Let the property given by the official meaning of *dachshund* denote by D. Denoting the other two properties Fol, Lb, respectively, we can see that following constructions construct T:

$$[{}^0\mathrm{Req}\ {}^*\mathrm{Fol}\ {}^*\mathrm{D}],\ [{}^0\mathrm{Req}\ {}^*\mathrm{Lb}\ {}^*\mathrm{D}].$$

Another important notion is *perspective* (or *point of view*), see a thorough analysis in [Hautamäki 1986]. In general, the type of a perspective is given by the scheme $(\alpha\iota)_{\tau\omega}$. For some examples consider perspectives *colour, age, behaviour, body form.* For *age*, α is τ (or ν, if you like): the number of years, that is. For the other perspectives α is $(o\iota)_{\tau\omega}$: in the given WT pair *colour* associates every individual with a particular colour (particular colours are, of course, properties of individuals, so the type is $(o\iota)_{\tau\omega}$); *behaviour* again associates every individual with some property that is a typical behaviour of it; similarly for *body form* where the typical shape is again presentable as a property (cf. above: *possessing ...*).

Thus the selection of particular requisites is always given in terms of some perspective.

Remark: The type of *perspective* could be generalised. We have seen above that such notions as *typical* have to be used. Perhaps this is not the best analysis; we could better speak of perspectives that connect particular kinds of object not only with individuals but also, e.g., with properties. So instead of speaking about the typical behaviour of an individual we could connect behaviour with *properties*, in our example with the property *(being a) dachshund*. Accepting this explication we would have to replace the type-theoretical scheme of perspectives by $(\alpha\beta)$, where β is a type of an intension, mostly of a property. Then we would have two kinds of perspective: one of them, type $(\alpha\iota)_{\tau\omega}$, e.g., *age*, *body form, behaviour,* etc., the other, type $(\alpha\beta)$, e.g., *the maximum age, typical body form, typical behaviour,* etc. The latter associates an intension with one of its requisites; therefore this kind of perspective is an extension.

A minor philosophical digression is now useful. TIL is anti-essentialist in the sense that individuals are construed as being 'bare', something like 'pegs' on which particular properties happen to hang. All the same, the notion of *essence* is not tabu in TIL: essences are associated with *intensions.* We define:

Definition 33 (*essence*)

Let A be an intension (an α-role, type $\alpha_{\tau\omega}$, or an α-property, type $(o\alpha_{\tau\omega})$. The *essence of* A is the set of all requisites of A, constructed by ($p \to (o\alpha)_{\tau\omega}$):

$$\lambda p\ [{}^0\mathrm{Req}\ p\ {}^0\mathrm{A}]$$

(See [Tichý 1979], where the notions of requisite and essence are explained and used to show that ontological proofs of the existence of God by St. Anselm and Descartes are not correct.) –

The essence of that intension which is denoted by an expression *E* is given by the official (default) meaning of *E. The phenomenon of metaphor can be then described as associating the original expression with one or perhaps more requisites ignoring the other members of the essence.* The selection of the apt requisites is determined by some *perspective*, see above.

Thus the expression *dachshund* — to return to Bartsch's example — becomes ambiguous (homonymous) and our question is: can we — in virtue of this fact — infer that the original **CS** has been enriched by a *primitive concept*, i.e., by the concept ${}^0\mathrm{dachshund}_1$, where $\mathrm{dachshund}_1$ is 'a dachshund under the perspective (say) *body form*'?

We know what a trivialisation means: it simply returns the object without any change. Philosophically (or: meta-logically) it means that there is a(n objective!) procedure that *identifies the object in question,* in our case the property of having a 'dachshund body form', *without being supported by another procedure.* On a most abstract level this is imaginable, of course, but is it necessary that the language that accepts the expression denoting the property $dachshund_1$ does so *via* accepting such an autonomous procedure?

It is certainly *not* necessary, but we can imagine a situation where it is *possible*: imagine a world where dachshunds have been for some reason exterminated but where the linguistic memory causes the expression *dachshund* to still be used just in the sense of the property $dachshund_1$. At first this usage can be unofficial, but later the children when learning the mother tongue learn just this usage. Yet there are no dachshunds in this world, only some people with a long back etc. Most children learn the expression *dachshund* in the similar way as they learn, e.g., the words *dog*, *moon, water* etc., i.e., by empirical generalisation. At the time when these children become adults the word *dachshund* already *officially* denotes the property $dachshund_1$; we can say that a **CS** underlying this 'experimental' phase of the development of such a language really contains the *primitive concept* ${}^0\mathrm{dachshund}_1$.

Even this story need not be too convincing: we can expect that a language that has accepted the changed meaning of the expression possesses also such expressions as *long, back, legs,* etc. But then the primitive concept ${}^0\mathrm{dachshund}_1$ is not necessarily part of the **PC** in question: the property $dachshund_1$ is definable in such a language, independently of the way the respective concept is acquired in the ontogenesis of the users. Hence our story is trustworthy only if the original language did not possess expressions for *long* or *back* or *legs,* etc.

Remark: An essential distinction between ontogenesis and phylogenesis consists in the fact that handling the former we confront particular phases of the development with the official language whereas the latter concerns relations between two phases of the development of one and the same official (ordinary) language. –

Our search for genuinely new primitive concepts that would accompany some enrichment of an ordinary language have so far only poor and doubtful results. No wonder. Artificial experiments should be supplemented by empirical research which is the competence of linguistic disciplines. The logical analysis of natural language (LANL – see 1.4.3.1) can evaluate the empirical data from the viewpoint of conceptual analysis. All the same, it should be clear that the tools offered by LANL make it possible to explain logically relevant fundamentals of what happens on the linguistic level.

Ad b) *Heteronymous development*.

Expressions borrowed from science and technology are elements of the development of an ordinary language. While such expressions are given semantics within the respective disciplines, their usage in ordinary language is adapted to colloquial practice. This fact has to be taken into account when the theory of **CS**s is to be applied.

For an illustration consider a special theoretical term like *database*. In computer science we can find an exact definition (or perhaps some more or less equivalent alternative definitions) of databases. This means that the **CS**(s) underlying computer science contain(s) concepts that identify the property *being a database*. (Observe that the language specific for computer science need not be based on the preconcepts ο, ι, τ, ω.) Since computers began to become pervasive the need to enrich *ordinary* language with some important expressions arose. Yet, whereas the professional worker in computer science has to possess the concept underlying the *professional* term *database* (to be able to design, use, maintain, update etc. databases) the layman, on the one hand, does not need this professional concept. But, on the other hand, he often has to exploit databases in his customary practice, and so should be able to *somehow identify* the respective property. Ordinary language offers a solution to this problem (*via* dictionaries or so): a *simplified* definition.

Maybe more convincing examples can be taken from the cases where the borrowed term is a technological expression. Consider the term *auto(mobile)*, which is surely part of contemporary *ordinary* language. Workers in the automobile industry know many requisites of the property *(being a contemporary) automobile* but other speakers of the ordinary language need not know all these requisites; they need not know the *essence* of this property. What they do need — and what is thus a part of the official ordinary language — is a concept that identifies any object that is an automobile. We could object that if we are to identify the property *(being an) automobile* then we have to know the *essence* of this property. So we should know all the requisites, not only those offered by a simplified definition.

We can at least artificially find some extreme cases where the fact that some requisites are 'concealed' causes the given property *not* to be identified. (I would call such cases 'swindlers'.) Example: A model instead of an automobile is sold; the swindler disappears before the buyer finds out that, e.g., having four wheels, coachwork, an exhaust pipe, etc. are only some of the requisites and that requisites like *possessing a tank* and *possessing spark-plugs* are necessary members of the respective essence.

We can see that the development of science and of technology calls for enrichment even of *ordinary* (even *colloquial*) language. Yet how can such an enrichment be analysed from the viewpoint of our theory of **CS**s? *An empirically creative extension of the given empirical **CS** requires that the new **CS** contain at least one new empirical primitive concept* (see Principle in 2.4 and Definition 31!). Our examples, however, show that borrowing scientific and technological expressions together with some simplification of their meanings is realised *via* some definitions, which means that such concepts are not simple. It looks like if the (simplified) terms borrowed from science & technology were definable in terms of the **CS** that was supposed to be creatively enlarged.

This is, indeed, impossible: we cannot assume that the **CS** that underlies English from the 16th century is sufficient for founding contemporary English. The development of (the lexical part of) any language does not deserve the name 'development' unless the respective **CS**(s) underwent some creative extensions.

Thus some new terms have to express *primitive empirical concepts.*

To find such particular examples is not a task for LANL. Data that could confirm our theoretical claim must come from empirical research in, e.g., comparative linguistics.

All the same, let us tr*y* to construct a more or less artificial example that would illustrate the presence of a *new primitive empirical concept.*

For this purpose try to imagine a very early, very primitive stage of mankind and of the language of a group of people. (We will translate this language into English.) At that time hammer was invented. What about the expression *hammer* that had to be (according to our story) a new expression of the respective language? Can we say that the concept that underlies this expression is a simple one? To answer this question we must be aware of an obvious empirical fact: prior to the introduction of the new expression *hammer* into the language there was some *activity*, some practice. There was no possibility (as we can claim without loss of generality) of *defining* the tool just invented. Because of the social practice it was *immediately clear* that there was some activity, viz. *hammering*, that is important and needs therefore some linguistic representation.

At this time many (perhaps most) theoreticians will say: This is how a new concept has come into being, because it was *created* by the language. What I believe is that this new concept, a simple procedure, was *discovered* since its *potential* (objective) presence was only *actualised*. Arguments can be found in Tichý's project of *meaning driven grammar*, see in particular the quotation from 1.4.3.5.

Thus the only important distinction between our theory of *objective concepts* and the theories of concepts as mental entities does not consist in our denying the role a social practice plays in the history of languages and in conceptual 'development'; it rather consists in the claim that *facts of the developing world help us to discover various objective procedures that deserve linguistic encoding.*

3.2.4.2. b) Languages of science

> [t]he idea that statements have their truth values *independent* of embedding theory is so deeply built into our ways of talking that there is simply no 'ordinary language' word or short phrase which refers to the theory-dependence of meaning and truth. [Putnam 1983, 430]

The most interesting problems connected with languages of science arise at the moment when the interplay of *language* and *theory* is taken into account. Putnam's remark suggests that there is a vexed question known as the problem of *theory ladenness* of concepts. We will try to analyse this problem from the viewpoint of our theory of concepts and conceptual systems.

Let us first formulate some consequences of our conception in connection with the relation between language and theory.

a) A **CS** is, of course, neutral with respect to truth: for any member of a **CS** that constructs a proposition P there exists another member that constructs the negation of P. In this sense we can agree with Kuhn that "there is no sense in which a lexicon may itself be true". (See [Sankey 1997, 76].) We have seen that **CS**s are in a sense collections of *problems*, and since we are analysing *empirical* **CS**s the solutions (i.e., empirical methods of finding extensions of particular intensions) are in the following sense not given *a priori*: the results of such methods are dependent on the state of the world. Therefore, while the members of a **CS** are associated with the members of the respective lexicon by *convention*, the theory *based* on the **CS** is (more or less, as a theory of *verisimilitude* would say) true dependently on the state of the world. As Sankey, c.d., p. 77, says:

> [w]hile theories may be expressed using the resources of a conventional lexicon, nothing follows from this about the nature of theories. To think otherwise is to confuse the language in which a claim is expressed with the claim itself.

b) What is a *theory* from our vantage point? We have two options: either it is a set of propositions closed under entailment, or it is the set of concepts that 'generates' this set of propositions. Choosing the latter option we can define a (consistent) theory $\mathbf{T_{CS}}$ as a (proper) subset of **CS**, i.e., as that part of the original **CS** whose members construct accepted propositions. Whether the accepted propositions are true or not is no longer a matter of convention. We have seen that a transition from a **CS** to another are creative only if the resulting **CS** contains some *new* empirical concepts, which in the case of **EE** presupposes that some new empirical concepts are *simple*. One of the main sources of problems with the analysis of the development of scientific theories is that

new concepts are not always associated with new expressions.

c) Comparing $\mathbf{T_{CS}}$ with $\mathbf{T_{CS'}}$, where $\mathbf{T_{CS'}}$ is what we wish to call a creative modification of the theory $\mathbf{T_{CS}}$, we can see that the most interesting cases are those ones where it is not the case that $\mathbf{CS} \subset \mathbf{CS'}$ (NB: this is a *correction of Definitions 31*): one of typical examples is the transition from phlogistic chemistry to oxygen chemistry; it is not so that the latter would contain the concept of phlogiston and the concept of oxygen — the former concept 'disappeared'. As Sankey, p. 14, says (emphasis mine):

> The phlogiston and oxygen theories are examples of different scientific theories which applied *distinct conceptual apparatus to a common set of phenomena.*

A very general scheme of creative modifications is then succinctly suggested in [Sankey 1997, 110]:

> New concepts are introduced and old concepts undergo modification.

Remark: Sankey's formulations, viz. "New terms with new meanings" and "old terms shift their meaning", suggest that the author is aware of the inaccuracy connected with the frequently used formulations like "some concepts change (shift etc.) their meaning". The latter formulations are from our viewpoint senseless, since concepts *are* meanings. –

The problems that are well-known from Kuhn's, Feyerabend's, Putnam's et alii works arise as soon as we begin to doubt *whether two or more theories based on distinct conceptual systems can be said to concern a common set of phenomena and, in consequence, whether they can be compared at all.* As applied to the development of a theory this problem — known as the problem of *incommensurability* — consists in questioning the comparability of two or more phases of development of one and the same (?) theory.

There are more than many articles and monographs handling the incommensurability problem (IP), cf. Bibliography on IP, which contains (up to January 24, 2001) about 400 titles. Naturally, my aim here cannot be an evaluation of these titles and an attempt to give a further analysis from the viewpoint of history and philosophy of science. Instead, what can be done which perhaps offers something new can be based on our theory of **CS**s. Let us therefore begin with a schematic ('laboratory') example whose analysis should elucidate some points that could be neglected if we relied on verbal means only.

Example

Let T_1 be a theory dealing with surfaces of Earth localities and classifying them w.r.t. the way of their exploitation. Let T_2 be a later (a more 'fine-grained', as we will see) phase of development of T_1. L_{T1}, L_{T2} be the languages of respectively T_1, T_2. Two vocabularies, $\mathbf{V}_1$, $\mathbf{V}_2$, translate respectively L_{T1}, L_{T2} into English. Fragments of them are below:

$\mathbf{V}_1$

bink	meadow_or_pastureland
bace	having the same surface as
Loc	locality A
Lok	locality B

$\mathbf{V}_2$

cink	meadow
cank	pastureland
bace	having the same surface as
e	it is not the case that
Loc	locality A
Lok	locality B

Due to the extremely simple grammar (shared by both the languages) the following sentences are translated into English as follows:

1. Bink Loc — A is a meadow or pastureland
2. Bink Lok — B is a meadow or pastureland
3. Loc bace Lok — A has the same surface as B
4. E cink Lok — B is not a meadow
5. Cank Lok — B is pastureland
6. Cink Loc — A is a meadow
7. E cank Loc — A is not pastureland
8. E bace Loc Lok — A does not have the same surface as B

Now suppose that reality contains localities A and B. A is a meadow, not pastureland, whereas B is pastureland, not a meadow. According to T_1 the surface of A is the same as the surface of B (see 3), whereas T_2 is the negation of 3. (see 8).

This contradiction calls for an answer to some questions:

- Does the transition from T_1 to T_2 mean progress in that T_2 *corrects* a mistaken claim made by T_1?
- Can L_{T1} be translated into L_{T2}?
- Can L_{T2} be translated into L_{T1}?

Let us begin with the two last questions; we will try to get a fragment of a vocabulary $\mathbf{V}_3$, translating L_{T1} into L_{T2}. (Without loss of generality we can assume that L_{T2} contains the word “or” with the same meaning as in English.)

$\mathbf{V}_3$

$\mathbf{L_{T1}}$	$\mathbf{L_{T2}}$
bink	cink or cank
bace	bace
Lok	Lok
Loc	Loc

At least one fragment of L_{T1} is thus translatable into $L_{T2.}$

Our attempt at making up an inverse vocabulary V_4 breaks down:

$\mathbf{V}_4$

$\mathbf{L_{T2}}$	$\mathbf{L_{T1}}$
cink	?
cank	?
bace	bace
Lok	Lok
Loc	Loc

Now let us check our vocabularies as to whether the translation they offer is *successful*. From our viewpoint the expression E is a successful translation of the expression

E' iff the *meaning* of E is the same as the *meaning* of E'. This *verbal* formulation will surely be agreed upon perhaps by all linguists/semanticists. There is, however, a snag: how would you define *meaning*?

This question could be formulated more specifically as follows: *What should a nice theory of meaning look like?* (Which is the title of a paragraph in [Newton-Smith 1981, 162]; Newton-Smith's answer (p.163-164) is:

> ...a theory of meaning that will help with the current problem [*viz. the non-holistic explanation of incommensurability* – P.M.] will have to be fine-grained. Theories of meaning according to which the meaning of a sentence is given by the truth-conditions or assertability-conditions of the sentence within which they occur would be a coarse-grained and not a fine-grained theory.

TIL can be construed as a correction and modification of Frege's famous scheme (but Tichý himself avoided adopting Fregean terminology in this respect); then *meaning*, as it is most frequently used in English, would correspond to Frege's *Sinn* rather than to his *Bedeutung*. For us meaning is therefore best construed as being a structured entity. Hence meanings — as *genuinely structured* (Cresswell's tuples are not satisfactory – see 1.2.2)— are *constructions* and in the case of expressions without indexicals of any kind they can be identified with our *concepts*. Thus checking the vocabularies above consists in comparing constructions that underlie the particular entries.

At least initially we will assume that all the simple expressions of L_{T1} and L_{T2} express simple concepts; thus we have ^{0}bink, ^{0}bace, etc. Then the constructions that underlie sentences 1 – 8 will be (types: bink, cink, cank/ $(o\iota)_{\tau\omega}$, bace/ $(o\iota\iota)_{\tau\omega}$, Loc, Lok/ ι):

1' $\lambda w \lambda t\, [{}^0\mathrm{bink}_{wt}\ {}^0\mathrm{Loc}]$

2' $\lambda w \lambda t\, [{}^0\mathrm{bink}_{wt}\ {}^0\mathrm{Lok}]$

3' $\lambda w \lambda t\, [{}^0\mathrm{bace}_{wt}\ {}^0\mathrm{Loc}\ {}^0\mathrm{Lok}]$

4' $\lambda w \lambda t\, [{}^0\neg\ [{}^0\mathrm{cink}_{wt}\ {}^0\mathrm{Lok}]]$

5' $\lambda w \lambda t\, [{}^0\mathrm{cank}_{wt}\ {}^0\mathrm{Lok}]$

6' $\lambda w \lambda t\, [{}^0\mathrm{cink}_{wt}\ {}^0\mathrm{Loc}]$

7' $\lambda w \lambda t\, [{}^0\neg\ [{}^0\mathrm{cank}_{wt}\ {}^0\mathrm{Loc}]]$

8' $\lambda w \lambda t\, [{}^0\neg\ [{}^0\mathrm{bace}_{wt}\ {}^0\mathrm{Loc}\ {}^0\mathrm{Lok}]]$

Naturally, we can write down these analyses in virtue of accepting the vocabularies $\mathbf{V}_1$ and $\mathbf{V}_2$. But without the assumption that these vocabularies offer successful translations into English we simply could not do anything.

Remark: The fact that L_{T2} is untranslatable into L_{T1} is compatible with the translatability of both L_{T1} and L_{T2} into English. For thorough argumentation see [Sankey 1997]. –

Now we come to our first question. Its presupposition is that T_{L1} is mistaken in that it makes it possible to accept a false claim (sentence 3).

But is this really so? Is the sentence 3, as formulated in L_{T1}, really false?

Let us once more compare the constructions 3' and 8': Obviously, in every ⟨W, T⟩ they construct opposite truth-values, so it looks like if sentence 8. actually *corrected* the false sentence 3. Yet there is a presupposition — that the meaning of the word *bace* is the same in the case of L_{T1} and in the case of L_{T2}; indeed, ^{0}bace in 3' is the same construction as ^{0}bace in 8'.

We will return to this question later; now it seems as if this presupposition were too strong: if it were correct, then claim 3, when translated into English, would say that both A and B are meadows or pasturelands and that their surface is the same, and this claim would be wrong. On our conditions, however, where ^{0}bink is a *simple concept*, the language L_{T1} is *insensitive* to the distinction between meadows and pasturelands (no separate concepts identify meadows as something distinct from pasturelands), which, by the way, makes a $\mathbf{V}_4$-like vocabulary impossible. Thus should (and could) a speaker of L_{T1} deny that A and B are indiscernible (as for their surface)?

It seems that we are confronted with the situation (suggested above in point c)) when a new concept is associated with an old expression. Claim 8 may be true without 3 being false.

Remark: Our example may bring to mind Field's theory of *denotational refinement* (see [Newton-Smith 1981, 176-178]). (We could perhaps associate *bink* with Newtonian *mass*, *cink* with *proper mass* and *cank* with *relativistic mass*; another real example (a 'safer' one) could be surely found, since it seems that the example with 'proper mass' and 'relativistic mass' is not just a refinement — see [Earman, Fine 1997].) According to Field (and his notion of partial denotation) claims made about objects determined by a coarse-grained concept are true if they are true about objects determined by both more fine-grained concepts, false if they are false about objects determined by both more fine-grained concepts, and without any truth-value otherwise. Our examples do not support the adequacy of this definition. For let us consider the following claims (formulated for simplicity's sake in a mixture of L_{T1} and English without any loss of intelligibility):

1. *There are n surfaces bink.*
2. *Binks are endangered by dry seasons.*
3. *Sheep graze on binks.*

Under normal conditions sentence 1 is (for a given number *n*) true of *bink* objects and false of both *cink* and *cank* objects. Hence it should be false according to Field. But why should it? The sentence 2 is obviously true of *bink, cink* and *cank* objects. This is in accordance with Field. Finally the sentence 3 can be interpreted in two ways: either it means *on all binks* or *on some binks*. In the former case it is false of *bink* objects, false of *cink* objects and (may be) true of *cank* objects. In the latter case it is true of *bink* objects, false of *cink* objects and true of *cank* objects. In both cases it should lack a truth-value according to Field. But why should it? –

We will try now to accommodate our analyses to our denying that *bace* has the same meaning in L_{T1} as in L_{T2}. There are two options here:

First, let both meanings remain *simple* concepts (very improbable, of course). Then we have to admit that the (simple, immediate) procedure connected with *bace* in L_{T1} – let the word *bace* be provided with an index, say, $bace_1$ – is distinct from such a procedure connected with $bace_2$. This option is not very intuitive, considering the character of the English translation (*having the same surface as*). Let us therefore consider the second option: the concepts underlying $bace_1$ and $bace_2$ are distinct *complex* concepts. Then the ontological definitions (see 2.6) will contain some other concepts, like SURFACE, and $^*bace_1$ would use another (more coarse-grained) classification of surfaces than $^*bace_2$. Does it however justify the claim that $^*bace_1$ is another concept than $^*bace_2$? Not at all: Let *bace* be defined as follows ($a, b \rightarrow \iota$, i.e., localities, Surfof/ $((o\iota)\iota)_{\tau\omega}$, i.e., the surface of (the locality)):

$$[^*bace_{wt}\, ab] \text{ iff } [^0\!= [^0Surfof_{wt}\, a]\, [^0Surfof_{wt}\, b]]$$

Now the transition from L_{T1} to L_{T2} means that the coarse-grained concept ^{0}Surfof has been decomposed into a more fine-grained concept ^{0}Surfof1, in correspondence with the transition from *bink* to *cink* and *cank*. Now $[^*bace_{wt}\, ^0Loc\, ^0Lok]$ holds (in the given ⟨W, T⟩-pair) for Surfof and does not hold for Surof1; *bace* simply determines whether the ‘input properties’ of the given localities are or are not distinct. Since *bink* is another property than *cink* or *cank* and *cink* is distinct from *cank* it would be absurd if $[^*bace_{wt}\, ^0Loc\, ^0Lok]$ did hold in the case of *bink* as well as in the case of *cink* and *cank*. The concept $^*bace_1$ is the same concept as the concept $^*bace_2$.

We have shown that T_2 does not *correct* T_1. So our elementary schematic example seems to corroborate the incommensurability thesis, i.e., that the content of two or more theories (most frequently phases of the development of a theory) cannot be compared, so that we cannot speak about a *progress* of a theory if some shifts of meanings take place.

Yet such a consequence does not obtain. This can be shown independently of any theory of verisimilitude (which is not to say that a good theory of verisimilitude must be irrelevant here). T_{L2} is more progressive than T_{L1} because it can pose more real problems than the latter: to speak informally, in T_{L2} we can pose the problems

How many meadows or pasturelands are there?

How many meadows are there?

How many pasturelands are there?

whereas T_{L1} can — *ceteris paribus* — pose only the first of them. (Our example falls under the EE-case, of course.) –

Now what can we learn from our long Example?

First, if the general scheme given in this example can be successfully applied to a real scientific theory then a holistic approach to semantics is unnecessary. The phenomena of untranslatability and incommensurability can be accounted for without taking refuge in Quinean holistic relativism. (The condition of applicability has to be fulfilled, of course, so that our rejection of holistic semantics is not as convincing as a genuine refutation would be.)

Further, what does applying *distinct conceptual apparatus to a common set of phenomena* mean? In other words, how do we know that two distinct **CS**s concern the common set of phenomena? In our Example we knew this because there was a background language, viz. English, and both L_{T1} and L_{T2} were translatable into it (not being symmetrically translatable one into the other). Thus: *We can claim that two distinct **CS**s concern a common set of phenomena if and only if a background (obviously natural) language **L** is at our disposal and all members of both the **CS**s are expressible in **L***. The 'if' is obvious; as for the 'only if', it is at least difficult to imagine a situation where we would have no support from natural language for our claim.

After all, natural language is always present whenever we handle *primitive,* i.e., *simple* concepts. To elucidate this fact let us have some thoughts about the role of *trivialisation.*

Intermezzo: specific character of trivialisation; analytic truths and logic

Trivialisation is the most controversial kind of construction. Whereas composition and closure represent intuitively clear procedures, and variables (even in our objectual version) are well-known as for what we can expect they do, trivialisation is too simple to let us imagine a particular procedure and too complex to be identified with some variable-like entity. This somewhat enigmatic character of trivialisation makes it possible, e.g., to avoid the Millian idea of proper names as being simply labels on objects (see [Jespersen 2000]). But because of the obviously important role trivialisation plays in our theory of **CS**s it deserves some more thorough comments.

One important point to be taken into account is that *any* theory of procedures (and TIL is one of such theories) is necessarily *finitist* in the sense that it has to start with some *elements* that are *simple* in that they are no more decomposable into more simple components. (Let me here once more quote a relevant formulation from [Fletcher 1998, 51]:

> If one had to define constructions in general, one would surely say that a type of construction is specified by some *atoms* and some *combination rules* of the form 'Given constructions $x_1,\ldots,x_k$ one may form the construction $C(x_1,\ldots,x_k)$, subject to certain conditions on $x_1,\ldots,x_k$'.

Trivialisations are such 'atoms', better perhaps 'the simplest molecules', since the atoms proper are variables (see [Tichý 1988, 63]); the only constituent part of a trivialisation is the object/construction under trivialisation. A pre-theoretical idea is that the respective object/construction is constructed *directly*, i.e., without any mediation by other constructions.

What has been here defined as *simple concept* is just a trivialisation: the object under trivialisation is either a variable (which is not an interesting case in the present contexts) or an object whose type is of order 1 (thus a *first order object*, 'FOO'). FOOs are objects, not constructions. Simple concepts identify (= construct) FOOs directly. Not taking into account variables (which depend upon a valuation) all other ways of conceptually identifying FOOs are compositions and closures, i.e., complex constructions that use various distinct concepts to get the respective FOO.

We have seen that the concepts expressed by simple expressions are frequently complex since simple expressions are frequently *abbreviations* (see also [Materna 2000]). The process of introducing new abbreviations can be studied by theoretical linguistics; it is – as a historical and hence an empirical phenomenon — outside LANL but LANL takes the fact of abbreviation into account. From our viewpoint the linguistic definitions that realise the abbreviations use as *definiens* an expression whose ontological counterpart is an ontological definition of the object (see 2.6). The ontological definition — as a non-simple concept — contains other simple concepts as its components. Theoretically the following case is possible: within a particular **CS** there are two or more concepts that are equivalent, i.e., construct one and the same object. On the linguistic level, this corresponds to the case where we can formulate two or more variant (but equivalent) definitions. (Bealer would rightly say that these definitions determine distinct concepts, see [Bealer 1982].) Yet a more interesting case can occur: Imagine a **CS** whose **PC** contains a concept ^{0}A and whose **DC** contains a (complex) concept B equivalent to ^{0}A among whose components ^{0}A does not occur. (This can happen even in the independent **CS**s (see 2.5).) How would we decide whether B is equivalent to ^{0}A?

This question is answered as follows: We *know* that the B is equivalent to ^{0}A, since

i) '^{0}A' (i.e., our artificial name of that construction) contains 'A', which is a name of an object *in our (background) natural language*,

ii) all the simple concepts that are components of B are also given *via* names in our language,

iii) the procedure given by the complex concept B is unambiguously given; thus

iv) we know the object constructed by ^{0}A and the object constructed by B and can see whether the object is the same in both cases.

On the other hand, logic — with its model-theoretical and proof-theoretical methods — cannot decide such equivalences where at least one side of the equivalence relation is (given by) a simple concept. Logically relevant properties of and relations between *complex* concepts (e.g., analyticity, entailment) are, in general, not inaccessible to logic. To exemplify our suggestions consider the following examples that should elucidate the classical problem of the relation between logical truth and analyticity.

Consider and compare following two sentences:

a) *Those individuals that are mammals and water animals are (belong to) mammals.*

b) *Bachelors are (belong to) men.*

Both sentences are analytically true. Our conceptual analysis gives us (we use here set-theoretical concepts without loss of generality, $\cap$ / ((οι)(οι)(οι)), $\subset$ / (ο(οι)(οι))):

a') $\lambda w \lambda t\ [^0\subset [^0\cap\ ^0\text{Mammal}_{wt}\ ^0\text{Water\_animal}_{wt}]\ ^0\text{Mammal}_{wt}]$

b') $\lambda w \lambda t\ [^0\subset\ ^0\text{Bachelor}_{wt}\ ^0\text{Man}_{wt}]$

Instead of a'), b') we could write

a'') $\lambda w \lambda t\ [^0\text{belong\_to}\ [^0\text{and}\ [^0\text{Mammal}_{wt}\ ^0\text{Water\_animal}_{wt}]\ ^0\text{Mammal}_{wt}]$,

b'') $\lambda w \lambda t\ [^0\text{belong\_to}\ ^0\text{Bachelor}_{wt}\ ^0\text{Man}_{wt}]$.

We have chosen a'), b'): as speakers of English we know that *and* and *belong to* behave here as $\cap$, $\subset$, respectively. Thus the analyticity of a) can be discovered by logic; here we can ignore the denotations of *mammal* and *water animal* — these expressions play the role of predicate symbols that can be interpreted in any way without affecting the truth-value of the sentence.

The analyticity of b) cannot be discovered by Logic if its analysis is b') (or b'')). The reason is that *bachelor* as well as *man* are extra-logical words (unlike *and* and *belong to*), i.e., their interpretation is variable. Thus if we choose (in some WT-pair) an interpretation that *bachelor* denotes, e.g., a class {a, b, c} and *man* denotes a class {c,d}, the resulting truth-value will be **F**.

The distinction between logical truth and analyticity is standardly defined as follows:

A sentence is *logically true* iff its truth-value is **T** in all interpretations that preserve the meanings of all logical words (connectives, quantifiers, identity).

A sentence **A** is *analytically true* iff its truth-value is **T** in that interpretation that preserves the meaning of all subexpressions of A.

Thus logically true sentences are analytically true but not *vice versa*. For if a sentence is logically true, then it is true in any interpretation (that preserves the meanings of logical words), and among such interpretations there will also be that one that preserves the meanings of all its subexpressions. On the other hand there are sentences (like b)) that are true in that interpretation that takes into account the meanings of all subexpressions and false in some other interpretations, including those ones where meanings of the 'logical' words are preserved.

This distinction is exemplified above (sentences a), b)). From the viewpoint of our conceptual analysis we can say something more.

First, any case of trivialisation of a first-order object (see Definition 1 and 2) exemplifies what we have said about the *presence of natural language*: trivialisation of such an object constructs *what is denoted by the name X in* ${}^0X$. (A banal warning: trivialisation does *not* construct this name! See also the *sky-blue* vs. *azure* example in 2.1.) So the simple concepts in our analyses are intelligible because of the banal fact that we understand expressions of our background language.

Second, the way to reconcile analyticity with logical truth has to be the way of transition from *an analysis* to *the analysis* (see Intermezzo: Parmenides Principle) based on finding more and more fine-grained concepts and thus on discovering such CSs that are relevant w.r.t. the given problem and make it possible to (ontologically) *define* objects given originally by simple or at least more coarse-grained concepts**.**

In our example, let us refine the concept ^{0}bachelor as follows:

$${}^0\text{bachelor} = \lambda w\lambda t\, [{}^0\cap\, [{}^0\text{Man}_{wt}\ {}^0\text{Unmarried}_{wt}]$$

Then we can replace b') by

c') $\lambda w\lambda t\, [{}^0\subset [{}^0\cap\, [{}^0\text{Man}_{wt}\ {}^0\text{Unmarried}_{wt}]]\ {}^0\text{Man}_{wt}]$.

This time logic can check that b) is logically true.

But what if we want to know whether the sentence

d) *Bachelors are adults.*

is analytic. Again, due to the banal fact that we understand the word *bachelor* we can immediately say: "Yes, this sentence is analytic", but logic is again silent. This time c') brings no solution either. But let us refine the concept ^{0}man as follows:

$${}^0\text{man} = \lambda w\lambda t\, [{}^0\cap\, [{}^0\cap\, [{}^0\text{Human}_{wt}\ {}^0\text{Male}_{wt}]\ {}^0\text{Adult}_{wt}]];$$

our analysis of d) is then

d‘) $\lambda w \lambda t\ [^0\subset [^0\cap [^0\cap [^0\cap [^0Human_{wt}\ ^0Male_{wt}]\ ^0Adult_{wt}]]\ ^0Unmarried_{wt}]\ ^0Adult_{wt}]$,

which means that logic can do its work again.

We know that Carnap tried to solve our problem *via* introducing *meaning postulates.* Quine’s well-known criticism of Carnap’s attempt is justified in some respects but independently of this criticism we can offer a more fundamental solution. In this connection we should cite Jackendoff, who in his [1995, in particular 38-41] criticises Fodor’s claim that all concepts are simple, and shows that

> a meaning postulate approach to inference either misses all generalizations across inferential properties of lexical items or else is essentially equivalent to a decomposition theory.

What is then a ‘decomposition theory’? We could say that it is a linguistic counterpart of the above conceptual (objectual) theory of conceptual refinement. So on the linguistic level it is said:

> The problem of lexical decomposition, then, is to find a vocabulary for decomposition that permits the linguistically significant generalizations of inference patterns to be captured formally in terms of schemas ...

(see [Jackendoff 1995, 39])

It is plausible to assume:

*If a natural language sentence gets the analysis in the sense of our explication of the Parmenides Principle and the respective **CS** is no longer decomposable, then every claim entailed by the sentence can be logically justified.*

This *principle of detecting all logical consequences of a sentence* works however with a somehow unclear (albeit intuitively perhaps intelligible) term *decomposable **CS***. Now we will try to characterise more precisely what we mean by *decomposing a **CS***.

End of Intermezzo

3.3 Decomposition

In 2.5 we have defined dependence of concepts. Now we have to introduce a more precise notion of dependence, which will be relevant w.r.t. the notion of decomposing empirical conceptual systems.

Definition 34 (*empirical content*)
The *empirical content of a concept C* (EC_C) is the set of all empirical subconcepts of C (see Definition 4a). –

Definition 35 (*dependent concepts, comparable concepts*)
Let C, C' be concepts. C' is *dependent on* C iff $EC_C \subset EC_{C'}$.
C' is *comparable with* C iff $EC_C \cap EC_{C'} \neq \varnothing$. –

We can formulate some easily provable claims:

i) If C' is dependent on C, then C, C' are comparable. (Not *vice versa*!)

ii) All members of $\mathbf{DC_{CS}}$ are dependent on some members of $\mathbf{PC_{CS}}$.

iii) All members of $\mathbf{DC_{CS}}$ are comparable with some other members of $\mathbf{DC_{CS}}$.

iv) No member of $\mathbf{PC_{CS}}$ is dependent on any other concept.

v) Every member of $\mathbf{PC_{CS}}$ is comparable with some members of $\mathbf{DC_{CS}}$.

vi) No member of $\mathbf{PC_{CS}}$ is comparable with any other simple concept.

vii) Dependence is antisymmetric; therefore it induces partial ordering of concepts.

viii) Comparability is reflexive, symmetric but not transitive.

(Ad vii): Let *c, c'* be variables ranging over concepts. A formulation of vii) — using a symbolism of predicate logic — is:

$$\forall cc' \, (EC_c = EC_{c'} \supset c = c')$$

It could seem that the Bolzanian examples like the pair FATHER_of_MOTHER and MOTHER_of_FATHER refuted vii), since the respective concepts are distinct and the empirical content seems to be the same. Yet the equality of the empirical contents would obtain only if empirical content contained only *simple* concepts (as it is obviously presupposed by Bolzano).

Point iv) is the core of all problems connected with trivialisation. A $\mathbf{CS}_2$ could be conceived of as a *decomposition* of a $\mathbf{CS}_1$ if some of its primitive concepts were dependent on some (distinct) primitive concepts of $\mathbf{CS}_2$, which is impossible due to point iv). Thus the following attempt to define decomposition fails:

***Definition** (*decomposition,* abortive)
A $\mathbf{CS}_2$ is a *decomposition of* a $\mathbf{CS}_1$ iff (*c, c'* range over concepts; a simplified notation)

$$\exists cc'(c \in \mathbf{PC_{CS1}} \wedge c \neq c' \wedge c' \in \mathbf{PC_{CS2}} \wedge c \text{ is dependent on } c') –$$

Our next attempt must take into account point iv); before we try again, let us once more consider the character of trivialisation.

Decompositions discover the ('hidden'?) procedure that is presupposed by trivialisation. Thus what *seemed* not to need other concepts *actually used* them. Hence decomposition is a sort of *discovery*. We are able to describe it in virtue of the fact that we use a (background) natural language. Our second attempt to define decomposition will exploit this fact; this time we cannot use the notion DEPENDENT (point iv)!) but *equivalence of concepts* (see Definition 17) will serve our purpose.

Definition 36 (*semi-dependence*)
c semi-depends on c' iff $\exists c''$ (*c* is equivalent to *c''* and *c''* is dependent on *c'*) –

Definition 37 (*decomposition*)
A $\mathbf{CS}_2$ is a *decomposition of* a $\mathbf{CS}_1$ iff

$$\exists cc' \, (c \in \mathbf{PC}_{\mathbf{CS}1} \wedge c \neq c' \wedge c' \in \mathbf{CS}_2 \wedge c \text{ semi-depends on } c') \text{ –}$$

Now we will illustrate the above definitions by an (artificial) example.
Let the $\mathbf{PC}_1$ be

$$\{{}^0\text{father}, {}^0\text{mother}, {}^0\text{son}, {}^0\text{daughter}, {}^0\text{husband}, {}^0\text{wife}, {}^0\text{brother}, {}^0\text{sister}\},$$

and let the $\mathbf{PC}_2$ be

$$\{{}^0\text{male}, {}^0\text{parent}, {}^0\text{spouse}\}.$$

Here an even stronger condition than that required by Definition 37 is fulfilled:

$$\forall c \, (c \in \mathbf{PC}_{\mathbf{CS}1} \supset \exists c' \, (c \neq c' \wedge c' \in \mathbf{CS}_2 \wedge c \text{ semi-depends on } c'))$$

Thus, for example,

$$[{}^0\text{Equiv} \; {}^{00}\text{father} \; \lambda w \lambda t \; \lambda xy \; [[{}^0\text{parent}_{wt} \, xy] \wedge [{}^0\text{male}_{wt} \, x]]]$$

constructs **T**, and

$$\lambda w \lambda t \; \lambda xy \; [[{}^0\text{parent}_{wt} \, xy] \wedge [{}^0\text{male}_{wt} \, x]]$$

depends, of course, on ^{0}parent and on ^{0}male.

This solution is viable: we cannot prove that $EC_C \subset EC_{C'}$ if C' is simple. On the other hand, equivalence can be checked due to translatability to our background language.

Thus the transition of a **CS** to a decomposed **CS'** means that at least some primitive concepts of the **CS** get an ontological definition in the **CS'** (see Definition 22).

Again, we have to stress that to define decomposition of a system is not to say that the languages of (empirical) scientific theories develop just in this way, i.e., by transitions to more and more decomposed conceptual systems. We have to take into account the cases where the primitive concepts underlying one language are replaced by *other* concepts that need not semi-depend on the original ones. Further, we have to take into account the frequent cases when the same expressions of a language are associated with distinct concepts.

Remark 1: There is a problem with the term 'expression'. To adduce an example, are we to say that the English term 'bank' is the same expression if it denotes a financial institution (or a respective building) and if it concerns that route along a river? We have accepted this viewpoint when defining homonyms, but perhaps it would be more precise if we admitted that there are *two* expressions here, since an expression is defined not only by its morphological form but also by its meaning. We will set aside this problem. –

Remark 2: One of the well-known cases where the same expressions are associated with distinct concepts is the discovery of non-Euclidean geometries: it should be clear that, e.g., the concept of Euclidean parallels is another concept than the concepts of Riemannian or Lobatschewskian parallels. Many misunderstandings — especially occurring in philosophy — would disappear. –

3.4 Incomparable conceptual systems

The core of the problems with incommensurability is — from the logical viewpoint — the problem of *incomparable conceptual systems,* which can become a sufficient condition of incommensurability based on such systems. Here we will concentrate on *incomparable empirical* ***CS****s.* (See Definition 35.)

First of all, incomparability of *concepts* is defined by Definition 35: Two concepts C, C' are *incomparable* iff $EC_C \cap EC_{C'} = \varnothing$.

Remark: In her [1967] R. Kauppi elaborated an analysis of concepts that is probably the most modern articulation of extensional (and traditional) conception. Kauppi defines incomparable concepts as follows (see p. 38):

> Zwei Begriffe heissen miteinander *vergleichbar* ... wenn beide wenigstens einen eigentlichen Begriff als ihr gemeinsames Merkmal enthalten.
>
> ('eigentlich' means *non-empty*, see p. 12.)

Thus to be comparable means to share at least one *Merkmal,* i.e., obviously, at least one member of the empirical content (see Definition 34). Yet our theory differs from that of Kauppi's in at least one essential point.

To see this let us consider Kauppi's example (*ibidem*). According to Kauppi, the concepts RED and YELLOW are comparable because they share the concept COLOURED. But how do we know that they share this concept? This is not clear in Kauppi: she introduces a notion of conceptual systems but her notion differs from ours; it is based on an intensional (in Kauppi's sense) relation *containment* ('*Enthalten'*). Using our terms we can obtain Kauppian conceptual systems like

{ DOG, ANIMAL, LIVING BEING, ...},

or some more complex systems (p.72f), obeying some axioms concerning properties of comparability, compatibility etc.; obviously, Kauppi's criteria of creating conceptual systems are distinct from ours. The reason is that although Kauppi strives for making concepts structured she cannot do so: her systems are based on an extensional (in our sense) conception of concepts, and the components of her concepts are actually members of their content ('intension'); the general notion of *construction* is what is needed. Our concepts-constructions are much more similar to Bolzanian concepts (as what links together the members of the respective content).

All the same, the idea of ordering concepts according to their empirical contents (Definition 35) is well realisable in our theory (see [Duží 2003a]). –

Our question is now: When can we say that two **CS**s based on *the same preconcepts* are incomparable?

True, the phenomenon of incomparability of *concepts* is mostly innocuous: most members of a **CS** are mutually incomparable without any influence on the problem of incommensurability. Consider, e.g., our example $\mathbf{PC}_2$, where the concept

$$\lambda w \lambda t\ \lambda x\ [{}^0\exists \lambda y\ [{}^0\wedge\ [{}^0\text{parent}_{wt}\ xy][{}^0\text{male}_{wt}\ x]]],$$

i.e., a concept of the property *being a father*, is incomparable, say, with the concept ^{0}spouse. Hence we have to be very careful when defining incomparability of conceptual systems so that the definition is relevant w.r.t. the problems with incommensurability.

These problems are always articulated in connection with various stages of the development of a scientific theory. (The most frequently adduced examples are mechanics — Newton vs. Einstein, impetus vs. momentum — chemistry — phlogiston vs. oxygen — classical physics vs. general relativity / quantum mechanics; see [Sankey 1997, 108].) A consequence of incommensurability is:

Let ***T***, ***T’*** be *incommensurable* theories. Let T, T’ be the sets of sentences that express, respectively, ***T***, ***T’***. *Then at least one of T, T’ is not translatable into the other.*
(Do not forget that *theory* has been construed here as a subset of a **CS**.)
For, suppose that T would be (correctly, of course) translatable to T’ and *vice versa.* Then the notion of incommensurability would be entirely obscure.

One of the consequences of mutual translatability of both T, T’ would be that the *area* of ***T***, ***T’*** (see 2.5, point 2), as well as their *expressive power* (see Def 30) would be the same. All examples of incommensurable theories seem to demonstrate that something (area, expressive power) has changed during the transition from a theory to its incommensurable counterpart: it seems that, e.g., theories of relativity are able to pose problems that cannot be posed by Newton’s physics. Our artificial ‘bink – cink’ example shows that this phenomenon can be demonstrated at least in simple cases.

To sum up, if two theories are mutually translatable then no progress in posing problems can be expected.

Now the questions arise:

Is the problem of incommensurability connected with incomparability of conceptual systems?
How can the incomparability of conceptual systems be defined?

The first question, cannot be answered without answering the second. Let us try.

Here we suppose that the preconcepts are shared. So we set aside cases like comparing **CS**s that are based on distinct types (thus, e.g., arithmetic of natural numbers vs. thermodynamics). Now take, e.g., the case where the new concept RELATIVISTIC_MASS has been introduced to physics. There are two options here. Either this is a *primitive concept* or it has been derived from primitive concepts. Let us suppose that the latter case is a case of EE, so that both options lead to the consequence that there is a *new primitive empirical* concept in the Einsteinian **CS**, new w.r.t. the original, say, Newtonian system; then there is no concept in the old system that would be comparable with this new concept. So we can generalise and formulate the required definition.

Definition 38 (*incomparable **CS**s*)
Conceptual systems **CS** and **CS**' are *incomparable* iff at least one of their members is incomparable with all concepts of the other. –

That such a case is in a sense anomalous can be seen from the point v) (following Definition 35).

It is clear that in the case of incomparability a concept that is incomparable with all concepts of the other **CS** is necessarily a member of **PC**. In our artificial example, there are no concepts in the poorer system that would contain *cink* or *cank.* Thus the two systems are incomparable according to Definition 38. From another viewpoint (in)comparability can be defined in another way: for example both *cink* and *cank* share *bink* as a requisite (see Definition 28 and [Duží 2003a]).

Incomparability is irreflexive and symmetric (and so cannot be transitive).

Let $\mathrm{INTAR}_{\mathbf{CS}}$ be the subset of the area of **CS** (see 2.5, point 2) that contains just intensions. Let the systems **CS** and **CS'** be such that the intersection $\mathrm{INTAR}_{\mathbf{CS}} \cap \mathrm{INTAR}_{\mathbf{CS'}} = \varnothing$ or is at least very small. Incomparability of such systems is obvious and trivial. (As an example consider some conceptual systems that underlie a part of physics and a part of biology.) The fact of incomparability of conceptual systems is interesting only in two cases:

a) The areas of the two systems essentially overlap

b) The systems underlie distinct stages of the development of one and the same science.

(Clearly, b) is a special case of a).)

Let us consider case b); we consider empirical **CS**s only.

When can we say that two or more **CS**s underlie *one scientific discipline* during its development? One possible answer would be: Let **CS**, **CS'** be two conceptual systems. If the $\mathrm{INTAR}_{\mathbf{CS}}$ is the same as $\mathrm{INTAR}_{\mathbf{CS'}}$ or if the former is a subset of the latter, then we say that **CS** and **CS'** underlie two stages of one and the same scientific discipline. Thus physics, for example, can be said to be one and the same discipline even when the areas of its conceptual systems obviously increase during its development. (The INTAR of our artificial example with *bink* is also a subset of the INTAR of the system with *cink, cank*, assuming that the latter contains, e.g., a concept of disjunction.) The problems with *incommensurability* obviously concern just this occurrence of *incomparability* of conceptual systems (on the linguistic level: incommensurability of theories). How should incommensurability be analysed (i.e., defined) from our viewpoint (from the viewpoint of logic, in particular of TIL and our theory of conceptual systems)?

Here we must make a choice: the term ‘incommensurability’ is not unambiguous. Newton-Smith in his [1981, p.149–150] adduces two spurious sources of incommensurability, viz. incommensurability due to value variance and incommensurability due to radical standard variance. These two possible interpretations we will dismiss here (as Newton-Smith does) and will take into account only incommensurability due to radical meaning variance.

Yet speaking about meaning variance we presuppose that some terms have changed their meaning so that they have become ambiguous. A new concept has not been associated with a new term. Newton-Smith’s example: Newton’s *mass* term has another meaning in the relativistic mechanics. If we claim “mass is invariant”, and then “mass is not invariant” we do not necessarily claim a contradiction — this is immediately clear if we write $mass_1$, $mass_2$ instead of *mass* in the first, second claim, respectively.

Let us model this situation by slightly changing our artificial ‘bink’ example. So let us write *bink* in $\mathbf{V}_2$ instead of *cink*, other entries unchanged. Let the resulting vocabulary be denoted by $\mathbf{V'}_2$. Thus the original *bink* denotes now meadows, whereas *cank* denotes pasturelands. Now *Loc bace Lok* is true in $\mathbf{V}_1$ and false in $\mathbf{V'}_2$. Moreover, *E bink Lok* is true in $\mathbf{V'}_2$, false in $\mathbf{V}_1$ but not translatable from $\mathbf{V'}_2$ to $\mathbf{V}_1$. Also, our example against Field, viz., *There are n surfaces bink*, is — under normal circumstances — true (for some *n*) in $\mathbf{V}_1$ and false in $\mathbf{V'}_2$ or true in $\mathbf{V'}_2$ and false in $\mathbf{V}_1$; it will possess the same truth-value in both vocabularies only in such worlds+times where either there are no pastures or there are no meadows (or, of course, there are neither pastures nor meadows).

A question arises: if the situation with $\mathbf{V}_1$, $\mathbf{V'}_2$ models in a simplified way incommensurability, can we model the incommensurability case also in the situation with $\mathbf{V}_1$, $\mathbf{V}_2$? Let the latter situation be denoted by S1 and the former by S2. S1 differs from S2 by not being — at first sight — connected with an equivocation. All the same, one point is common to both S1 and S2: the (fragments of the) underlying **CS**s are incomparable in the sense of Definition 38. The *cink* in S1 is incomparable with any concept from $\mathbf{V}_1$, which also holds of the *bink* in $\mathbf{V'}_2$ in S2. Incommensurability in the former case means that sentences that contain *cink* cannot be translated to $\mathbf{V}_1$ (which should model the earlier stage of the theory), which could suggest that the stage of the theory that corresponds to $\mathbf{V}_2$ does not make it possible to compare both theories. The latter case is more interesting: there it seems as if no development of the theory could be stated, since the new theory does not correct the claims made by the old theory — it simply formulates other claims that are incomparable (rather than incompatible) with the old claims.

Let us return to the question whether the transition from *bink* to *cink* in S1 or to *bink* in **V'**$_2$ in S2 has influenced a change of meaning of the word *bace*. It looks so, because — as we have already suggested — if *bace* did not change its meaning the new theory could be said to *correct* the old one: sentence 8 would be the negation of sentence 3. (An analogy in the case of sentences 2 and 4 does not hold because *cink* is incomparable with *bink*, and similarly so if *bink* would occur in the situation S2.) Yet we could insist on the view that the new theory *does not correct* the old one even if *bace* did not change its meaning. (We have shown that actually it *did not*: indeed, in translating *bace* into English we presupposed that predicating *bace* in some ⟨W,T⟩ pair about a pair of surfaces will give True just if the two surfaces are of the same kind. Now the transition to the new theory did not change this presupposition; what did change was only that due to the greater sensitivity of the new theory the (new) names of kinds of surface denoted sometimes other kinds than the old names.

To sum up: incommensurability is always connected with the incomparability of the respective conceptual systems. Therefore it cannot be claimed that *due to new concepts* the old theory is corrected by the new, 'incommensurable' theory. (The new theory can, of course, correct the old one — just like the hypotheses formulated in the stage of 'normal science', i.e., within one and the same conceptual system, can be corrected within this very stage — but such corrections will be independent of the fact that some incomparable **CS** has come into being and some untranslatable expressions have been added.)

So we can accept the phenomenon of incommensurability, both if new concepts are associated with new terms and if this is not the case. What we do *not* accept are some inferences from the fact that some stages of development of a theory are incommensurable. I mean such false inferences as some form of relativism. Sankey's [1997] and Newton-Smith's [1981] as well as many participants of the *Incommensurability (and Related Matters)* discussion have adduced strong arguments against relativism in this respect. Here I would like to stress only two points connected with our main topic, i.e., theory of conceptual systems.

First, our theory is obviously realistic and anti-holistic. This feature satisfies a necessary condition of overcoming relativistic hypotheses around the incommensurability phenomenon.

Another important feature of our theory is that it strives to be as fine-grained as possible; concepts as 'structured meanings' are good explanatory tools. This last point can be documented as follows:

To sum up: The fact — accepted here — that distinct stages of the development of a theory are incommensurable, so that the 'progress' of the theory cannot be defined simply as a correction of the preceding stages, is compatible with the fact that the progress can be stated using another criterion: over a wider area the new stage can pose (and dependently on reality solve) more problems. Schematically we can distinguish two forms (or 'degrees') of creative development of theories. The 'lower degree' consists in mathematically creative extensions (Definition 31'): new logico-mathematical tools are used; the 'IEE case'. The 'higher degree' consists in empirically creative extensions (Definition 31): new empirical concepts enrich the PC part of the given CS; the 'EE case'.

This hypothesis, whose plausibility has been argued for above, could have been articulated due to the way the concepts have been defined, since the connection between concepts and problems has been made explicit (which no set-theoretical conception of concepts could make possible).

3.5 Empirical vs. non-empirical

We have shown that distinguishing *empirical* and *non-empirical* concepts and **CS**s is important (for example when **IEE** and **EE** case are distinguished). Can the above scheme guarantee that this distinction is detectable?

Empirical concepts have been defined as those concepts that identify *non-trivial* intensions. Thus there are two kinds of non-empirical concepts: those ones that identify extensions and those that identify trivial intensions. Any non-empirical concept P of the former kind is automatically detected: if t in $P \rightarrow t$ is a type of some extension, then P is a non-empirical concept. However, not *vice versa*: t may be $\alpha_{\tau\omega}$ for some α but the respective P will construct a trivial (or an almost trivial) intension.

Remark: An intension is *almost trivial* iff its value is the same in all WT-pairs where it exists. See the Remark accompanying Definition 28. For details, see also [Duží 2003c]. –

Let us illustrate this case by an (artificial) example. Our **CS** will contain some logico-mathematical primitives and following empirical primitives (let I be some individual):

$$^0\text{mother} \rightarrow (\iota\iota)_{\tau\omega},\ ^0\text{father} \rightarrow (\iota\iota)_{\tau\omega},\ ^0\text{I} \rightarrow \iota,\ ^0\text{woman} \rightarrow (o\iota)_{\tau\omega}.$$

The respective **DC** certainly contains the concept

$$\lambda w\lambda t\ [^0\text{woman}_{wt}\ [^0\text{mother}_{wt}\ ^0\text{I}]],$$

say, D_i , which constructs the proposition *I's mother is a woman*. Clearly, $D_i \rightarrow o_{\tau\omega}$. We know, however, that the proposition is an almost trivial intension — its value is True in all WT-pairs where the mother of the respective individual exists. Is this fact detectable by the means that our **CS** has at its disposal?

On the one hand, possessing the above concepts means that the respective user should not admit the possibility of falsity of that proposition. On the other hand, this fact obviously holds because one of the *requisites* (see Definition 28) of the property *mother* is the property *female* or *not male*. In our **CS** this holds only implicitly: the **CS** does not contain the concept ^{0}female (or ^{0}male, which would suffice if **CS** contained a concept of negation).

Let **CS**' be a decomposition of **CS**, for example with following empirical primitives instead of the primitives above:

$$^0\text{human} \rightarrow (o\iota)_{\tau\omega},\ ^0\text{female} \rightarrow (o\iota)_{\tau\omega},\ ^0\text{parent} \rightarrow (o\iota\iota)_{\tau\omega}.$$

CS' is a decomposition of **CS** according to Definition 37. We know that ^{0}woman is equivalent to

$$\lambda w\lambda t\ \lambda x\ [^0\wedge\ [^0\text{human}_{wt}\, x][^0\text{female}_{wt}\, x]];$$

also, ^{0}mother is equivalent to

$$\lambda w\lambda t\lambda x\ [^0\iota\ \lambda y\ [^0\wedge\ [^0\wedge\ [^0\text{human}_{wt}\, y][^0\text{female}_{wt}\, y]]\ [^0\text{parent}_{wt}\, yx]]].$$

True, when we say that $\lambda w\lambda t\lambda x\ [^0\wedge\ [^0\text{human}_{wt}\ x][^0\text{female}_{wt}\ x]]$ is an ontological definition of the property whose linguistic *abbreviation* is the word 'woman', and when we say the same of $\lambda w\lambda t\ \lambda x\ [^0\iota\ \lambda y\ [^0\wedge\ [^0\text{human}_{wt} y]\ [^0\wedge\ [^0\text{parent}_{wt}\ yx][^0\text{female}_{wt} y]]]]$ and the word 'mother', it seems a very unnatural claim; we usually learn first the word 'mother' — the phrase 'the human female parent' comes much later, similarly for 'woman'. So it looks as if we claimed that we first learn abbreviations and only afterwards what has been abbreviated. This would be absurd, of course, but we do not intend to describe the temporal course of the learning process. (See, however, the Remark below.) What is important here are the logical relations between such expressions like 'woman' and 'female human' etc. The direction of some learning process is not relevant; after all, we hardly can speak about *the* direction — sometimes we go 'from abbreviations' (like here), sometimes 'to abbreviations' (like when we learn some professional definitions).

From the viewpoint of **CS**' the sentence 'I's mother is a woman' can be analysed as follows:

$$\lambda w\lambda t\ [\ [\lambda w\lambda t\ [\lambda x\ [^0\wedge\ [^0\text{human}_{wt}\, x][^0\text{female}_{wt}\, x]]]]_{wt}$$
$$[\lambda w\lambda t\ [^0\iota\ \lambda y\ [^0\wedge\ [^0\wedge\ [^0\text{human}_{wt}\, y][^0\text{female}_{wt}\, y]]\ [^0\text{parent}_{wt}\, y\ ^0\text{I}]]]]_{wt}\ ],$$

or, using abbreviated infix notation:

$$\lambda w\lambda t\ [\ \lambda x\ [[^0\text{human}_{wt}\, x] \wedge [^0\text{female}_{wt}\, x]]\ ^0\iota y\ [[^0\text{human}_{wt}\, y] \wedge [^0\text{female}_{wt}\, y] \wedge [^0\text{parent}_{wt}\, y\ ^0\text{I}]]\ ]$$

Standard logic immediately discovers the trivial character of the constructed proposition. (See Intermezzo in 3.2.4.2.b).)

The transition from one **CS** to another one, that is a *decomposition* of **CS,** is at the same time a transition from a system whose concepts are connected with their *requisites* only implicitly to a system in which the requisites of the old concepts are discovered and made explicit, being themselves concepts of the new system.

Remark: The seeming absurdity of the idea that a child first learns concepts underlying some *abbreviations* and only later the concepts in terms of which the objects identified by these primarily learned concepts are defined can be now explained away. The new interpretation could be: not *first abbreviations* but *first requisites implicitly* (*later explicitly*). The concept ^{0}mother is, of course, connected with its requisites from the very beginning but when a child knows only this concept not knowing concepts like FEMALE, PARENT etc. then it means only that his or her learning language (and thus the respective **CS**(s)) is connected with verbally fixed discovering requisites of the originally learned concepts, so with *semantic dependencies of particular expressions of the respective language*. –

Similarly what is *semantically nonsense* need not be explicitly checked in conceptual systems that do not take into account requisites of the respective intensions. Consider the following sentence:

‘Some tables speak English’

Grammatically and type-theoretically the sentence is correct. It is nonsensical in the semantic sense; *table* as well as *speak English* are properties of individuals but among the requisites of *table* there is the property *not being a living creature* and among the requisites of *speak English* there is the property *being a living creature*, so what we feel to be semantically nonsensical gets the form ‘to be inconsistent’. This remains implicit unless the respective conceptual system contains the concept BEING A LIVING CREATURE.

There are, however, nonsensical sentences in a more radical sense (cf. *Wiener Kreis*). As an example we can adduce

‘Some numbers are red’

Why do we feel that this sentence is ‘more nonsensical’ than the preceding one?

This intuition can be perhaps supported by the fact that we could somehow imagine a table that would speak whereas there is no way to *imagine* a red number.

In TIL the preconcepts are based on atomic types among which two (disjoint) types are ι and τ. According to Definition 4 the above sentence cannot be analysed: *red* is a property of individuals whereas the variables ranging over numbers *v*-construct τ-objects. Thus in TIL (given the distinct preconcepts ι and τ) *the sentence does not say anything*.

Imagine, however, a **CS** whose preconcepts would not distinguish between numbers and individuals. The fact that a separate type for numbers would be missing would have to be compensated by activating a requisite of *red*, viz., e.g., *not being a number*. As soon as this would be done our sentence could be analysed and shown to be a contradiction — so its nonsensical character would be on the same level as our first example. By the way, since this result is not in harmony with our intuition we can see that the preconceptual distinction between ι and τ (as in TIL) is justified.

In any **CS** where some members of $\mathbf{DC_{CS}}$ construct some requisites of some concepts from **CS** we should *mark* the pairs of concepts standing in the requisite relation. The constructions of the form $[{}^0Req\ {}^0C_1\ {}^0C_2]$, where C_1, C_2 are members of such marked pairs, can be associated with any **CS** that contains the marked pairs; they are not members of some part of the given **CS**, they are above such a system and can be said *to be accepted* by the respective **CS** — independently of any *theory* that may be a part of it: they would rather correspond to Carnap Carnap Carnap's *meaning postulates.* So we could say that a **CS** together with its marked pairs (that determine the requisite claims) defines *which propositions constructed by its members are analytic;* derivatively, it defines *which sentences of a language based on it are analytic.* See also the *Relativity of analyticity* claim in the next paragraph.

3.6 Comparing conceptual systems

Let us consider following **CS**s:
CS1:

$$\langle \{P_1 \rightarrow t_1, ..., P_k \rightarrow t_k\}, V, Triv, Comp, Clos, HDef \rangle$$

with t_i from T,
and **CS**2, **CS**2'

$$\langle \{P'_1 \rightarrow t'_1, ..., P'_m \rightarrow t'_m\}, V, Triv, Comp, Clos, HDef \rangle$$

with t_i from T',
where a) for some or all *i,* $P_i \neq P'_i$ and T = T' (**CS**2)
b) as above, but T ≠ T' (**CS**2')

Our question is: To what extent are we justified to claim that the areas of **CS**1, **CS**2, **CS**2' are the same, or disjoint, or overlapping?

Let us begin with the more simple case of comparing **CS**1 with **CS**2. These systems share at least their preconcepts. All the same the comparison is difficult, since at least some primitives of both systems are incomparable (Definition 35). Is there any way out?

At least in some simple cases one way out can be easily found: see 3.2.4.2.b), Intermezzo. All simple concepts in such simple systems identify objects that the user of the respective *natural (ordinary) language* knows. Therefore, the areas of the respective systems are accessible to the (ideal) user of this background language.

What about the pair **CS**1, **CS**2'? Here even the *preconcepts* are distinct. Imagine the case where a type in **CS**1 — unlike in **CS**2' — is ι and a type in **CS**2' — unlike in **CS**1 — is, say, π, i.e., the set of 'material points' (which is thinkable in some **CS** for a fragment of physics). The propositions that are conceptually identified in **CS**1 can be — let us admit — denoted by some *sentences* of the natural (ordinary) language. Similarly some sentences of the natural language will denote the propositions that are conceptually identified in **CS**2'. Let the set of the former sentences be denoted by *S1* and the set of the latter sentences by *S2*. Not all members of one of the sets *S1, S2* are translatable to the members of the other set. We assume, however, that all members of both sets are translatable into the given natural language. So it looks like the following situation: in order to swallow both *S1* and *S2* the given natural language accepts both **CS**1 and **CS**2', i.e., their union. The areas of both systems should then be accessible for such a stage of a natural language.

Unfortunately, things are never that simple.

Our solutions presuppose that expressions of scientific theories may be mutually untranslatable but that they all are translatable into natural language(s) (see [Sankey 1997]).

The problem is, however, that natural languages can be construed either as languages that accept every sublanguage including professional jargons, or else as ordinary languages (see 3.2.4.2). In the former case our solutions are no solutions at all: mechanically adjoining any professional sublanguage to the preceding stage preserves the phenomenon of untranslatability. In the latter case the translatability is an illusion only: take, e.g., a theory of subatomic particles and the concept, say, *spin. We can say that ordinary English has the expression 'spin', which should denote the entity identified in an exact way by the concept *spin. But any speaker of English who is not a physicist and says that (s)he understands this expression obviously lies or is simply mistaken. Thus it seems that comparing conceptual systems of the kind **CS**1 and **CS**2' (maybe even **CS**2) is connected with essential problems.

But the presence of these problems is not surprising. When people say that they do not understand what the particular (notably the exact) sciences talk about it is not primarily because people, in general, have not learned the claims made by such sciences: they have not mastered the respective sublanguages and so the respective concepts. And mastering a scientific language means reconciling ourselves to at least partial untranslatability also w.r.t. ordinary language.

Imagine the situation where a man A, who has learned quantum physics tries to explain some claims made by this science to B, who does not understand its basic terms. It seems that the only way — if the explanation should be real, not metaphorical — consists in making B *learn the language* of quantum physics. There is an analogy with somebody's learning another natural language. True, the translatability of the new language to the, say, mother tongue is mostly unproblematic, but first, also in this case it surely sometimes happens that an exact translation is impossible —imagine English vs. Chinese—and, most importantly, *learning foreign languages is independent of whether the learned language is translatable into the mother language.* As Sankey in his [1997, p.89] says:

> Bilingual speakers do not translate 'in their heads' while conversing in a foreign language, so a bilingual may understand a foreign expression not translatable into his home language.

This is the key point.

Its consequence is: The particular scientific sublanguages of a natural language L need not be translatable to L. The resulting *pluralism* of conceptual systems is compatible with using L on the one hand and professional sublanguages on the other hand.

Summing up: *All distinct conceptual systems cannot be compared w.r.t. their areas. The systems incomparable in this sense induce radically distinct classifications of objects*.

Are we bound therefore to accept the relativistic interpretation of untranslatability, incommensurability etc.? Not at all.

The problems with evaluating (various stages of) theories with respect to progress or verisimilitude (and so the problems with incommensurability) always concern the development of a scientific discipline. These problems never arise when observing radically distinct disciplines: we never ask whether quantum physics or biology is closer to the truth or more progressive. But comparing various stages of development of one and the same discipline is not connected with the *radically* distinct classification of objects, at least (but not only) the preconcepts are the same during the respective development, or perhaps the set of preconcepts is enlarged.

Remark: The later stages of the development of a discipline may change the earlier classification of objects, but in an obvious sense we would not call such a change 'radical'. To illustrate this claim imagine the stage of astronomy where the concept PLANET, say *P, identified such a property that its value in the real world was the set {Sun, Mercury, Venus,...,...}(some of which were not yet known). In the later stages of astronomy two changes are observable: *P changed to *P', which caused that its value in real world was {Mercury,...}, i.e., Sun ceased to be a planet. Second, some other members of the set (being the members independently of the given state of the knowledge, of course) have been discovered. The former change can be characterised as a change of the original classification of objects but we will hardly call such a change 'a radical change'. –

Let us now return to Putnam's criticism of the incommensurability thesis (see [Sankey 1997, p.83], where the core of this criticism is quoted):

> [I]f this thesis were really true then we could not translate other languages — or even past stages of our own language — at all. ... [t]he members of other cultures,

> including seventeenth-century scientists, would be conceptualizable by us only as animals producing responses to stimuli... [T]o tell us that Galileo had 'incommensurable' notions and then to go on to describe them at length is totally incoherent.

Sankey's response to this criticism is based on the idea that natural language is

> a conglomerate of terminologies or local idioms with special areas of application. Untranslatability between theoretical languages constitutes a relation between sublanguages within a total language. (*ibidem*, p.87)

Thus our claims concerning untranslatability of some sublanguages are not necessarily incoherent since the total language can serve as a metalanguage for those sublanguages.

Our artificial 'bink-cink languages' were an illustration of this idea. L_{T2} was not translatable into L_{T1} but this fact could have been formulated in English as a 'background language' since L_{T1} as well as L_{T2} were translatable into English.

We have however seen that in the real cases of scientific languages ordinary language cannot play the role of the metalanguage. Scientific jargons are based on such primitives and – do not forget – such sophisticated members of the $MATH_{CS}$ that the only way to master the respective jargon is to learn it *directly*, not by translating from the natural language in question.

Thus we have to live with various mutually irreducible conceptual systems, i.e., tools for defining objects *from various distinct points of view.* Indeed, the situation where our knowledge of the world is scattered and becomes more and more specialised can be described in terms of a theory of conceptual systems as follows:

Every concept—with the exception of strictly empty and (in a sense also) quasi-empty concepts —identifies (constructs) some object. Grouping concrete things according to various criteria is what the empirical concepts do. They construct intensions and in virtue of this the concrete things can be seen as bearers of individual roles, of properties and relations. Some empirical concepts construct propositions; these become true or false, which is dependent on the state of the world. People are — in general — interested in truth. Languages are codes of concepts (in general, of constructions — see [Tichý 1988, §44]). The way we usually say this is that concepts are meanings of linguistic expressions. Thus detecting truth happens due to the verification of sentences, i.e., due to the verification of propositions, i.e., due to concepts that construct propositions. The emergence and development of particular sciences means that our view of objects had to become more fine-grained, which leads to the origin of particular groups of concepts whose constructing potential defined what we have called *area*. (See 2.5, point 2.) The resulting pluralism of conceptual systems serves the pluralism of areas.

Every empirical (theoretical) discipline can be characterised by an *initial area*; perhaps it happens that the initial area is shared by more disciplines. Anyway, the development of a discipline can be conceived of as extending this initial area (or at least of a core thereof). This process of extending is realised *via* two forms: **I**n**E**ssential **E**xtension (**IEE**$_{(\mathbf{CS'}\ \mathbf{CS})}$) and **E**ssential **E**xtension (**EE**$_{(\mathbf{CS'}\ \mathbf{CS})}$): see Definition 26. The 'essent' root of these names should not be taken to signal a positive feature: it is rather a neutral characteristic. Indeed, in some cases **IEE** is clearly not very radical (when compared with **EE**) — remember our example with *wild cats*. On the other hand, considering such disciplines as theories of modern physics, where the role of the **MATH** part of the given **CS** is 'very essential', we have to state that the changes of the area caused by extending the **LOG** ∪ **MATH** part of the given **CS** can be surprisingly radical.

Remarks:

1. We use here the singular "the (given) **CS**", although the developing discipline is necessarily connected with various **CS**s. We can accept this way of speaking and support it by the following convention: a sequence of **CS**s that underlie particular stages of the development of one and the same (empirical) discipline can be called 'the **CS** underlying' this discipline.
2. One could ask: why is the **LOG** ∪ **MATH** part of a **CS** not included in the machinery of this **CS**? After all, the members of this part are typical *tools* neutral to the objects investigated. We can answer as follows: All (logical and) mathematical functions and constructions are objects that can be handled *via* constructions. Let **CS** be a conceptual system in the sense of Remark 1. Then we can state that various stages of **CS** contain various mathematical concepts, mostly their number increases. That part of **CS** which we want to call *machinery* is however the same in all the stages of development. Thus the core of machinery is made up of *detecting* (rather than *creating*) *functions* and *applying functions to arguments*. If the members of the (variable!) **LOG** ∪ **MATH** part of **CS** were part of the machinery then within one stage thereof we would have, e.g., an addition and subtraction operation, in some following stage the operations of multiplying and dividing would be added, later powers and extracting roots etc. etc. Now all these operations can be constructed *via* primitive or derived concepts and the constant machinery would take care of the rest. After all, once these mathematical objects are at our disposal their use is reducible to creating and applying functions. –

The particular areas are not absolutely isolated one from the other. We have rightly said that as for radically distinct disciplines we never ask which is 'more progressive', etc., but there is one moment that explains why the plurality of areas does not lead to atomised *membra disiecta* of our knowledge. Some disciplines are able to *explain* some claims made by other disciplines. Let us analyse this interesting phenomenon from the viewpoint of our theory of conceptual systems.

First of all one example (see [Rosenberg 2000, 76-77]):

> [t]he balanced equations of *chemical stoichiometry* (for example $2H_2 + O_2 \rightarrow 2H_2O$) are explained by assumptions the *chemist* makes about electron-sharing between hydrogen and oxygen atoms. But these laws, underived in chemistry, are the derived, explained generalizations of *atomic theory*. (Emphasise mine. – P.M.)

Thus we compare *chemical stoichiometry, chemistry, atomic theory.* Each of these disciplines is based on some conceptual system. At the same time, could we draw from the above quotation the conclusion that the sequence *chemical stoichiometry, chemistry, atomic theory* makes up *one discipline*, whose stages of development are the members of this sequence? If this were the case, then with chemistry every development of chemical stoichiometry would disappear, with atomic theory the same would hold for chemistry. Since this is not the case we can certainly claim that the members of the mentioned sequence are (in this sense) independent disciplines. What about the particular **CS**s?

Here we meet a difficulty, which Rosenberg (*ibidem*) from another angle characterises as follows:

> No one suggests that scientists actually present theories as axiomatic systems, still less that they explicitly seek the derivations of less fundamental laws from more fundamental ones. It is important to remember that ... the axiomatic account of theories is a 'rational reconstruction' of scientific practice designed to reveal its underlying logic.

An analogous consideration concerns the fact that no scientist explicitly reveals the primitive concepts of his/her discipline. Moreover, our rational reconstruction is complicated by the following:

a) Our explication of the term *concept* is not well known;
b) using such an explication is not a matter for (empirical) science, but for philosophical logic.

What can be done, then?

Certainly nothing what would try to describe the real process of developing particular disciplines. 'Laboratory', artificial assumptions are the only option that would remain in the framework of philosophical logic and be useful for a general theory of science as far as philosophical logic can be.

Let us return to the problem that could be formulated as follows:

Consider two **CS**s that underlie two distinct disciplines T1 and T2: **CS**1 and **CS**2. Let **A** be a general law-like claim accepted by T1. Our problem is:

What does it mean (in terms of our theory of conceptual systems) when we say that T2 offers an explanation of ***A****?*

(It suffices to presuppose that **A** is a *general* sentence; explaining particular events can be derived from explaining general claims *via* adding respective *boundary conditions.*)

Ignoring — for the sake of simplicity — some criticisms of the classical **D**(eductive) **N**(omological) model of explanation [Hempel 1965] (see, e.g., [Rosenberg 2000]) we will apply its scheme to solving our problem.

To make our solution easier suppose first that T2 developed from a later stage of T1. $T_{\mathbf{CS}1}$ contains concepts that construct accepted propositions of T1. Among these propositions is that one denoted by **A**. $T_{\mathbf{CS}2}$ contains concepts that construct accepted propositions of T2. Since T2 is later than T1 we can suppose that it is more fine-grained than T1; thus some (primitive) concepts of **CS**2 are decompositions of some concepts of **CS**1. A consequence thereof is that T1 is translatable to T2 but *not vice versa.* (Remember our laboratory example in 3.2.4.2.b, T1 and T2.) Even then the claims made by T1 can be derived from T2. To use our example mentioned above for illustration, we can formulate a correct derivation:

There are *m* surfaces cink
There are *k* surfaces cank
bink = cink or cank
$m + k = n$

$\therefore$ There are *n* surfaces bink

(Observe that the third premise, necessary for the derivation, 'or' exclusive, is realised due to the translatability from L_{T1} to L_{T2}. Since this translatability is one-direction only, no symmetric derivation is possible, i.e., from the fact that there are *n* surfaces bink nothing can be deduced concerning the number of cink and cank surfaces.)

Now if what is claimed by **A** is explained by T2, then using the DN model of explanation we get

$$\mathbf{V}^2$$
$$\therefore \mathbf{A}$$

where $\mathbf{V}^2$ is a sentence (perhaps a conjunction of sentences) accepted as a law (as laws) in T2.

This is the easier case; it can be characterised as a *fine-grained justification of a coarse-grained (or less fine-grained) claim.*

The case where T2 is simply *another* discipline is more complicated. Here we cannot suppose that the *area* of the respective conceptual systems is the same or at least that the two areas overlap. In general, $T_{\mathbf{CS}2}$ may contain other primitive concepts (or even preconcepts) than $T_{\mathbf{CS}1}$. As for the Rosenberg example cited above, we can ask: is this the case of chemistry and atomic theory? Of atomic theory and quantum theory?

A plausible working conjecture (at least for some cases):

Let **CS**1 contain primitives $C_1,...,C_k$, and **CS**2 (possibly incomparable, hence distinct) $D_1,...,D_m$. Among the members of $\mathbf{D}_{\mathbf{CS}1}$ there is a construction Φ (an ontological definition, see Definition 22) the simple subconcepts of which are members of $\{C_1,...,C_k\}$. It can happen that **CS**2 *adds* (see Remark 1 above) a new primitive D_{m+1} to its $\mathbf{P}_{\mathbf{CS}2}$ so that it holds

$$D_{m+1} = \Phi.$$

(A necessary presupposition is that the sets of the preconcepts of both systems at least overlap.)

This means that T2 speaks about an object that has been defined in T1 but ignores the way this object has been defined. *Now the necessary condition of communication between both systems has been fulfilled: both areas now overlap sharing at least the object identified by the simple D_{m+1} and defined by Φ.*

Now the explanation can be realised. Φ has defined an object, and the respective *theory* describes its behaviour in terms of **CS**1. An explanation of this behaviour is needed, be it a causal or another kind of explanation. The conceptual means of the theory T1 (i.e., the *theory* as a subset of **CS**1—see 3.2.4.2b, consequence b)) are not sufficient; the conceptual means of T2 suffice. Thus our **DN** model scheme (see above) can be used where **A** is the description of the behaviour of the object defined by Φ and $\mathbf{V}^2$ is some set of law-like sentences of T2 (the underlying set of concepts being a subset of the theory as a subset of **CS**2). Here we can presuppose that both theories are mutually untranslatable; all the same the explanation will work, since whoever uses this explanation will behave like a bilingual person: he will understand both languages directly, without translating.

This schematic conjecture obviously implies that a necessary condition for explaining some general claims of a theory T in terms of a theory T' is that the areas of the respective conceptual systems are not disjoint. This condition is, of course, not a sufficient condition.

We have already suggested that *real* examples that would exploit our theoretical framework are difficult to find (see the points a), b) above). We can only try to offer some schematic examples within our theoretical reconstruction. Thus consider the physiological concept (HUMAN) LIVER. We can suppose that a conceptual system of physiology (in a given time span) can be theoretically reconstructed. It is highly probable that within this system, say, $S_{ph,}$ the concept LIVER is an ontological definition, i.e., it is a complex concept rather than a simple one. Now imagine that we have a conceptual system of anatomy, say, S_{an}. Among the concepts of the latter system we can certainly find a concept LIVER', expressed by the same (English, German, etc.) expression, but this time we are justified in believing that

LIVER and LIVER' are equivalent but all the same *distinct* concepts. There is probably no need to take over the definition from S_{ph} and make it an element of S_{an}. It is not important to repeat such parts of the physiological definition as, e.g., the concept GLAND in handling LIVER', where such concepts like those which identify the relative position of the liver in the (human) body are relevant. Thus LIVER' is either a simple concept in S_{an} or another ontological definition, distinct from LIVER.

Naturally, this example is not very good, in particular because the relation between physiology and anatomy is not comparable with the relation between a more coarse-grained and a more fine-grained discipline. (So we would hardly say that what we know of the liver from physiology is in some way *explained* by what we know from anatomy, or *vice versa.*) Some afterthoughts, however, connected with considering this example may be illuminating. First of all, we cannot talk about any communication between the conceptual systems (here S_{ph} and S_{an}) if the respective areas are disjoint. In our example this cannot happen since we stated that LIVER and LIVER' are distinct but *equivalent*: there is some property (*being a liver*, that is) that is constructed by LIVER as well as by LIVER'.

So which property is it? Ignoring the professional jargons we just use the term *liver* in the natural language and speak obviously about this property; also, when communication takes place between a physiologist and an anatomist, when they speak about *liver* they understand each other although using distinct concepts. In still other words: the set of objects that possess this property in a world W at the time T is unambiguously determined for any W, T independently of whether LIVER or LIVER' is used. A simpler example from mathematics: equilateral triangles are equiangular triangles.

Now we can try to tackle a similar semantic problem frequently handled in the contemporary literature.

Consider the sentence

All drops of water are drops of a substance with molecular composition H_2O

Is it necessary? Is it analytic? Is it *a priori*?

We will answer all these questions from our viewpoint.

1. *Necessity*: As we are told by modal logicians, there are many kinds of necessity. As for logical or analytic necessity, we will say more in 2. It seems that nomological necessity is not relevant for our sentence and for our problem. (Although the 'twin-water' problem seems to suggest some relevance.)

2. *Analyticity.* This problem seems to reduce to the problem whether WATER and (SUBSTANCE WITH MOLECULAR COMPOSITION) H_2O are equivalent concepts. Thus: can water possess some other molecular composition than H_2O? (The twin-water problem.) Now let WATER be a simple concept (so: ^{0}water). The concept

(SUBSTANCE WITH MOLECULAR COMPOSITION) H_2O is, of course, not simple, since it has other concepts (HYDROGEN, OXYGEN, MOLECULE etc.) as components. Thus the two concepts are not identical; they are really *two*, but the property of some wholes *being water* is one and the same. Thus it looks as if the sentence above is analytic. The discussions about this problem show, however, that the simplicity of this solution is at least suspicious. Actually, the problem is more complicated because it cannot be solved in terms of pure concepts only. Our sentence contains the simple *expression* 'water'. It is not obvious that the simple expression 'water' expresses the simple concept ^{0}water. We can see this in Putnam's twin-water analysis (see [Putnam 1975]), where the expression 'water' is connected with a complex concept (surely not the only one possible, taking into account various (even individual) idiolecta, see, e.g., 3.1). *Whereas ^{0}water should simply identify the objective property identical with what is identified by (...)H_2O, some of the complex concepts connected with the expression 'water' may be not equivalent to the latter. Hence the question of analyticity cannot be unambiguously answered unless we choose some concept to be attached to the word 'water'.* But this choice of concept can be construed as dependence on a particular **CS**.

This approach to deciding whether a sentence is analytic can be generalised as follows:

(Relativity of Analyticity)

*Let **A** be a sentence (in any natural language). Let **CS**1, ...,**CS**n be various distinct conceptual systems and let C_A be an analysis of A (see INTERMEZZO: PARMENIDES PRINCIPLE). Further, let C_A be based on the assumption that all the subexpressions of **A** express members of some **CS**i. If the truth-value of **A** can be unambiguously determined under C_A we say that **A** is analytically definite (i.e., analytically true or analytically false) with respect to **CS**i.*

As our last example shows, one sentence can be analytically definite w.r.t. one **CS** and synthetic w.r.t. another. A classical instance of this fact can be found in [Frege 1892a], where Frege shows that one and same sentence about Aristotle can be informative and at the same time analytic dependently on the meaning connected with the name 'Aristotle'.

Remark: Observe what Quine in his [1953, 33] writes about analyticity:

> The notion of analyticity about which we are worrying is a purported relation between statements and languages: a statement *S* is said to be *analytic for* a language *L,* and the problem is to make sense of this relation generally, that is, for variable 'S' and 'L'.

The relativity considered by Quine concerns *statements* (= sentences) and *languages*. If we consider instead *(constructions of) propositions* and *conceptual systems* we get a non-circular relativity, whereas Quine's approach seems to prove that a non-circular definition of analyticity is impossible. –

3. *A priori?* The answer is dependent on an answer to 2. The following claim holds, however: If the sentence is analytic (due to a suitable choice of the concept attached to 'water'), then it is *a priori* as well. Otherwise an experience (investigating the state of the world) is necessary for verification.–

Remark: The claim that analyticity is relative is connected with the well-known problem of natural kinds and the theory of direct reference. It is therefore very instructive to quote from [Marti 2002, p.3], a review of Soames' book *Beyond Rigidity*, where the author says:

> Going back to the conclusion of chapter 9, it seems to me that Soames' approach provides the basis for a definition of rigidity for kind predicates. A compound predicate like 'is a substance with molecular composition H_2O' expresses a complex property that determines a kind as designatum; in this case the kind designated is arguably (for metaphysical reasons) the same with respect to every index, and thus the predicate is rigid. But if the predicate is 'fills rivers and lakes' or 'is Mary's favorite substance' the kind designated may well vary from index to index.

We will show formulations that have to be changed from our viewpoint; thereafter it will be clear that what the author wanted to say is in harmony with our approach to 'water vs. H_2O' problem (as well as to other similar problems).

Ad 'complex property': according to the PWS (and our) definition of properties there are no 'complex properties'. Here 'concept' instead of 'property' should stand.

Ad 'the predicate is rigid': The Reader is surely able to see that the expression 'H_2O' is rigid in the sense that it denotes a property; it is *not* rigid if it should mean that the class of objects possessing the property *being water* would be the same in all worlds. (See [Tichý 1996].)

Ad 'the kind designated may well vary from index to index': what actually varies is the population of such properties so that there is no essential distinction between this case and the case with H_2O. In both cases there is a property conceptually identified and in both cases the class of the bearers of the given property varies from world to world. So what connects these observations of the reviewer with our claim is that the attaching of the *concept* H_2O to the *expression* 'H_2O' is essentially unambiguous whereas there are many concepts that can be attached to the simple expression 'water'.

One more comment: This quotation shows how indeterminate expressions are used by the proponents of the 'natural kinds' theory. The way the term 'kind' is used in the quoted review means that kinds are simply classes that are determined by a property in particular worlds (so: 'populations'). Here the Occam's razor should be applied –

We can, of course, raise a general question: when we want to speak about one and the same object (property, role, relation, magnitude etc.) in terms of distinct **CS**s and use therefore distinct concepts, can we be sure that these distinct concepts will be equivalent? The answer is: No! There is no automatic guarantee, of course. Particularly interesting cases can be sought when a theory (in the linguistic sense, as a set of sentences) is said to *develop.* Thus we come to a frequently discussed and misunderstood topic.

3.7 The development of concepts

What kind of entities can be said to develop?

We do not intend to articulate here a particular theory of development (or even a theory of evolution). We would, however, like to take up a definite standpoint to such formulations that talk about developing concepts. Thus we should first of all answer the question above.

To say that

(The entity) X develops

means bearing in mind the following points:

i) There is some *process.*

ii) There is some entity (X) that takes part in this process and is identified as such an object that is recognisable during the process.

iii) There are some *stages* of the process.

iv) In distinct stages of the process X possesses distinct sets of properties.

We could define the scheme of constructions that construct particular developments; here we only illustrate a particular case. When saying that *somebody* develops during some period of his/her life from the viewpoints of properties $P_1,...,P_n$, we mean that there is an interval I when an individual *i* has the property *being a person* and that there are some subintervals of I such that *i* possesses in each of them distinct subsets of the set $\{P_1,...,P_n\}$. The same individual *i* can be said to develop (even in the same interval I) with respect to distinct sets of properties. Thus we can evaluate his/her development from the viewpoint of sexual maturity, education, sociability, etc. The X (see the points i) through iv)) is in this case the individual *i as a person*. (Individuals as bare individuals cannot develop, of course.)

To come closer to our problem, consider (scientific) theories. Can a *theory* develop in our sense? We have introduced two notions of theory. A theory in the first sense is a set of sentences closed w.r.t. entailment. A set — and *a fortiori* a set of sentences — cannot develop. What *can* develop is *a set of claims accepted by a theory T*. The process of development of T (in the given world W) consists in changing, i.e., adding and/or rejecting claims during the process. This time the constant X (see the points above) could be the property *being accepted by T*. Let $\{C_{11},...,C_{1n}\}$ be the set of (axiomatic) claims accepted by T during a time interval I_1, and $\{C_{11'},...,C_{1m'}\}$ such a set during a time interval I2. The two sets are distinct (otherwise no development is present) but both share a property *being accepted by T*. Yet we can see that we are threatened by contradiction: if a theory is a set of sentences and if the sentences accepted by it in I1 differ from the sentences accepted in I2, then our notion of theory is inconsistent.

This problem is a nice example of the untenability of extensionalism. We have said — repeating the routine formulation — that a theory is a *set* of sentences etc. It is not, it cannot be. First of all, it is not *sentences* as specific linguistic units that determine whether something is a theory: what really counts are the *propositions* denoted by the respective sentences. Thus we could say that a theory is a set of *propositions* (or of *constructions* of propositions). This definition is also not correct: being a theory is an *intension*: something is a theory in a world-time, and even not taking worlds into account we get a contradiction when ignoring the temporal factor, since the set of propositions that is a theory at a time *t* is mostly another set than the set of propositions that is a theory at another time. Thus we have to say:

A theory (in the first sense) is a *property of a set of propositions*. Thus

$$Theory_1 \,/\, (o(o o_{\tau\omega}))_{\tau\omega}.$$

Returning to our example, let T be the value of some theory in a world W, so that it is a *chronology* of sets of propositions. Thus T remains T even when the sets of accepted claims by T in distinct times differ. Thus the X (the constant entity, that what develops) is in any world W the chronology T_W, and the 'changing' properties are just the distinct sets of accepted claims (propositions).

A theory in the second sense has been defined as a subset of the given **CS** (see 3.2.4.2.b), point b)). This definition is also an extensionalist simplification. Actually, it is a *property of sets of constructions* (of the given order), so — most frequently —

$$Theory_2 \,/ (o(o{*_1}))_{\tau\omega}.$$

The explication of *development* proceeds then similarly.

Applying this style of explication to the problem of the development of concepts we can proceed as follows:

First of all, concepts themselves — like sets — cannot develop: they are abstract, hence they are not temporally localisable. On the other hand, the process w.r.t. which we could speak about development can be described as a process during which a developing discipline (see Remark 1 in 3.6) *accepts* a sequence of distinct concepts, say, $C_1,...,C_n$ (with at least one concept simple) that share one feature — this is our constant X — viz. they construct objects that are *similar* in a following sense: all of them are in the area that is studied by this discipline and are *intended* to explicate one and the same "intuitive or pre-analytic idea" (see [Brown 2000, p.109]). This fact is often accompanied by another fact, viz. that all the concepts are expressed in the given language by one and the same expression, which means that there is often an undetected homonymy.

The expressions in italics show that the notion of development of concepts is not a logical/semantic notion. It is an empirical notion characterised, if not defined, in terms of pragmatics. All the concepts $C_1,...,C_n$ are simply *distinct concepts*, their similarity is given by the pragmatic context of a developing discipline. If however the concepts themselves were defined only pragmatically, then we could understand Lakatos' "proof-generated concepts" that "erase" the naive concepts so that they "disappear without any trace". (See *Proofs and Refutations,* 1976, quoted by [Brown 2000, 110].) Our concepts as abstract (prescriptions of) procedures cannot disappear (neither can they be born).

Thus what develops when we say (as we often do) that a concept underwent a change, a development? It is the pragmatic context of being exploited by a discipline in the process of the development of the latter. So when we say that the concept MASS underwent development during the transition from Newton's mechanics to relativistic mechanics then actually we have got two distinct concepts covered by one and the same expression; their developmental connection can be seen only in the pragmatic context of developing mechanics. (Whether this example is adequate may be dubious — see [Earman, Fine 1977]; a better example could be surely found.)

A nice example of a *local* 'development': I cannot help quoting Lakatos as does Brown in his [2000, 107-109]. There was evidently an intuitive or pre-analytic idea of a polyhedron, connected with the Descartes-Euler claim that the numbers of vertices V, edges E, and faces F was given by the equality

$$V - E + F = 2$$

Now the first approximation (the first *concept*!) (A SOLID WHOSE SURFACE CONSISTS OF POLYGONAL FACES) is opposed by a counterexample (a nested cube), where $V - E + F = 4$. A new concept is offered (A SURFACE CONSISTING OF A SYSTEM OF POLYGONS) which again meets a counterexample (tetrahedra having an edge or a vertex in common), where $V - E + F = 3$; the definitive (?) concept also eliminates this counter-example.

This micro-example is very instructive. The illusion that concepts themselves develop is supported by the fact that the same expression ('polyhedron') is used, moreover, that instead of saying that the same expression is gradually connected with distinct concepts it is (usually) said that the concept gradually develops. We could see, however, that unless concepts are identified with expressions (and even then with some difficulty) the development characterises an empirical process rather than concepts themselves.

3.8 Different theorems, different concepts?

Up to now, distinct but artificial conceptual systems have been tested as for the comparability of their concepts. The conceptual systems we have so far considered have been encoded by some linguistic means. Brown in [Brown 2000] formulates a much more radical question. Comparing 'verbal' proofs of the three "intermediate theorems" (as presented in a Bolzanian spirit) with "pictorial proofs" Brown shows that both kinds of proof 'explain' the theorem and tries to react to the following objection (p. 29):

> [t]hat we actually have different concepts of continuity at work: one is the ε-δ concept, which is more or less Bolzano's; the other is so-called pencil identity, a geometrical notion.

Brown agrees, but he adds:

> However, it would be mistake to infer that the results of the two proofs are *incommensurable.*

A convincing argument is given (p.30):

> Even if the picture merely does psychological work, that in itself would be only explicable by assuming that ε-δ continuity and pencil continuity are somehow deeply related. If they are completely unrelated, then what is the picture doing there? It would be like a dictionary giving a verbal description of apples but illustrating the definition with a picture of a banana.

Discussing the role of diagrams in mathematics Brown says in the same monograph (p.174):

> If, as Platonism maintains*, there is more to mathematical reality than mathematical language (which is merely an instrument to represent non-linguistic mathematical reality),* then pictures might be another way to represent that reality. (Emphasis mine. P.M.)

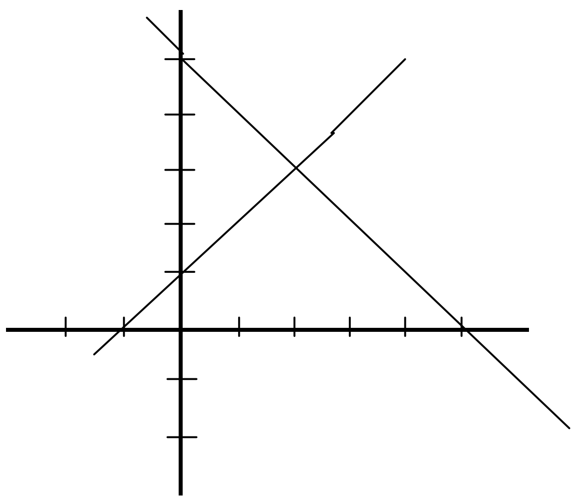
Now this is a good topic for theory of conceptual systems: let us compare two conceptual systems: Let **CS**1 be a **CS** reconstructed from natural language (including geometrical expressions) and **CS**2 a conceptual system reconstructed from pictorial language. Can we compare **CS**1 and **CS**2? (From the last but one quotation it follows that we should!).

Once more: When speaking about concepts and conceptual systems in connection with languages we start from the assumption that there is some *language L*, which *encodes* what has been defined as *constructions.* The latter are abstract (prescriptions of) procedures, which enjoy objective (i.e., subject-independent) character and may become — if encoded — *meanings* of the expressions of L. Our problem can be formulated as follows: Considering a system of geometric pictures G and, on the other hand, a system of expressions of a language L, where particular claims formulated in L can be unambiguously associated with some members of G we pose the following question: *Can the system G be reconstructed so as to make it possible to compare the conceptual system connected with L with the conceptual counterparts derivable from the system G?*

The most transparent instance of this problem is probably the invention of analytic geometry. L is the language of analytic geometry, G is a system of geometric (planimetric) pictures characterized by *xy* coordinates. It is not difficult to associate particular claims of L that can be translated into sentences of the form

The straight line satisfying the equation $y = -x + 5$ *intersects the straight line satisfying the equation* $y = x + 1$ *in the point* $\langle 2,3 \rangle$

and the like with particular pictures. (The corresponding picture would be:)

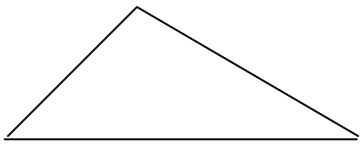


The problem begins when *general claims* of analytic geometry are to be (unambiguously) associated with pictures. A simple example: Let us for the time being suppose that our set of types is extended to contain 'tuple types' (see [Zlatuška 1986]). Where $\beta_1, ..., \beta_m$ are types, we denote the Cartesian product of types $\beta_1,...,\beta_m$ by $(\beta_1,...,\beta_m)$. There is a general claim in L that describes the calculation of the point of intersection of two straight lines. Simplifying a little (omitting trivialisations, using infix notation and the usual way of writing quantifiers) we can connect that claim with the following concept (Intersect / $((\tau,\tau)$ $(o\tau\tau)$ $(o\tau\tau))$, $k,k',n,n' \rightarrow \tau$, $x,y \rightarrow (\tau,\tau)$, $p_1, p_2 \rightarrow (o(\tau,\tau))$, $\iota/$ $(\tau,\tau)(o(\tau,\tau))$, identities and arithmetic operations of obvious types)

$$\forall kk'nn' \; [[\mathrm{Inters}\; p_1p_2] = [\iota\lambda(x,y)\; [[p_1 = \lambda xy\; [y = kx + n]] \wedge [p_2 = \lambda xy\; [y = k'x + n']]]] \supset$$

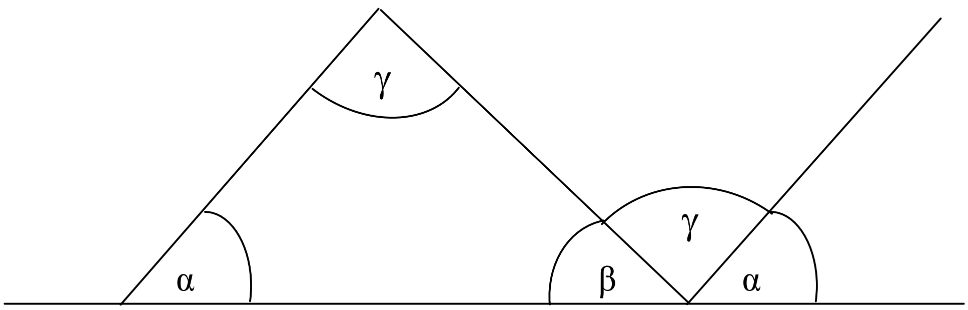
$$x = [n' - n] : [k - k']].$$

No picture at all can be 'translated' so that the above concept would become the *meaning* of such a picture. An attempt at a generalisation can be made, of course: let us say about such a particular picture that it is '*paradigmatic'*. This method is usual in geometry when some claims of trigonometry are explained (and 'visually proved'). The picture below



represents "any triangle", as we are often told. Some 'pictorial operations' realised on such a picture can be then interpreted either so that they *psychologically support* the proof given in L, or that they can be (unambiguously?) associated with some expressions of L. It is only this latter case that is of interest for *logical* analysis of natural language.

The general claims illustrated by a particular picture can be taken to be 'translations' from G if the picture is *paradigmatic,* i.e., when the situation that results from applying the pictorial operations to the picture can be described independently of some idiosyncratic properties of the picture. Thus the 'pictorial proof' of the claim (SAT) *that the sum of internal angles of a (planar Euclidean) triangle is* 180^0 can be given as follows:



and you can see that the situation does not change as for the desired claim if the kind or the size of the triangle changes. But, this 'you can see' I s just this suspicious point: you always can test your hypothesis on a finite number of examples only; the generalisation looks then like an inductive process. The analytic character of SAT, as formulated in L, seems to be lost. (Therefore so many mathematicians deny 'pictorial proofs' genuinely proof-theoretical character — see [Brown 2000].) On the other hand, even if we gave up any effort to prove that G, in general, were able to offer the means of distinguishing analytic claims from the empirical ones, one important point obviously holds: the L-like and the G-like systems can be meaningfully compared and the respective **CS**s are not unrelated. The respective translation is in principle possible and the respective L- and G- expressions will be *synonymous*: we should get the same construction, so that L and G would differ only by the vocabulary and rules that would lead to one and the same construction/ concept.

The link between a 'normal' and a 'pictorial' language can be more easily seen when we compare the languages of geography with the 'languages' of *geographical maps*. Understanding ('ability to read') geographical maps means that a 'vocabulary' and some grammatical rules are given, so that looking at a place in the map we can say, e.g., "Well, London is a larger town than Prague, and it is situated in the west of Prague"; applying a respective **CS** we can write down the respective construction (a concept of the proposition in question), which is the meaning of the above sentence and, at the same time, of the respective part of the map (of a 'pictorial sentence'). Clearly, maps at the scale 1:1000 represent a richer language than maps at the scale, say, 1:75000; the languages represented by distinct specialised maps (for example, political maps vs. the other kinds) are distinct; we could continue showing the clear correspondence of the two kinds of language. Some special features characterise, of course, the 'map languages'. For example, the proposition denoted by the sentences of the form "A is in the west of B" and "B is in the east of A" is one and the same — which can be immediately seen when inspecting the map — but the sentences are not synonymous but only weakly equivalent (see Definitions 18 and 19), which cannot be seen when inspecting the map (unless a prescription of the way the reader has to move his/her head when reading the map is a part of the grammatical rules for the 'map languages').

3.9 Once more analyticity

(See also [Duží, Materna 2004])

The claim of the relativity of analyticity can be generalised in a most natural way: we can ask whether a given *concept*, i.e., not only a sentence (or: propositional concept) is analytic. To give a non-trivial answer we begin by stating that no empirical, i.e., *a posteriori* concept can be analytic, let our intuition of analyticity be however broad. Thus it could seem that any non-empirical, i.e., *a priori* concept is *eo ipso analytic*: Kant's 'proof' that arithmetical claims are *synthetic a priori* has been long ago shown to be incorrect (see, e.g., Couturat's analysis, [Couturat 1908]), at least for examples à la 7 + 5 = 12.

Yet there seems to be some rational intuition contained in Kant's view. Being aware of the fact that neither Kant nor his critics had at their disposal logical means we can use nowadays, we hope that this (suspected) rational core could be formulated in a way not accessible to Kant or his later opponents. First however let Kant speak [1781, Einleitung II, quoted from http://www.Gutenberg2000.de/kant/krva/krva004.htm]:

> Entweder das Prädikat B gehört zum Subjekt A als etwas, was in diesem Begriffe A (versteckterweise) enthalten ist; oder B liegt ganz ausser dem Begriff A, ob es

zwar mit demselben in Verknüpfung steht. Im ersten Fall nenne ich das Urteil analytisch, im andern synthetisch.

From the viewpoint of contemporary logic Kant's definition is vulnerable first due to his reducing sentential structure to subject-predicate. Ironically, this reduction (routine in Kant's time) is most unnatural just in the case of mathematical (in particular, arithmetical) statements, which Kant hoped to prove to be synthetic. To see this let us observe the famous example adduced by Kant: the statement 7 + 5 = 12 is (of course, *a priori* but at the same time) *synthetic*, since in the 'subject', i.e., according to Kant, '7 + 5' the number 12 is not contained.

Now let us apply our theory of analysis to the statement 7 + 5 = 12. For the sake of simplicity let us use trivialisations, setting aside the well-justified doubts concerning their universal applicability; here their use will not influence our arguments. So we have

$$[{}^0\!= [{}^0\!+ {}^0 7\ {}^0 5]\ {}^0 12].$$

A concept that represents what could be called *predicate* here is the concept of identity, so ${}^0\!=$. The *pair* of concepts that together represent the '*subject*' is then the *pair* $\langle [{}^0\!+ {}^0 7\ {}^0 5], {}^0 12 \rangle$. All these concepts are *used*, which is in harmony with what Kant wanted to tell us. Now we will modify Kant's question.

If Kant had (*per impossibile*) accepted that it is identity that plays the role of predicate, then his question would not be whether the number determined by the concept ${}^0 12$ is contained in what is determined by the concept $[{}^0\!+ {}^0 7\ {}^0 5]$ but whether the pair of these numbers is contained in the relation =. The answer is, however, positive, so that the statement is analytic.

(The situation will not change if the role of predicate will play the expression '= 12'.)

All the same, we can show that not all arithmetic statements can be evaluated in this simple way. Consider again Fermat's Last Theorem (variables ranging over natural numbers):

$$\forall\, a\, b\, c\, n\, (\, n > 2 \supset \neg\, (\, a^n + b^n = c^n\,))$$

Trying to save the paradigm of the subject-predicate structure of a sentence would be extremely unnatural here. Even so, Kant's vague idea proves to cover a rational intuition. Let us compare the construction that underlies Fermat with the construction $[{}^0\!= [{}^0\!+ {}^0 7\ {}^0 5]\ {}^0 12]$.

The object constructed is in both cases a truth-value. In the latter case, however, to get this truth-value it is sufficient to use the concepts contained — as proper subconstructions — in the whole construction. Let us check the former case. Here the relevant concepts are ${}^0\neg$, ${}^0\supset$, ${}^0\forall$, ${}^0\!=$, ${}^0\!+$, ${}^0\!>$, ${}^0 2$, ${}^0\mathrm{Exp}$ (where $\mathrm{Exp}(x,y) = x^y$). Under the finitist assumption it is impossible to construct the truth-value in question working with those concepts only. The proof had to use some further concepts.

This comparison — together with our explicating concepts as constructions (and so as problems) – leads us to the following definition:

Definition 39: Let C be a non-empirical concept the simple concepts–subconstructions of which are $C_1,...,C_k$. C is an *analytic concept* iff a finitary method of identifying the object constructed by C is definable in terms of $C_1,...,C_k$ only. The non-analytic (non-empirical) concepts will be called *synthetic concepts.* –

Since non-empirical concepts identify their extensions *a priori* we can formulate a pseudo-Kantian

Claim: *There are synthetic concepts a priori.* –

Remark: A similar thought —based on intuitionistic principles —can be found in [Martin-Löf 1992]. –

In general the following statement can be formulated:

Finding an algorithmic solution to a problem can be considered to be discovering an analytic (a priori) concept that is equivalent to the respective synthetic (a priori) concept (= problem, see 3.2.2).

A classical example illustrating this statement is the "case π"): By now we should know that "π" is an abbreviation only: the respective definiens ('ratio of ...') expresses the ontological definition of π in a **CS** that contains concepts like CIRCLE, RADIUS and not necessarily many other mathematical concepts. This ontological definition does not enable us to use an *algorithmic* method of calculating any member of the infinite expansion of π. What can be called a mathematical *discovery* was finding an equivalent concept that does make it possible; more such algorithms have been defined, e.g.

$$\sum_{k=0}^{\infty}\frac{(-1)^k}{4^k}\left(\frac{2}{4k+1}+\frac{2}{4k+2}+\frac{1}{4k+3}\right)$$

(The case of explicating the term *algorithm* (likewise all the cases of a genuine *explication*) is distinct from the case π: The term 'algorithm' means roughly 'a mechanical method' and is markedly vague—unlike the term 'π'.)

The transition from a synthetic (*a priori*) concept to its equivalent analytic counterpart(s) can be described in terms of *conceptual systems*: We can distinguish between two situations:

1) The given problem is formulated in a **CS** that contains all the concepts necessary for the transition. This situation is analogous to Kuhn's stage of *normal science*, the possibility of solution is already given, what remains to be done is to become aware that such and such already known concepts make the solution possible.

2) The given problem is formulated in a **CS** that does not contain the concepts necessary for the transition. Then something like a *change of* (maybe *local mini-*)*paradigms* is realised – a Kuhnian (mini-)revolution: new *primitive* concepts are needed.

(What decides which of these situations takes place is, of course, whether the concepts necessary for solution are (situation 1) or are not (situation 2) derivable from the given $\mathbf{PC_{CS}}$.)

Remark: So it holds — in harmony with our *Relativity of Analyticity* claim — that, e.g., the (concept of) Fermat's Last Theorem is analytically definite (true) with respect to such **CS**s that contain all concepts necessary for finite decision; with respect to other **CS**s it is synthetic. –

But: ***Most a priori concepts are incurably synthetic.*** This follows since there are uncountably infinitely many functions but only countably infinitely many recursive functions. Thus, e.g., any concept of the class of 1st order theorems of predicate logic is synthetic: this class is undecidable in the class of all 1st order formulas of predicate logic.

4. Concluding essay: Concepts and objectivity

We have seen that conceptual systems determine various *areas*. At the same time we have argued that the variety of conceptually determined areas does not imply an insurmountable confusion of languages (a Tower-of-Babel phenomenon). The relativist moods characteristic of the post-wave lead however to questions of the following kind:

Isn't it so that the area described by a conceptual system is ***created*** *by this system?*

In other words, some people suspect that the functions, properties, relations etc. that we are able to talk about due to a conceptual system S come into being with S only; a change of S (i.e., transition to another system) as if caused that those functions, properties, relations etc. cease to 'exist'.

(In [Davidson 1984, p.183] this view (not shared by him) is characterised as follows:

> Reality itself is relative to a scheme: what counts as real in one system may not in another.

This is a very succinct characterisation. Davidson's views are well-known but we must not forget that his notion of conceptual scheme differs from ours, and, last but not least, that Davidson construes concepts as "words with fixed meanings", which must lead to other results (comparable, not 'incommensurable') than our definition. Some of Davidson's considerations can be interpreted from our point of view and then accepted. Take, for example, the Davidsonian principle of *charity*, i.e., his recommendation to suppose that what our partner claims is (mostly) true. Applying this principle we can detect (with some higher degree of probability) which concepts our partner associates with respective expressions.)

Let us now investigate the views of this kind.

To illustrate these views with a particular example, imagine the situation where a language cannot express the concept of *spider*. (The users of such a language live — in some isolation, to be a little bit realistic — in a country where no spiders live, and they do not use the language of zoology.) Does it mean that *for the speakers of such a language* there is no such property as (*being a*) *spider*? And that as soon as that language (better: its underlying conceptual system) is enriched by the respective concept the property comes into being *for the speakers of this enriched language*?

Still in other words: does the property *spider(hood)* exist just *for such languages* (and their speakers) *whose underlying conceptual system contains the concepts necessary for defining spider(hood)*?

First of all, we must be aware that the elements of any conceptually determined area are either mathematical objects or other *abstract* objects, empirical or non-empirical. Setting aside mathematical objects we can see that the empirical objects whose existence we investigate are intensions. Thus we have to distinguish two questions: *Do spiders exist?* and *Does the property 'to be a spider' exist?* The latter question is independent of answering the former question: once we specify what the term 'exist' means we can easily admit that the latter question can be positively answered even then when there are no spiders. Some more details (see also Tichý 1979]): If the attribute 'to exist' stands before an occurrence of an empirical expression E (in English, modifications for other languages can be always realised) then it can mean either

i) that the intension denoted by E is in an intelligible way 'occupied' in the given world-time (by an individual, by a non-empty class etc.) — see the Remark following Definition 28, or

ii) that the intension is objective (then the expression denoting the intension is as a rule preceded by expressions like 'the property' or 'the role' or 'the proposition' etc. Thus the distinction between the two questions above can be fixed just as above. An alternative in some cases is attaching the ending 'hood', so: *Does spiderhood exist?*

Thus *Colours exist* means: *The class Colour is not empty;* thus this kind of existence is simply the existential quantifier. Not only that: saying *The class of colours exists* we say: *The class containing the class of colours is not empty*. If we intend to say more then we have to say only that (like in ii) above) the given extension is objective (which is — for Carnap as well as for most post-philosophers — a metaphysical claim). In our example we can even suppose that there are *no spiders* but our second question is still here: for a realist even empty properties (or properties empty in the given world) do exist in the sense of objectivity. (Bolzano would add: they — as well as the non-empty ones — do *not* exist in the sense of spatio-temporal localisability.)

Now we can return to our question. This question has introduced a new category: *to exist for...*. We can immediately see that the phrase

A *exists for sth/sb*

is only a paraphrase of

sth/sb (a language, a language user...) knows (is acquainted with, has access to...) **A**.

Thus ('absolute') existence in our sense is *presupposed* by phrases containing this 'existence for...'. Therefore our question, i.e.,

> does the property *spider* exist just *for such languages* (and their speakers) *whose underlying conceptual system contains the concepts necessary for defining spiderhood*?

only *seems* to be 'revolutionary', provocative: actually the (expected) positive answer is a *tautology*, since its sense is given by the formulation

> Just those languages whose *underlying conceptual system contains the concepts necessary for defining spider(hood) have the access to the property spider(hood).*

(We can see that the objective property *spiderhood* is presupposed by this answer.)

Remark: A recapitulation is here maybe useful: the property *spider* is identical with the property *spiderhood*. The evident distinction in our using these terms is given by the distinction *de re* and *de dicto*. 'spiderhood' can be used only if the concept is in the *de dicto* supposition. Compare

Some spiders live in water. vs. *Spiderhood is a zoological property.*

See 1.4.2.3, example A. and B. –

Our approach implies that to claim 'existence' of abstract objects means to claim that they are definable independently of any subject (in this way Bolzano defines objectivity) and that they are not created by the mind of a subject but rather discovered by it. The development of a language is connected with discovering new abstract entities that have to be fixed by expressions of the language because we need them for our description of the world and explanations of particular phenomena. *Language is needed for fixing abstract entities; the latter are needed as criteria that are used for determining which (important) properties can be attributed to which empirically found concrete objects*. When Adam was asked by God to give names to animals etc., it did not mean that he should give names to *particular* concrete animals: it meant that the property *elephant* had to be distinguished from the property *dog* etc.: *Adam had to invent language, not to fix contingent ostensions.*

Which rival approach could contribute to the question of objectivity? It seems that negative solutions have to defend extremely counterintuitive claims; for example, that to be is to be known – a new version of Berkeley's form of solipsism (*esse est percipi*).

Further, we can observe that the fact of various (even incomparable) conceptual systems — 'conceptual plurality' — does not at all prove that various areas are not objective. On the contrary, to conceptually determine an area would not be possible if the abstract objects were not objective; some of them proved to be useful, some of them led to a blind alley — e.g., the property *phlogiston*, which is objective but as being empty in the actual world is not useful. Anyway, usefulness and objectivity are distinct categories. We can 'fabricate' the concept A DOG WHOSE TAIL CONTAINS JUST 357 PIECES OF HAIR; the impression that this is only our invention arises due to the fact that such a concept is of no use at all. Yet the respective property surely 'exists', independently of whether some individual does or does not possess it, and independently of our speaking or thinking of it. Moreover, we can logically guarantee that the property is not the empty property: no contradiction is derivable from the assumption that there is such a dog so that there are such world-times pairs where the class of individuals possessing this property is not empty.

(A question for a relativist: Does the above property exist for the language of astronomy? And if not, does it mean that this property is not objective? Solution: see the preceding text.)

To sum up*: **Concepts discover abstract entities that may be useful for describing and explaining the world. Conceptual systems define particular areas (of interest) and make it possible to distinguish truth from falsity; the pure logico-mathematical systems do so a priori, empirical systems, using the logico-mathematical part as a priori tools, pose empirical problems whose solution is to be found via empirical procedures (which are a sort of questions to be answered by the world).***

Appendix 1: Symbols

1. TYPES

<u>1st order:</u>

Atomic:

ο (truth-values)

ι (individuals)

τ (time moments, real numbers)

ω (possible worlds)

Complex:

$(\alpha\ \beta_1 ... \beta_m)$ $(\beta_1 \times \ldots \times \beta_m \rightarrow \alpha)$

<u>Order *n, n > 1*</u>

$*_{n\text{-}1}$

2. CONSTRUCTIONS

$x_1, x_2, \ldots,$ (*variables*)

^0X *trivialisation*

$[X\ X_1 ... X_m]$ *composition*

$[\lambda x_1 ... x_m\ X]$ *closure*

3. LOGICAL SYMBOLS

Connectives: $\neg, \wedge, \vee, \supset$

Quantifiers: $\forall, \exists$

Identity: =

'Descriptive operator': ι (context determines when ι is not the type of individuals)

Appendix 2: Some specific features of TIL

1. ***Frege's 'semantic triangle' corrected***

expression

|

'sense' (meaning) = construction (concept)

|

denotation (intension in the empirical case)

---

under this line experience necessary

reference (in the empirical case)

2. ***Transparency****: meaning is the same in all contexts.*

   (What seems to be a change of meaning is actually a change of *supposition*.)

3. ***No formal language in the standard sense is introduced.***

   ('Formal means' for fixing *constructions* — see Appendix 1 — are not formal in the sense that they would 'wait for interpretation': in terms of them we simply speak about constructions themselves, which are prescriptions of extra-linguistic abstract procedures given unambiguously by these 'formal means' ('language of constructions', if you like).)

4. ***An objectual notion of variable.***

   (Variables — as a kind of construction — are extra-linguistic; the letters used for them are *names of variables*.)

5. ***The ramified hierarchy*** *makes it possible to avoid the distinction object language – metalanguage.*

   (In TIL we can *mention,* not only *use* constructions. Mentioning constructions means that the 'hyperintensionality' is present.)

6. ***Explicit use of variables w, t for possible worlds and times.***

   (The 'standard' way of handling intensionality consists in 'translating' the respective expressions of a natural language to an artificial formal language and interpreting the result in a metalanguage. In TIL translating is replaced by a (direct) analysis, i.e., by finding a construction that constructs the intension in question *via* $\lambda w \lambda t$ [...].)

7. ***Intensionality*** *is a universal phenomenon: it is present not only in modal contexts but wherever an empirical expression stands.*
   (See point 1.)

8. ***Using Fregean terminology: 'sense' has to be structured***.

   (Cresswell's 'structured meanings', i.e., tuples, are replaced by constructions.)

9. ***Functions (as mappings) are taken to be partial, i.e., returning <u>at most</u> one value***.

   (Which corresponds to a natural phenomenon, e.g. truth-gaps.)

10. ***Anti-essentialism****: individuals are 'bare', they possess no empirical property necessarily (i.e., in all possible worlds-times).*

    ('Essential properties', 'requisites', concern no particulars, only intensions. 'being a man' is a requisite of 'being a philosopher' — i.e., the former property is a requisite of the latter; if an individual happens to be a philosopher then 'being a man' is not a requisite of that individual.)

11. It is ***impossible*** to logically determine *which of the possible worlds is the actual one.*

Appendix 3: Some principles of the theory of concepts based on TIL

1. Concepts are abstract language independent entities.
2. (Meaningful) expressions are codes of concepts. Particular languages differ in the way they encode concepts.
3. Concept acquisition, concept possessing are mental entities unlike concepts themselves.
4. As Bolzano already suspected, concepts are structured. Under this assumption only we can explain why various distinct concepts can identify one and the same object.
5. Concepts can be modelled as closed TIL constructions.
6. Constructions that are α- or η-equivalent do not represent distinct concepts. The relation Quasi-identity (definable in terms of α- and η-equivalence) enables us to define a distinguished, 'normalised' construction among all quasi-identical constructions and let it represent the respective concept (while the other quasi-identical constructions can be said to point to this concept).
7. No empirical concept identifies a particular. Every empirical concept identifies a non-trivial intension.
8. Where X is an object of order 1 the trivialisation ^0X is a *simple concept.*
9. Any improper closed construction is a *strictly empty concept*.
10. Any closed construction that constructs an empty class/relation is a *quasi-empty concept.*
11. Any closed construction that constructs an intension whose value in $\langle W, T\rangle$ is either missing or an empty class/relation is a *concept empirically empty in* $\langle W, T\rangle$.
12. *Synonyms* are expressions that express one and the same concept.
13. (*Weakly*) *equivalent* expressions express (distinct but) equivalent concepts.
14. *Coincident* expressions express distinct non-equivalent empirical concepts that identify distinct intensions whose value is the same in the actual world-time. (I.e., their denotations are distinct unlike their reference.)

Appendix 4: A solution of a Putnam's problem

('Carnapian vs. Polish Language')

The classical problem formulated in [Putnam 1990] consists in handling the following situation:

Let us have a 'world' with three individuals *a, b, c* (Putnam uses x_1, x_2, x_3, we use the latter characters as names for variables.); let us ask: "How many *objects* are there in this world?" Now Putnam constructs a confrontation between a 'Carnapian' language, in which we can answer the question as follows: 'There are just three individuals here, viz. *a, b, c*.' and a 'Polish logicians' language', obviously a language of a (Leśniewskian) mereology, where we would have (omitting the 'null' object) seven objects, viz. $a, b, c, a + b, a + c, b + c, a + b + c$. Further: supposing that, say, *a* is red and *b* is black and considering sentences

(1) There is an object which is partly red and partly black.

(2) There is an object which is red and an object which is black.

we can state that (2) is true in both 'Carnapian' and 'Polish logicians' ' world whereas (1) seems to be true only in the Polish version. On the other hand, we can easily prove (?) that (1) implies (2) and *vice versa*, which leads to the following question: "What is the point of treating (1) as an abbreviation of (2) if it doesn't, in fact, have the same *meaning* as (2)?" (p.100). (By the way, using the term 'meaning' should be suspicious under Quine's admonishing finger, but 'to preserve meaning' has to be understood as 'to obey translation practice'; so no 'ontological commitment' arises... . *Difficile satiram non scribere.*)

Putnam reproduces a probable Quine's solution (*façon de parler*) and he himself defends a kind of (Carnapian) conceptual relativity.

We can show that our approach makes it possible to justify not only verbally a simple (and verbally easily expressible) solution based on plurality of conceptual systems.

In the following analysis we will again use following abbreviations: infix notation for truth functions and identity. The resulting symbolic expressions should be read as (names of) constructions.

We have a, b, c / ι, ∨ / (οοο), $=_1$ / (οττ), $=_2$ / (οιι), ∃ / (ο (οι)), *Card* / (τ (οι)), + / (ιιι), *Red, Black* / $(οι)_{τω}$, ι / (τ (οτ); $k \rightarrow τ$, $x, y, z \rightarrow ι$; (Distinguish, please, ι as a type and ι as a function — a 'singulariser' — the distinction should be clear from context.)

First : How many *objects* are there in that 'world' ?

Clearly, the problem is not unambiguously defined unless the word 'object' is specified. To specify this word means to decide which concept is associated with it; this decision is again dependent on a particular conceptual system. To show this let us consider two conceptual systems (defining their primitive part only and admitting that there is no loss of generality connected with this assumption):

CS1 $\{ {}^0a, {}^0b, {}^0c, {}^0\vee, {}^0{=_1}, {}^0{=_2}, {}^0Card, {}^0Red, {}^0Black, {}^0\iota \}$

CS2 **CS**1 $\cup \{ {}^0+ \}$

The function + constructs from two individuals a new individual (their 'mereological sum'). We can assume that + is associative, i.e., $[{}^0+ [{}^0+ x\ y]\ z]$ v-constructs the same individual as $[{}^0+ x\ [{}^0+ y\ z]]$ for all valuations v.

Now in **CS**1 our question encodes the construction

$${}^0\iota\ \lambda k\ [\ k =_1 [{}^0\mathrm{Card}\ \lambda x\ [[x =_2 a] \vee [x =_2 b] \vee [x =_2 c]]]]$$

whereas in **CS**2 the construction is

$${}^0\iota\ \lambda k\ [\ k =_1 [{}^0\mathrm{Card}\ \lambda x\ [[x =_2 a] \vee [x =_2 b] \vee [x =_2 c] \vee \exists\ \lambda y\ \exists\ \lambda z\ [\ x =_2 [{}^0+ y\ z]]]]]].$$

Now everything is clear as for the first question. The **CS**1 construction (concept) constructs the number 3, the **CS**2 construction (concept) constructs the number 7. Furthermore, even if we used the vague 'identification' of *meaning* with 'translational practice' we would not say that the respective answers shared a meaning. Our approach shows it precisely — the constructions above are distinct (and, moreover, are not equivalent). Verbally expressed, the reason is that **CS**2 is a creative extension of **CS**1, expanding the area by adding the primitive ${}^0+$, so that the extension of the term 'object' changes. Thus both answers are true, only that they do not use the same *concept* associated with the term 'object'.

This result determines the solution of the problem of semantic interrelation between the sentences (1) and (2). The analyses of these sentences are (in TIL; a little bit simplified without loss of generality):

(1') $\lambda w \lambda t\ [{}^0\exists\ \lambda x\ [{}^0Red_{wt}\, x] \wedge [{}^0Black_{wt}\, x]]$

(2') $\lambda w \lambda t\ [[{}^0\exists\ \lambda x\ [{}^0Red_{wt}\, x]] \wedge [{}^0\exists\ \lambda x\ [{}^0Black_{wt}\, x]]]$

(Indeed, a degree of 'Davidsonian' Charity is needed: *Red*, as well as *Black* is now supposed to be predicable even in the sense of 'partly red', 'partly black'; a conceptual refinement, which would distinguish 'partly red' from 'red' is feasible, but our results would be the same.)

Now it indeed holds (as Putnam states) that the proposition constructed by (2') is true (assuming that *a* is red and *b* is black) in both systems, but in the case of **CS**1 the term 'object' means *from the viewpoint of* ***CS2*** and from the viewpoint of our ('background') language 'a simple (i.e., no parts having) individual' whereas its extension in **CS**2 embraces *any* individuals. And it indeed holds (as Putnam also states) that the proposition constructed by (1') is true in **CS**2. What to do now with the 'logical proof' that (2) implies (1) *within*

CS2? It works, of course, but again the price to be paid is that the term 'object' used within **CS**1 has to be semantically changed when used in **CS**2.

The whole problem is thus rather simple. This can be demonstrated by using *two* terms where the formulation of the problem contains *one* term. Thus let as transcribe both sentences as follows ('$object_1$' denotes the Carnapian objects, '$object_2$' denotes the "Polish" objects):

(1''') There is an $object_1$ which is partly red and partly black.

(2''') There is an $object_2$ which is red and an $object_2$ which is black.

Under our assumptions the sentence (1''') cannot be true (perhaps it lacks any truth-value). The sentence (2''') is true. But there are two other sentences whose truth-value we can check assuming the facts holding in the particular 'worlds' (i.e., there are three simple individuals, *a* being red and *b* being black):

(3) There is an $object_2$ which is partly red and partly black.

(4) There is an $object_1$ which is red and an $object_1$ which is black.

Now it is clear that the sentence (3) is true as well as the sentence (4).

The replacing the term 'object' by the two terms '$object_1$' and '$object_2$' is possible only in such languages that are based on **CS**2 or on a system that contains **CS**2. See also the 'language games' presented in the main text of the present book ('bink', 'cink', 'cank', etc.).

Remark: After having written the present Appendix 4, I became aware of [Brueckner 1998]. I have to appreciate Brueckner's analysis and am happy with the harmony of his analysis with what I just presented within the framework of my theory of conceptual systems.

References

[Bar-Hillel 1950] Bar-Hillel, Yehoshua: Bolzano's Definition of Analytic Propositions. *Methodos II.,5,* 32-55

[Bartsch 1998] Bartsch, Renate: *Dynamic Conceptual Semantics.* Center for the Study of Language and Information, Stanford, California, **CS**LI Publications & FoLLI

[Bealer 1982] Bealer, George: *Quality and Concept.* Clarendon Press, Oxford

[Benacerraf, Putnam 1983] Benacerraf, P. and Putnam, H. (eds.): *Philosophy of Mathematics. Selected Readings, 2nd ed.*, Cambridge UP

[Black 1937] Black, Max: Vagueness:An Exercise in Logical Analysis. *Philosophy of Science 4*, 427-455

[Bolzano 1837] Bolzano, Bernard: *Wissenschaftslehre I.,II.,* Sulzbach

[Brown Brown 1999] Brown, James R.: *Philosophy of Mathematics.* Routledge, London

[Brueckner 1998] Brueckner, A.: Conceptual Relativism. *Pacific Philosophical Quarterly 79,* 295-301

[Carnap 1947] Carnap, Rudolf: *Meaning and Necessity*. University of Chicago Press, Chicago

[Childers, Palomäki 2000] Childers, Timothy and Palomäki, Jari, eds.: *Between Words and Worlds. A Festschrift for Pavel Materna*. Filosofia, Praha

[Childers, Svoboda 2003] Childers, Timothy and Svoboda, Vladimír: On the Meaning of Prescriptions. In: Peregrin, J., ed.: *Meaning: The Dynamic Turn.* Elsevier

[Church 1940] Church, Alonzo: A formulation of the simple theory of types. *The Journal of Symbolic Logic 5,* 56-68

[Church 1951] Church, Alonzo: Intensional isomorphism and identity of belief. *Philosophical Studies V*

[Church 1956] Church, Alonzo: *Introduction to Mathematical Logic I,* Princeton

[Coffa 1991] Coffa, J.Alberto: *The Semantic Tradition from Kant to Carnap.* Cambridge UP

[Couturat 1908] Couturat, L.: *Die philosophischen Prinzipien der Mathematik*. Leipzig

[Cresswell 1975] Cresswell, Max J.: Hyperintensional Logic. *Studia Logica XXXIV, 1,* 25-38

[Cresswell 1985] Cresswell, Max J.: *Structured Meanings,* MIT Press, Cambridge, Mass.

[Davidson 1984] Davidson, Donald: On the Very Idea of a Conceptual Scheme. In*: Inquiries into Truth and Meaning.* Oxford, Clarendon Press

[Davidson, Harman 1972] Davidson, Donald and Harman, Gilbert, eds.: *Semantics of Natural Language, 2nd ed.*, D.Reidel

[Duží 1999] Duží, Marie: Existential Quantification into 'Intentional Contexts'. In: T. Childers, ed: *The Logica Yearbook 1999*. Filosofia, Prague 2000, pp. 258-272.

[Duží 2000] Duží, Marie: Using Materna's Theory of Concepts in Conceptual Modelling. In [Childers, Palomäki 2000, 111-129]

[Duží 2003a] Duží, Marie: Concepts, Language and Ontologies (from a Logical Point of View). In: *Information Modelling and Knowledge Bases XV*, ed. by Y. Kiyoki, E. Kawaguchi, H. Jaakkola, H. Kangassalo. IOS Press Amsterdam, pp. 193-209.

[Duží 2003b] Duží, Marie: Notional Attitudes (On wishing, seeking and finding). *Organon F,* X, SAV Bratislava, 2003, pp. 237-260, 412-416.

[Duží 2003c] Duží, Marie: Do we have to deal with partiality? *Miscellania Logica,* Tom V, ed. by K.Bendova a P. Jirku, Karolinum Praha 2003, pp. 45-76. Available also at: http://www.cs.vsb.cz/duzi/

[Duží 2004] Duží Marie: Intensional Logic and the Irreducible Contrast between *de dicto* and *de re*. In *ProFil* (volume) 5, (number) 1, 2004, 1-34, http://profil.muni.cz/01_2004/duzi_de_dicto_de_re.pdf

[Duží, Jespersen, Müller 2004] Duží, Marie, Jespersen Bjørn, Müller Jaroslav: Epistemic Closure and explicit Knowledge. In: Childers, T. and Majer, O.,eds.: *The Logica Yearbook 2004*, to appear.

[Duží, Materna 2001] Duží, Marie, Materna, Pavel: Propositional Attitudes Revised. In [Majer 2001], pp. 163-174

[Duží, Materna 2002] Duží, Marie, Materna, Pavel: Intensional Logic as a Medium of Knowledge Representation and Acquisition in the HIT Conceptual Model. In: *Information Modelling and Knowledge Bases XIV*. Ed. Hannu Kangassalo, Eiji Kawaguchi, Amsterodam, Netherlands. IOS Press, 2003, vol. Vol. 94, p 51-65.

[Duží, Materna 2003] Duží, Marie, Materna, Pavel: Parmenides Principle (An analysis of aboutness). In: Childers, T. and Majer, O.,eds.: *The Logica Yearbook 2002*, 159-178

[Duží, Materna 2004]Duží, Marie and Materna, Pavel: A Procedural Theory of Concepts and the Problem of Synthetic *a priori.* In: *Korean Journal of Logic*, Vol. 7, No. 1, March, 2004, pp. 1-22. ISSN 1598-7493 9

[Earman, Fine 1977] Earman, John and Fine, Arthur: Against Indeterminacy. *The Journal of Philosophy 74,* 9, 535-538

[Fletcher 1998] Fletcher, Peter: *Truth, Proof and Infinity*. Kluwer AP, Dordrecht/Boston/ London

[Fodor 1998] Fodor, Jerry A.: *Concepts. (Where Cognitive Science went wrong.)* Clarendon Press, Oxford

[Frege 1884] Frege, Gottlob: *Die Grundlagen der Arithmetik.* W.Koebner, Breslau § 88

[Frege 1892] Frege, Gottlob: Über Begriff und Gegenstand. Vierteljahrschrift für wissenschaftliche Philosophie 16, 192-205

[Frege 1892a] Frege, Gottlob: Über Sinn und Bedeutung. *Zeitschrift f. Philosophie und philosophische Kritik 100,* 25-50

[Geach, Black 1952] Geach, P. and Black, M., eds.: *Translations from the Philosophical Writings of Gottlob Frege*. Blackwell, Oxford.

[Gödel 1990] Gödel, Kurt: Russell's Mathematical Logic. In: S.Feferman et alii (eds): *Kurt Gödel Collected Works II,* Oxford UP

[Hautamäki 1986] Hautamäki, Antti: *Points of View and their logical analysis.* Acta Philosophica Fennica 41, Helsinki

[Hempel 1965] Hempel, C. G.: *Aspects of Scientific Explanation.* New York: Macmillan

[Horák 2001] Horák, Aleš: *The Normal Translation Algorithm in Transparent Intensional Logic for Czech. PhD Thesis,* Brno

[Jackendoff 1995] Jackendoff, Ray: *Semantic Structures.* MIT Press, Fourth Printing

[Jespersen 2000] Jespersen, Bjørn: Proper Names and Primitive Senses. In [Childers, Palomäki 2000], 70-92

[Jespersen 2003] Why the tuple theory of structured propositions isn't a theory of structured propositions, *Philosophia*, Vol. 31, Nos. 1-2 (2003), 171-83.

[Kant 1781] Kant, Immanuel: *Kritik der reinen Vernunft*. Königsberg.

[Kaplan 1978] Kaplan, David: On the Logic of Demonstratives. *Journal of Philosophical Logic 8*, 81-98

[Kauppi 1967] Kauppi, Raili: *Einführung in die Theorie der Begriffssysteme*, Acta Universitatis Tamperensis A/15, Tampere

[King 1997] King, Jeffrey C.: Structured Propositions. http://plato.stanford.edu/entries/propositions-structured/

[Köhler 2000] Köhler, Eckehart: Logic Is Objective *and* Subjective. In [Childers, Palomäki 2000, 13-20]

[Lakatos, Musgrave Musgrave 1995] Lakatos, Imre and Musgrave, Alan, eds.: *Criticism and the Growth of Knowledge.* Cambridge UP

[Lewis 1972] Lewis, David: General Semantics. In: [Davidson, Harman 1972, 169-218]

[Majer 2001] O.Majer, ed.: *The Logica Yearbook 2000.* Filosofia, Prague

[Marti 2002] G.Marti: Soames, Scott, *Beyond Rigidity: The Unfinished Semantic Agenda of* Naming and Necessity. *Notr Dame Philosophical Reviews 2002,* 1-4

[Martin-Löf 1992] Martin-Löf, P. "Analytic and Synthetic Judgements in Type Theory." Talk given at the workshop on Kant and Contemporary Epistemology, Florence

[Materna 1997] Materna, Pavel: How Many Concepts Are There? In: T.Childers, P.Kolář, Vl.Svoboda, eds.: *LOGICA '96,* Filosofia, Praha, 277-283

[Materna 2000] Materna, Pavel: Simple Concepts and Simple Expressions. In: T. Childers, ed.: *The logica yearbook 1999*, Filosofia, Praha, 245-257

[Materna 2002] Materna, Pavel: Horwich's Conception of Meaning: How to Change His "Deflationary View" Into a Non-trivial Conception. In: T.Childers and O.Majer, eds.: *The logica yearbook 2001,* Filosofia, Praha, 153-162

[Materna 2003] Materna, Pavel: Actuality and Possibility. In: A. Rojszczak, J.Cachro, G. Kurczewski, eds.: *Philosophical Dimensions of Logic and Science.* Kluwer AP, 289-296

[Montague 1974] *Formal Philosophy.* Edited by Richmond H.Thomason, Yale U.P., New Haven

[Moschovakis 1990] Moschovakis, N. Yiannis: Sense and Denotation as Algorithm and Value. *Logic Colloquium'90,* ASL meeting, Helsinki.

[Newton-Smith 1981] Newton-Smith, W.H.: *The Rationality of Science.* Routledge & Kegan Paul Ltd, London and New York

[Orilia 1999] Orilia, Francesco: *Predication, Analysis and Reference.* CLUEB, collana Heuresis, Linguaggio, Logica, Scienza, diretta da Alberto Pasquinelli & Giorgio Sandri, Bologna

[Palomäki 1994] Palomäki, Jari: *From Concepts to Concept Theory*, Acta Universitatis Tamperensis A/416, Tampere

[Palomäki 2001] Palomäki, Jari: The Subject Matter of Mathematics. Tichý's View Considered. In: O.Majer, ed.: *The Logica Yearbook 2000.* Filosofia, Prague 193-204

[Peacocke 1992] Peacocke, Christopher: *A Study of Concepts.* The MIT Press

[Peregrin 2000] Peregrin, Jaroslav: Constructions and Concepts. In [Childers, Palomäki 2000, 34-48]

[Popper 1986] Popper, Karl: *The Open Society and Its Enemies.* Routledge & Kegan, London

[Putnam 1975] Putnam, H.: The meaning of 'meaning'.In: *Mind, Language and Reality*, Cambridge University Press, 215-271

[Putnam 1983] Putnam, H.: Models and reality. In: [Benacerraf, Putnam 1983], 421-446

[Putnam 1990] Putnam, H.: Truth and Convention. In: *Realism with a Human Face.* Harvard UP, 96-104

[Quine 1953] Quine, W.v.O.: Two Dogmas of Empiricismus. In *From a Logical Point of View*, Harvard UP, Cambridge, Mass.

[Rey 1998] Rey, Georges: Concepts. *Routledge Encyclopedia of Philosophy, Version 1.0*, London and New York: Routledge (1998)

[Rosenberg 2000] Rosenberg, Alex: *Philosophy of Science.* Routledge, London & New York

[Russell 1903] Russell, Bertrand: *Principles of Mathematics*. New York, Norton (2nd ed.)

[Sandu, Hintikka 2001] Sandu,G., Hintikka,J.: Aspects of Compositionality. *Journal of Logic, Language,and Information 10*, 49-61

[Sankey 1997] Sankey, Howard: *Rationality, Relativism and Incommensurability.* Ashgate, Aldershof, Brookfield USA, Singapore, Sydney

[Shapiro 1997] Shapiro, Stewart: *Philosophy of Mathematics*. Oxford UP, New York

[Stechow, Wunderlich 1991] A.v.Stechow, D.Wunderlich, eds.: *Semantik/Semantics*. De Gruyter – Berlin – New York

[Sundholm 2000] Sundholm, Göran: Virtues and Vices of Interpreted *Classical* Formalisms: Some impertinent questions for Pavel Materna…, in: Childers 2000, 3-12]

[Tarski 1956] Tarski, Alfred: The Concept of Truth in Formalized Languages. In: *A.Tarski: Logic, Semantics, Metamathematics* Oxford at the Clarendon Press, 152-278

[Tichý 1972] Tichý, Pavel: Plantinga on Essence: A Few Questions. *The Philosophical Review 81,* 82-93

[Tichý 1978] Tichý, Pavel: Question, Answers, and Logic. *Amer. Philos. Quarterly 15,* 275-284

[Tichý 1978a] Tichý, Pavel: Two Kinds of Intensional Logic. *Epistemologia I*, 143-164

[Tichý 1978b] Tichý, Pavel: De dicto and de re. *Philosophia 8*, 1-16

[Tichý 1979] Tichý, Pavel: Existence and God. *The Journal of Philosophy 76*, 403-420

[Tichý 1980] Tichý, Pavel: The Logic of Temporal Discourse. *Linguistics and Philosophy 3*, 343-369

[Tichý 1982] Tichý, Pavel: Foundations of Partial Type Theory. *Reports on Mathematical Logic 14,* 52-72

[Tichý 1986] Tichý, Pavel: Indiscernibility of Identicals. *Studia Logica* 45, 257-273

[Tichý 1988] Tichý, Pavel: *The Foundations of Frege's Logic.* de Gruyter, Berlin / New York

[Tichý 1992] Tichý, Pavel: The Scandal of Linguistics. *From the Logical Point of View 3,* 70-80, Prague

[Tichý 1995] Tichý, Pavel: Constructions as the Subject Matter of Mathematics. In: W.Depanti-Schimanowich, E.Köhler, and Fr.Stadler: *Foundational Debate,* Kluwer AP, 175-186

[Tichý 1996] Tichý, Pavel: The Myth of non-rigid Designators. *From the Logical Point of View 2/94,* 20-30

[Tichý 1996a] Tichý, Pavel: The Analysis of Natural Language. *From the Logical Point of View 2/94*, 42-80

[Tichý 2004] Tichý, Pavel: *Collected Papers in Logic and Philosophy.* Filosofia Prague, University of Otago, Dunedin, 2004

[Whitehead, Russell 1964] Whitehead, A.N., Russell, Bertrand: *Principia Mathematica*. 4th. Ed., Cambridge at the University Press

[Yourgrau 1990] Yourgrau, Palle, ed.: *Demonstratives.* Oxford University Press

[Ziehen 1920] Ziehen, Theodor: *Lehrbuch der Logik*. Bonn 1920. (A.Marcus, E.Webers)

[Zlatuška 1986] Zlatuška, Jiří: Data Bases and Lambda Calculus. In: *Proc.IFIP'86 World Computer Congress,* Dublin 97-104

Index

F

G

H

I

J

K

L

M

N

O

P

Q

R

S

T

V

W

Y

Z

Bereits erschienene und geplante Bände der Reihe

Logische Philosophie

Hrsg.: H. Wessel, U. Scheffler, Y. Shramko, M. Urchs

ISSN: 1435–3415

In der Reihe „Logische Philosophie“ werden philosophisch relevante Ergebnisse der Logik vorgestellt. Dazu gehören insbesondere Arbeiten, in denen philosophische Probleme mit logischen Methoden gelöst werden.

Uwe Scheffler/Klaus Wuttich (Hrsg.)

Terminigebrauch und Folgebeziehung

ISBN: 978-3-89722-050-8 Preis: 30,- €

Regeln für den Gebrauch von Termini und Regeln für das logische Schließen sind traditionell der Gegenstand der Logik. Ein zentrales Thema der vorliegenden Arbeiten ist die umstrittene Forderung nach speziellen Logiken für bestimmte Aufgabengebiete - etwa für Folgern aus widersprüchlichen Satzmengen, für Ersetzen in gewissen Wahrnehmungs- oder Behauptungssätzen, für die Analyse von epistemischen, kausalen oder mehrdeutigen Termini. Es zeigt sich in mehreren Arbeiten, daß die nichttraditionelle Prädikationstheorie eine verläßliche und fruchtbare Basis für die Bearbeitung solcher Probleme bietet. Den Beiträgen zu diesem Problemkreis folgen vier diese Thematik erweiternde Beiträge. Der dritte Abschnitt beschäftigt sich mit der Theorie der logischen Folgebeziehungen. Die meisten der diesem Themenkreis zugehörenden Arbeiten sind explizit den Systemen F^S bzw. S^S gewidmet.

Horst Wessel

Logik

ISBN: 978-3-89722-057-7 Preis: 37,- €

Das Buch ist eine philosophisch orientierte Einführung in die Logik. Ihm liegt eine Konzeption zugrunde, die sich von mathematischen Einführungen in die Logik unterscheidet, logische Regeln als universelle Sprachregeln versteht und sich bemüht, die Logik den Bedürfnissen der empirischen Wissenschaften besser anzupassen.

Ausführlich wird die klassische Aussagen- und Quantorenlogik behandelt. Philosophische Probleme der Logik, die Problematik der logischen Folgebeziehung, eine nichttraditionelle Prädikationstheorie, die intuitionistische Logik, die Konditionallogik, Grundlagen der Terminitheorie, die Behandlung modaler Prädikate und ausgewählte Probleme der Wissenschaftslogik gehen über die üblichen Einführungen in die Logik hinaus.

Das Buch setzt keine mathematischen Vorkenntnisse voraus, kann als Grundlage für einen einjährigen Logikkurs, aber auch zum Selbststudium genutzt werden.

Yaroslav Shramko

Intuitionismus und Relevanz

ISBN: 978-3-89722-205-2 Preis: 25,- €

Die intuitionistische Logik und die Relevanzlogik gehören zu den bedeutendsten Rivalen der klassischen Logik. Der Verfasser unternimmt den Versuch, die jeweiligen Grundideen der Konstruktivität und der Paradoxienfreiheit durch eine „Relevantisierung der intuitionistischen Logik“ zusammenzuführen. Die auf diesem Weg erreichten Ergebnisse sind auf hohem technischen Niveau und werden über die gesamte Abhandlung hinweg sachkundig philosophisch diskutiert. Das Buch wendet sich an einen logisch gebildeten philosophisch interessierten Leserkreis.

Horst Wessel

Logik und Philosophie

ISBN: 978-3-89722-249-6 Preis: 15,30 €

Nach einer Skizze der Logik wird ihr Nutzen für andere philosophische Disziplinen herausgearbeitet. Mit minimalen logisch-technischen Mitteln werden philosophische Termini, Theoreme und Konzeptionen analysiert. Insbesondere bei der Untersuchung von philosophischer Terminologie zeigt sich, daß logische Standards für jede wissenschaftliche Philosophie unabdingbar sind. Das Buch wendet sich an einen breiten philosophisch interessierten Leserkreis und setzt keine logischen Kenntnisse voraus.

S. Wölfl

Kombinierte Zeit- und Modallogik. Vollständigkeitsresultate für prädikatenlogische Sprachen

ISBN: 978-3-89722-310-3 Preis: 40,- €

Zeitlogiken thematisieren „nicht-ewige“ Sätze, d. h. Sätze, deren Wahrheitswert sich in der Zeit verändern kann. Modallogiken (im engeren Sinne des Wortes) zielen auf eine Logik alethischer Modalbegriffe ab. Kombinierte Zeit- und Modallogiken verknüpfen nun Zeit- mit Modallogik, in ihnen geht es also um eine Analyse und logische Theorie zeitabhängiger Modalaussagen.

Kombinierte Zeit- und Modallogiken stellen eine ausgezeichnete Basistheorie für Konditionallogiken, Handlungs- und Bewirkenstheorien sowie Kausalanalysen dar. Hinsichtlich dieser Anwendungsgebiete sind vor allem prädikatenlogische Sprachen aufgrund ihrer Ausdrucksstärke von Interesse. Die vorliegende Arbeit entwickelt nun kombinierte Zeit- und Modallogiken für prädikatenlogische Sprachen und erörtert die solchen logischen Systemen eigentümlichen Problemstellungen. Dazu werden im ersten Teil ganz allgemein multimodale Logiken für prädikatenlogische Sprachen diskutiert, im zweiten dann Kalküle der kombinierten Zeit- und Modallogik vorgestellt und deren semantische Vollständigkeit bewiesen.

Das Buch richtet sich an Leser, die mit den Methoden der Modal- und Zeitlogik bereits etwas vertraut sind.

H. Franzen, U. Scheffler

Logik. Kommentierte Aufgaben und Lösungen

ISBN: 978-3-89722-400-1 Preis: 15,- €

Üblicherweise wird in der Logik-Ausbildung viel Zeit auf die Vermittlung metatheoretischer Zusammenhänge verwendet. Das Lösen von Übungsaufgaben — unerläßlich für das Verständnis der Theorie — ist zumeist Teil der erwarteten selbständigen Arbeit der Studierenden. Insbesondere Logik-Lehrbücher für Philosophen bieten jedoch häufig wenige oder keine Aufgaben. Wenn Aufgaben vorhanden sind, fehlen oft die Lösungen oder sind schwer nachzuvollziehen.

Das vorliegende Trainingsbuch enthält Aufgaben mit Lösungen, die aus Klausur- und Tutoriumsaufgaben in einem 2-semestrigen Grundkurs Logik für Philosophen entstanden sind. Ausführliche Kommentare machen die Lösungswege leicht verständlich. So übt der Leser, Entscheidungsverfahren anzuwenden, Theoreme zu beweisen u. ä., und erwirbt damit elementare logische Fertigkeiten. Erwartungsgemäß beziehen sich die meisten Aufgaben auf die Aussagen- und Quantorenlogik, aber auch andere logische Gebiete werden in kurzen Abschnitten behandelt.

Diese Aufgabensammlung ist kein weiteres Lehrbuch, sondern soll die vielen vorhandenen Logik-Lehrbücher ergänzen.

U. Scheffler

Ereignis und Zeit. Ontologische Grundlagen der Kausalrelationen

ISBN: 978-3-89722-657-9 Preis: 40,50 €

Das Hauptergebnis der vorliegenden Abhandlung ist eine philosophische Ereignistheorie, die Ereignisse über konstituierende Sätze einführt. In ihrem Rahmen sind die wesentlichen in der Literatur diskutierten Fragen (nach der Existenz und der Individuation von Ereignissen, nach dem Verhältnis von Token und Typen, nach der Struktur von Ereignissen und andere) lösbar. In weiteren Kapiteln werden das Verhältnis von kausaler und temporaler Ordnung sowie die Existenz von Ereignissen in der Zeit besprochen und es wird auf der Grundlage der Token-Typ-Unterscheidung für die Priorität der singulären Kausalität gegenüber genereller Verursachung argumentiert.

Horst Wessel

Antiirrationalismus

Logisch-philosophische Aufsätze

ISBN: 978-3-8325-0266-9 Preis: 45,- €

Horst Wessel ist seit 1976 Professor für Logik am Institut für Philosophie der Humboldt-Universität zu Berlin. Nach seiner Promotion in Moskau 1967 arbeitete er eng mit seinem Doktorvater, dem russischen Logiker A. A. Sinowjew, zusammen. Wessel hat großen Anteil daran, daß am Berliner Institut für Philosophie in der Logik auf beachtlichem Niveau gelehrt und geforscht wurde.

Im vorliegenden Band hat er Artikel aus einer 30-jährigen Publikationstätigkeit ausgewählt, die zum Teil nur noch schwer zugänglich sind. Es handelt sich dabei um logische, philosophische und logisch-philosophische Arbeiten. Von Kants Antinomien der reinen Vernunft bis zur logischen Terminitheorie, von Modalitäten bis zur logischen Folgebeziehung, von Entwicklungstermini bis zu intensionalen Kontexten reicht das Themenspektrum.

Antiirrationalismus ist der einzige -ismus, dem Wessel zustimmen kann.

Horst Wessel, Klaus Wuttich

daß-Termini

Intensionaliät und Ersetzbarkeit

ISBN: 978-3-89722-754-5 Preis: 34,- €

Von vielen Autoren werden solche Kontexte als intensional angesehen, in denen die üblichen Ersetzbarkeitsregeln der Logik nicht gelten. Eine besondere Rolle spielen dabei *daß*-Konstruktionen.

Im vorliegenden Buch wird gezeigt, daß diese Auffassungen fehlerhaft sind. Nach einer kritischen Sichtung der Arbeiten anderer Logiker zu der Problematik von *daß*-Termini wird ein logischer Apparat bereitgestellt, der es ermöglicht, *daß*-Konstruktionen ohne Einschränkungen von Ersetzbarkeitsregeln und ohne Zuflucht zu Intensionalitäten logisch korrekt zu behandeln.

Fabian Neuhaus

Naive Prädikatenlogik

Eine logische Theorie der Prädikation

ISBN: 978-3-8325-0556-1 Preis: 41,- €

Die logischen Regeln, die unseren naiven Redeweisen über Eigenschaften zugrunde liegen, scheinen evident und sind für sich alleine betrachtet völlig harmlos - zusammen sind sie jedoch widersprüchlich. Das entstehende Paradox, das Russell-Paradox, löste die sogenannte Grundlagenkrise der Mathematik zu Beginn des 20. Jahrhunderts aus. Der klassische Weg, mit dem Russell-Paradox umzugehen, ist eine Vermeidungsstrategie: Die logische Analysesprache wird so beschränkt, daß das Russell-Paradox nicht formulierbar ist.

In der vorliegenden Arbeit wird ein anderer Weg aufgezeigt, wie man das Russell-Paradox und das verwandte Grelling-Paradox lösen kann. Dazu werden die relevanten linguistischen Daten anhand von Beispielen analysiert und ein angemessenes formales System aufgebaut, die Naive Prädikatenlogik.

Bente Christiansen, Uwe Scheffler (Hrsg.)

Was folgt

Themen zu Wessel

ISBN: 978-3-8325-0500-4 Preis: 42,- €

Die vorliegenden Arbeiten sind Beiträge zu aktuellen philosophischen Diskussionen – zu Themen wie Existenz und Referenz, Paradoxien, Prädikation und dem Funktionieren von Sprache überhaupt. Gemeinsam ist ihnen der Bezug auf das philosophische Denken Horst Wessels, ein Vierteljahrhundert Logikprofessor an der Humboldt-Universität zu Berlin, und der Anspruch, mit formalen Mitteln nachvollziehbare Ergebnisse zu erzielen.

Vincent Hendricks, Fabian Neuhaus, Stig Andur Pedersen, Uwe Scheffler, Heinrich Wansing (Eds.)

First-Order Logic Revisited

ISBN: 978-3-8325-0475-5 Preis: 75,- €

Die vorliegenden Beiträge sind für die Tagung „75 Jahre Prädikatenlogik erster Stufe“ im Herbst 2003 in Berlin geschrieben worden. Mit der Tagung wurde der 75. Jahrestag des Erscheinens von Hilberts und Ackermanns wegweisendem Werk „Grundzüge der theoretischen Logik“ begangen.

Im Ergebnis entstand ein Band, der eine Reflexion über die klassische Logik, eine Diskussion ihrer Grundlagen und Geschichte, ihrer vielfältigen Anwendungen, Erweiterungen und Alternativen enthält.

Der Band enthalt Beiträge von Andréka, Avron, Ben-Yami, Brünnler, Englebretsen, Ewald, Guglielmi, Hajek, Hintikka, Hodges, Kracht, Lanzet, Madarasz, Nemeti, Odintsov, Robinson, Rossberg, Thielscher, Toke, Wansing, Willard, Wolenski

Pavel Materna

Conceptual Systems

ISBN: 978-3-8325-0636-0 Preis: 34,- €

We all frequently use the word “concept”. Yet do we know what we mean using this word in sundry contexts? Can we say, for example, that there can be several concepts of an object? Or: can we state that some concepts develop? What relation connects concepts with expressions of a natural language? What is the meaning of an expression? Is Quine’s ‘stimulus meaning’ the only possibility of defining meaning? The author of the present publication (and of “Concepts and Objects”, 1998) offers some answers to these (and many other) questions from the viewpoint of transparent intensional logic founded by the late Czech logician Pavel Tichý (†1994 Dunedin).

Johannes Emrich

Die Logik des Unendlichen

Rechtfertigungsversuche des *tertium non datur* in der Theorie des mathematischen Kontinuums

ISBN: 978-3-8325-0747-3 Preis: 39,- €

Im Grundlagenstreit der Mathematik geht es um die Frage, ob gewisse in der modernen Mathematik gängige Beweismethoden zulässig sind oder nicht. Der Verlauf der Debatte – von den 1920er Jahren bis heute – zeigt, dass die Argumente auf verschiedenen Ebenen gelagert sind: die der meist konstruktivistisch eingestellten Kritiker sind erkenntnistheoretischer oder logischer Natur, die der Verteidiger ontologisch oder pragmatisch. Die Einschätzung liegt nahe, der Streit sei gar nicht beizulegen, es handele sich um grundlegend unterschiedliche Auffassungen von Mathematik. Angesichts der immer wieder auftretenden Erfahrung ihrer Unverträglichkeit wäre es aber praktisch wie philosophisch unbefriedigend, schlicht zur Toleranz aufzurufen. Streiten heißt nach Einigung streben. In der Philosophie manifestiert sich dieses Streben in der Überzeugung einer objektiven Einheit oder Einheitlichkeit, insbesondere geistiger Sphären. Im Sinne dieser Überzeugung unternimmt die vorliegende Arbeit einen Vermittlungsversuch, der sich auf den logischen Kern der Debatte konzentriert.

Christopher von Bülow

Beweisbarkeitslogik

– Gödel, Rosser, Solovay –

ISBN: 978-3-8325-1295-8 Preis: 29,- €

Kurt Gödel erschütterte 1931 die mathematische Welt mit seinem Unvollständigkeitssatz. Gödel zeigte, wie für jedes noch so starke formale System der Arithmetik ein Satz konstruiert werden kann, der besagt: „Ich bin nicht beweisbar." Würde das System diesen Satz beweisen, so würde es sich damit selbst Lügen strafen. Also ist dies ein wahrer Satz, den es nicht beweisen kann: Es ist unvollständig. John Barkley Rosser verstärkte später Gödels Ergebnisse, wobei er die Reihenfolge miteinbezog, in der Sätze bewiesen werden, gegeben irgendeine Auffassung von „Beweis". In der Beweisbarkeitslogik werden die formalen Eigenschaften der Begriffe „beweisbar" und „wird früher bewiesen als" mit modallogischen Mitteln untersucht: Man liest den notwendig - Operator als beweisbar und gibt formale Systeme an, die die Modallogik der Beweisbarkeit erfassen.

Diese Arbeit richtet sich sowohl an Logik-Experten wie an durchschnittlich vorgebildete Leser. Ihr Ziel ist es, in die Beweisbarkeitslogik einzuführen und deren wesentliche Resultate, insbesondere die Solovayschen Vollständigkeitssätze, präzise, aber leicht zugänglich zu präsentieren.

Niko Strobach

Alternativen in der Raumzeit

Eine Studie zur philosophischen Anwendung multidimensionaler Aussagenlogiken

ISBN: 978-3-8325-1400-6 Preis: 46.50 €

Ist der Indeterminismus mit der Relativitätstheorie und ihrer Konzeption der Gegenwart vereinbar? Diese Frage lässt sich beantworten, indem man die für das alte Problem der futura contingentia entwickelten Ansätze auf Aussagen über das Raumartige überträgt. Die dazu hier Schritt für Schritt aufgebaute relativistische indeterministische Raumzeitlogik ist eine erste philosophische Anwendung der multidimensionalen Modallogiken.

Neben den üblichen Zeitoperatoren kommen dabei die Operatoren „überall" und „irgendwo" sowie „für jedes Bezugssystem" und „für manches Bezugssystem" zum Einsatz. Der aus der kombinierten Zeit- und Modallogik bekannte Operator für die historische Notwendigkeit wird in drei verschiedene Operatoren („wissbar", „feststehend", „beeinflussbar") ausdifferenziert. Sie unterscheiden sich bezüglich des Gebiets, in dem mögliche Raumzeiten inhaltlich koinzidieren müssen, um als Alternativen zueinander gelten zu können. Die Interaktion zwischen den verschiedenen Operatoren wird umfassend untersucht.

Die Ergebnisse erlauben es erstmals, die Standpunkt-gebundene Notwendigkeit konsequent auf Raumzeitpunkte zu relativieren. Dies lässt auf einen metaphysisch bedeutsamen Unterschied zwischen deiktischer und narrativer Determiniertheit aufmerksam werden. Dieses Buch ergänzt das viel diskutierte Paradigma der verzweigten Raumzeit („branching spacetime") um eine neue These: Der Raum ist eine Erzählform der Entscheidungen der Natur.

Erich Herrmann Rast

Reference and Indexicality

ISBN: 978-3-8325-1724-3 Preis: 43.00 €

Reference and indexicality are two central topics in the Philosophy of Language that are closely tied together. In the first part of this book, a description theory of reference is developed and contrasted with the prevailing direct reference view with the goal of laying out their advantages and disadvantages. The author defends his version of indirect reference against well-known objections raised by Kripke in Naming and Necessity and his successors, and also addresses linguistic aspects like compositionality. In the second part, a detailed survey on indexical expressions is given based on a variety of typological data. Topics addressed are, among others: Kaplan's logic of demonstratives, conversational versus utterance context, context-shifting indexicals, the deictic center, token-reflexivity, vagueness of spatial and temporal indexicals, reference rules, and the epistemic and cognitive role of indexicals. From a descriptivist perspective on reference, various examples of simple and complex indexicals are analyzed in first-order predicate logic with reified contexts. A critical discussion of essential indexicality, de se readings of attitudes and accompanying puzzles rounds up the investigation.

Magdalena Roguska

Exklamation und Negation

ISBN: 978-3-8325-1917-9 Preis: 39.00 €

Im Deutschen, aber auch in vielen anderen Sprachen gibt es umstrittene Negationsausdrücke, die keine negierende Kraft haben, wenn sie in bestimmten Satztypen vorkommen. Für das Deutsche handelt sich u.a. um die exklamativ interpretierten Sätze vom Typ:

Was macht sie nicht alles! Was der nicht schafft!

Die Arbeit fokussiert sich auf solchen Exklamationen. Ihre wichtigsten Thesen lauten:

- Es gibt keine Exklamativsätze aber es gibt Exklamationen.
- *Alles* und *nicht alles* in solchen Sätzen, haben semantische und nicht pragmatische Funktionen.
- Das „nicht-negierende“ *nicht* ohne *alles* in einer Exklamation ist doch eine Negation. Die Exklamation bezieht sich aber trotzdem auf denselben Wert, wie die entsprechende Exklamation ohne Negation.
- In skalaren Exklamationen besteht der Unterschied zwischen Standard- und „nicht-negierenden“ Negation im Skopus von *nicht.*

Die Analyse erfolgt auf der Schnittstelle zwischen Semantik und Pragmatik.

August W. Sladek

Aus Sand bauen. Tropentheorie auf schmaler relationaler Basis

Ontologische, epistemologische, darstellungstechnische Möglichkeiten und Grenzen der Tropenanalyse

ISBN: 978-3-8325-2506-4 (4 Bände) Preis: 198.00 €

Warum braucht eine Tropentheorie zweieinhalbtausend Seiten Text, wenn zweieinhalb Seiten ausreichen, um ihre Grundidee vorzustellen? Weil der Verfasser zuerst sich und dann seine Leser, auf deren Geduld er baut, überzeugen will, dass die ontologische Grundidee von Tropen als den Bausteinen der Welt wirklich trägt und sich mit ihnen die Gegenstände nachbilden lassen, die der eine oder andere glaubt haben zu müssen. Um metaphysischen, epistemologischen Dilemmata zu entgehen, sie wenigstens einigermaßen zu meistern, preisen viele Philosophen Tropen als „Patentbausteine“ an. Die vorliegende Arbeit will Tropen weniger empfehlen als zeigen, wie sie sich anwenden lassen. Dies ist weit mühseliger als sich mit Andeutungen zu begnügen, wie brauchbar sich doch Tropen erweisen werden, machte man sich die Mühe sie einzusetzen. Lohnt sich die Mühe wirklich? Der Verfasser wollte zunächst nachweisen, dass sie sich nicht lohnt. Das Gegenteil ist ihm gelungen. Zwar sind Tropen wie Sandkörner. Was lässt sich schon aus Sand bauen, das Bestand hat? Wenn man nur genug „Zement“ nimmt, gelingen gewiss stabile Bauten, doch wie viel und welcher „Zement“ ist erlaubt? Nur schwache Bindemittel dürfen es sein; sonst gibt man sich mit einer hybriden Tropenontologie zufrieden, die Bausteine aus fremden, konkurrierenden Ontologien hinzunimmt. Die vier Bände bieten eine schwächstmögliche und damit unvermischte, allerdings mit Varianten und Alternativen behaftete Tropentheorie an samt ihren Wegen, Nebenwegen, Anwendungstests.

Mireille Staschok

Existenz und die Folgen

Logische Konzeptionen von Quantifikation und Prädikation

ISBN: 978-3-8325-2191-2 Preis: 39.00 €

Existenz hat einen eigenwilligen Sonderstatus in der Philosophie und der modernen Logik. Dieser Sonderstatus erscheint in der klassischen Prädikatenlogik – übereinstimmend mit Kants Diktum, dass Existenz kein Prädikat sei – darin, dass „Existenz“ nicht als Prädikat erster Stufe, sondern als Quantor behandelt wird. In der natürlichen Sprache wird „existieren“ dagegen prädikativ verwendet.

Diese andauernde und philosophisch fruchtbare Diskrepanz von Existenz bietet einen guten Zugang, um die Funktionsweisen von Prädikation und Quantifikation zu beleuchten. Ausgangspunkt der Untersuchungen und Bezugssystem aller Vergleiche ist die klassische Prädikatenlogik erster Stufe. Als Alternativen zur klassischen Prädikatenlogik werden logische Systeme, die sich an den Ansichten Meinongs orientieren, logische Systeme, die in der Tradition der aristotelischen Termlogik stehen und eine nichttraditionelle Prädikationstheorie untersucht.

Sebastian Bab, Klaus Robering (Eds.)

Judgements and Propositions

Logical, Linguistic, and Cognitive Issues

ISBN: 978-3-8325-2370-1 Preis: 39.00 €

Frege and Russell in their logico-semantic theories distinguished between a proposition, the judgement that it is true, and the assertion of this judgement. Their distinction, however, fell into oblivion in the course of later developments and was replaced by the formalistic notion of an expression derivable by means of pureley syntactical rules of inference. Recently, however, Frege and Russell's original distinction has received renewed interest due to the work of logicians and philosophers such as, for example, Michael Dummett, Per Martin-Lf, and Dag Prawitz, who have pointed to the central importance of o both the act of assertion and its justification to logic itself as well as to an adequate theory of meaning and understanding.

The contributions to the present volume deal with central issues raised by these authors and their classical predecessors: What kind of propositions are there and how do they relate to truth? How are propositions grasped by human subjects? And how do these subjects judge those propositions according to various dimensions (such as that of truth and falsehood)? How are those judgements encoded into natural language, communicated to other subjects, and decoded by them? What does it mean to procede by inference from premiss assertions to a new judgement?

Marius Thomann

Die Logik des Könnens

ISBN: 978-3-8325-2672-6 Preis: 41.50 €

Was bedeutet es, einer Person eine praktische Fähigkeit zu attestieren? Und unter welchen Umständen sind derartige Fähigkeitszuschreibungen wahr, etwa die Behauptung, Max könne Gitarre spielen? Diese Fragen stehen im Zentrum der vorliegenden Untersuchung. Ihr Gegenstand ist die philosophisch-logische Analyse des Fähigkeitsbegriffs. Als Leitfaden dient eine Analyse normalsprachlicher Fähigkeitszuschreibungen, gemäß der Max genau dann Gitarre spielen kann, wenn er dies unter dafür angemessenen Bedingungen normalerweise erfolgreich tut. Drei in der Forschungsliteratur vorgeschlagene Systeme werden diskutiert, die zwar wertvolle Impulse für die formale Modellierung geben, als Vertreter des so genannten modalen Ansatzes aber von der Diagnose ontologischer Inadäquatheit betroffen sind: Die Entitäten, die als Fähigkeiten attribuiert werden, lassen sich nicht über Propositionen individuieren; ohne die explizite Referenz auf Handlungstypen, die eben gekonnt oder nicht gekonnt werden, bleibt Max' Fähigkeit, Gitarre zu spielen, unterbestimmt. Um diesen Einwand zu vermeiden, liegt demgemäß der hier vorgestellten Logik des Könnens ein Gegenstandsbereich zugrunde, dessen Struktur an der Ontologie von Handlungen orientiert ist.

Christof Dobieß

Kausale Relata

Eine Untersuchung zur Wechselbeziehung zwischen der Beschaffenheit kausaler Relata und der Natur der Kausalbeziehung

ISBN: 978-3-8325-5083-7 Preis: 57.00 €

Dieses Buch macht nachdrücklich klar, daß die Thematik „Kausale Relata" kein Nebenschauplatz der Kausalitätsdiskussion ist und sich die Analyse von Kausalität nicht auf die bloße Betrachtung der Kausalrelation selbst beschränken darf. Zwischen der Metaphysik der kausalen Relata und der Natur der Kausalbeziehung, so die Hauptthese dieses Werks, besteht eine enge theoretische Wechselbeziehung.

Untersucht wird diese These anhand zentraler kausaler Problembereiche: (1) der kausalen Präemption, (2) der Transitivität der Kausalität, (3) der dispositionalen Verursachung, (4) der negativen Verursachung und (5) der Konzeption von Verursachung als „qualitativem Fortbestand" („qualitative persistence").

Während die Probleme der Präemption und des qualitativen Fortbestands in der Auseinandersetzung zwischen kontrafaktischen Kausalkonzeptionen und Transfertheorien Bedeutung entfalten, betreffen die Transitivität der Kausalität sowie negative und dispositionale Verursachung nahezu alle Kausaltheorien. Der Forderung nach der Transitivität der Kausalität kann nur durch eine hinreichend präzise und eindeutig gefaßte Konzeption der kausalen Beziehungsträger entsprochen werden. Ob Dispositionen oder Negativereignisse in kausale Beziehungen treten können, hängt entscheidend davon ab, inwiefern Entitäten dieser Art ein ontologisches Bleiberecht zugestanden wird.

Logische Philosophie

Eds.: H. Wessel, U. Scheffler, Y. Shramko and M. Urchs

The series "Logische Philosophie" introduces philosophically relevant results of logical research. In particular, treatises are issued in which logical means are employed to solve philosophical problems.

We all frequently use the word "concept". Yet do we know what we mean using this word in sundry contexts? Can we say, for example, that there can be several concepts of an object? Or: can we state that some concepts develop? What relation connects concepts with expressions of a natural language? What is the meaning of an expression? Is Quine's 'stimulus meaning' the only possibility of defining meaning? The author of the present publication (and of "Concepts and Objects", 1998) offers some answers to these (and many other) questions from the viewpoint of transparent intensional logic founded by the late Czech logician Pavel Tichý (†1994 Dunedin).

Logos Verlag Berlin

ISBN 3–8325–0636–5